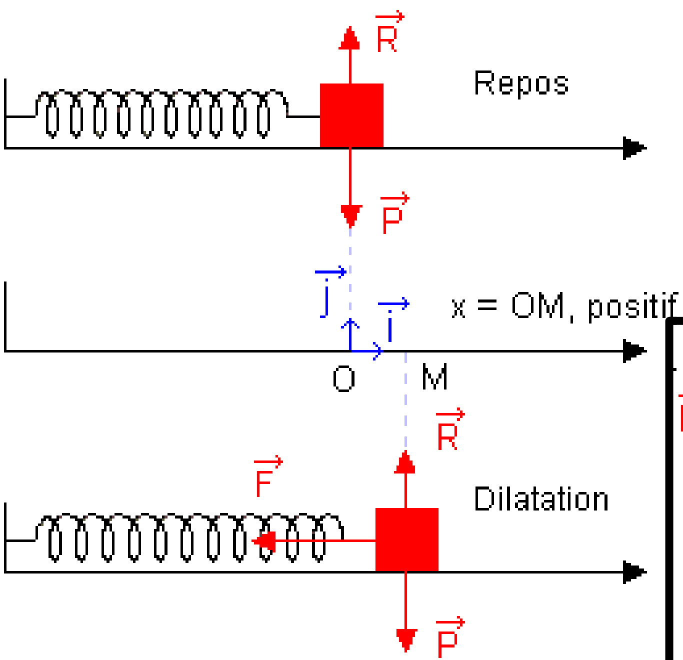
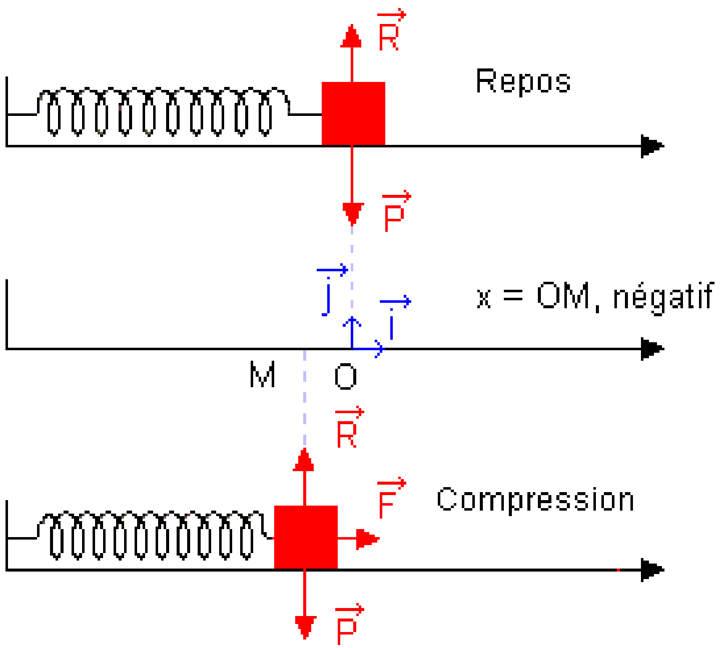
OSCILLATEUR HARMONIQUE



$\vec{F} = k\,\overrightarrow{MO} = -\,k\,\overrightarrow{OM} = -\,k\,x\,\vec{i}$

$\vec{P} + \vec{R} = \vec{0}$ (2)



$\vec{F} = k\,\overrightarrow{MO} = -\,k\,\overrightarrow{OM} = -\,k\,x\,\vec{i}$

Forces :

$\vec{P} + \vec{R} + \vec{F} = m.\vec{a}$ (3)

$\vec{0} + \vec{F} = m.\vec{a}$

$-\,k\,x\,\vec{i} = m\,\ddot{x}\,\vec{i}$

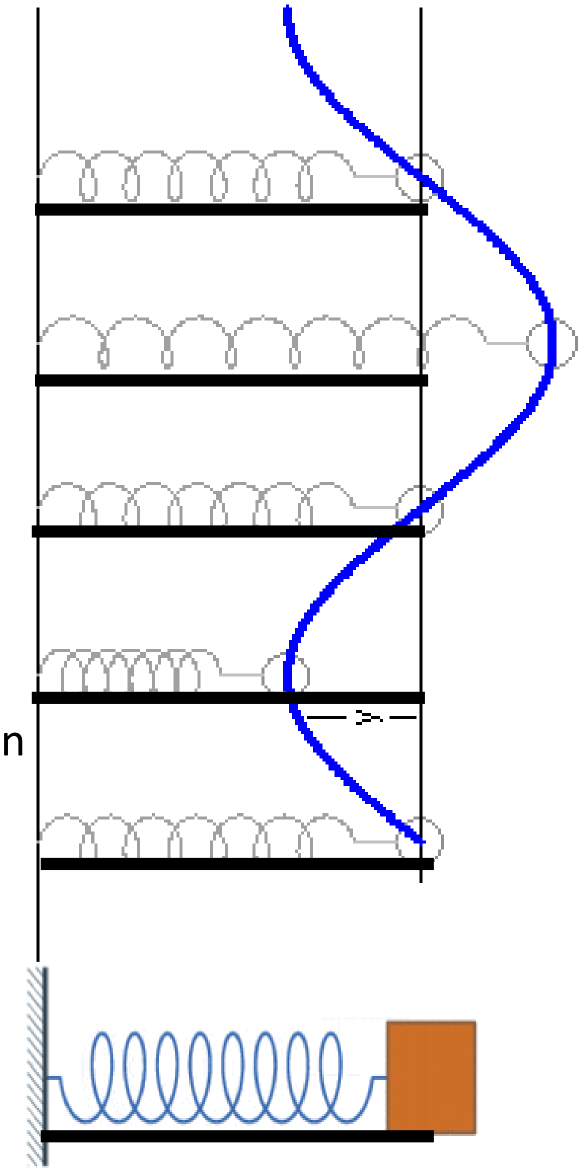
$(m\,\ddot{x} + k\,x)\,\vec{i} = \vec{0}$

$m\,\ddot{x} + k\,x = 0$

Energie potentielle

$V = 1/2kx^2$

OSCILLATEUR HARMONIQUE

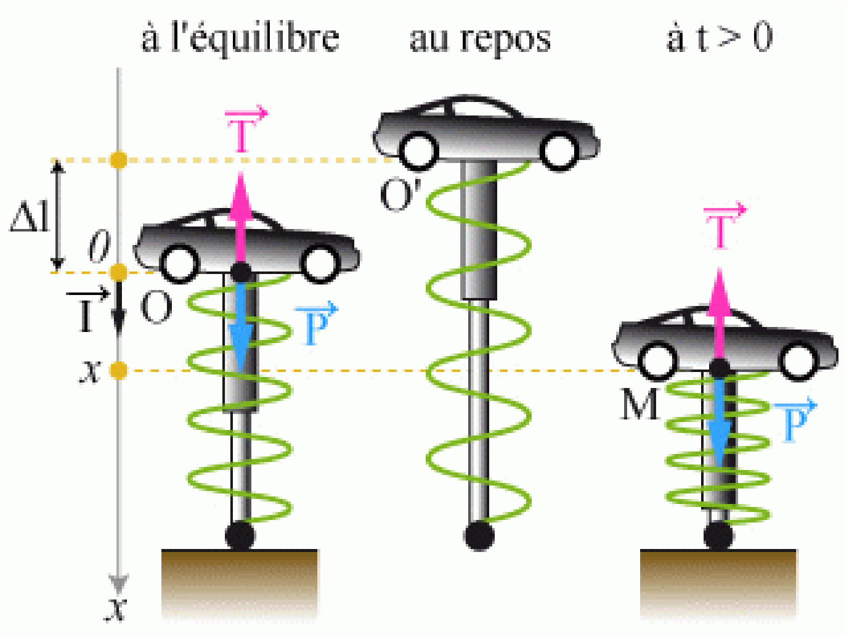


Oscillations sinusoidales de pulsation

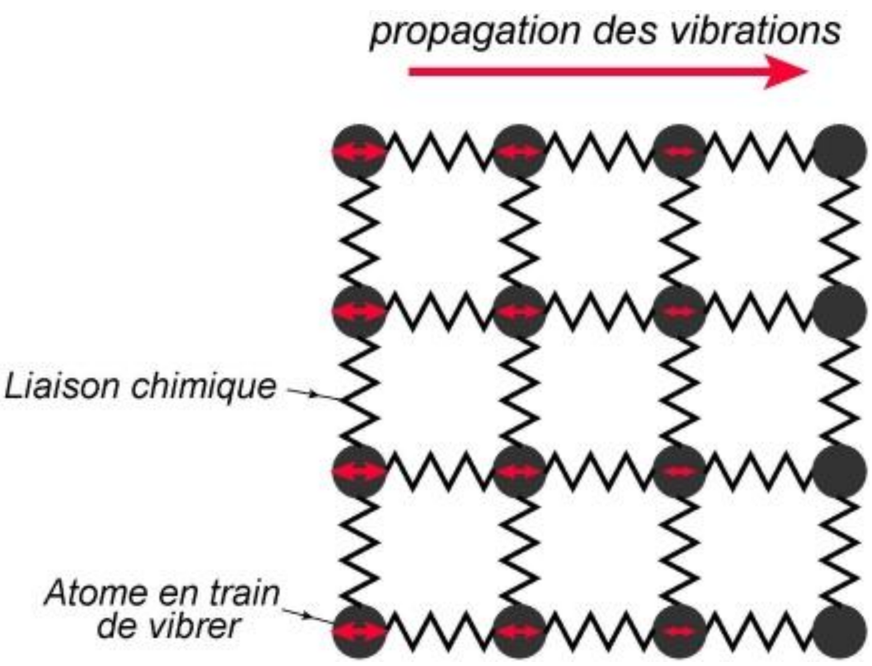
$$\omega = \sqrt{\frac{k}{m}}$$

Exemples :

En mécanique classique : vibration d'une auto

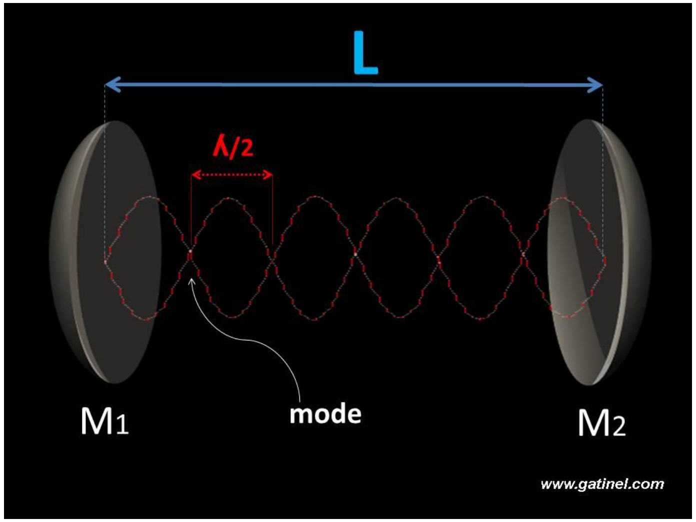


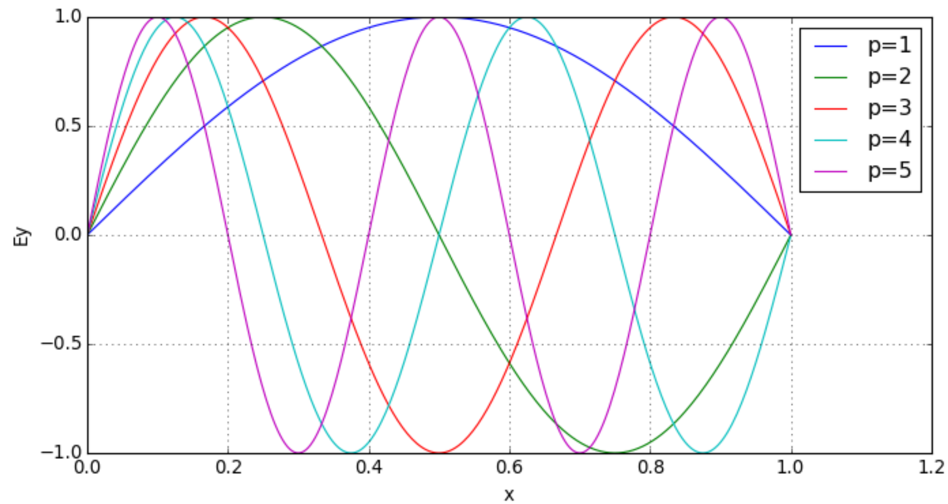
En physique du solide : vibration des atomes



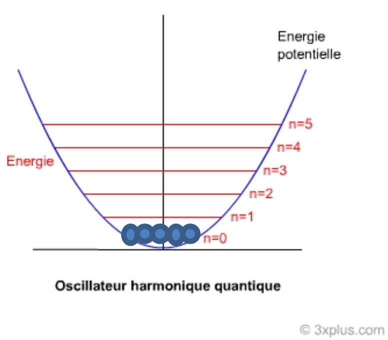
Exemples suite :

En électromagnétisme: modes dans cavité





Description quantique de l'énergie de particules identiques : les bosons



Niveaux d'énergie équidistants de $\hbar\omega$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

1. a) $[\hbar] = ML^2T^{-1}$ (en utilisant $E = \hbar\omega$ par exemple)

$[\omega] = T^{-1}$ $\quad$ $[E] = MT^{-2}L^2$

$$[k] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

b) $[\omega] = T^{-1} = m^{\alpha}k^{\beta} = M^{\alpha}M^{\beta}T^{-2\beta}$ d'où $\alpha + \beta = 0$ et $-2\beta = -1$

soit $\alpha = -1/2$ et $\beta = 1/2$

Donc $\omega = \sqrt{\dfrac{k}{m}}$

$$[E] = ML^2T^{-2} = m^{\alpha}k^{\beta}\hbar^{\gamma}$$

$$= M^{\alpha}M^{\beta}T^{-2\beta}M^{\gamma}L^{2\gamma}T^{-\gamma}$$

→ d'où $\alpha+\beta+\gamma = 1$

$-2\beta-\gamma = ½$

$2\gamma = 2$

$$\alpha+\beta+\gamma = 1$$
$$-2\beta-\gamma = \tfrac{1}{2}$$
$$2\gamma = 2$$

soit $\alpha = -1/2$, $\beta = \tfrac{1}{2}$ et $\gamma = 1$

Donc $\boxed{E = \hbar\sqrt{\frac{k}{m}} = \hbar\omega}$

c) $\boxed{\omega = \sqrt{\frac{k}{m}}}$ $\omega = 10^{15} s^{-1}$

$$\Rightarrow E = \hbar\omega = 1{,}05.10^{-34}.10^{15} = 1{,}05.10^{-19} J = 0{,}56 eV$$

$$\vec{F} = -\overrightarrow{grad}V(x) \Rightarrow -kx = -\frac{dV}{dx}$$

$$\Rightarrow V(x) = \frac{1}{2}kx^2 + cste$$

Si V(0) = 0 alors $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$

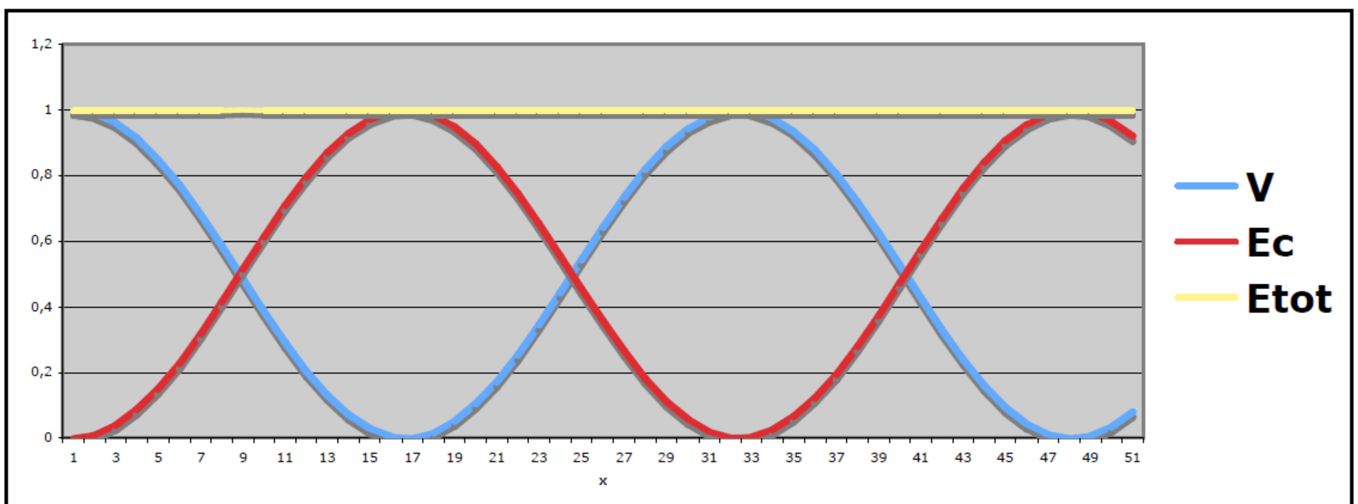
3. a) Le PFD donne $m\frac{d^2x}{dt^2} = -kx$

$$\boxed{x(t) = A\cos(\omega t + \varphi) \text{ avec } \omega^2 = \frac{k}{m}}$$

b) $V = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \varphi)$

$$E_c(t) = \frac{1}{2} m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2} A^2 m\omega^2 \sin^2(\omega t + \varphi)$$

d'où $E = V + E_p(t) = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} kA^2$



4. Traitement quantique

a) L'équation de Schrödinger à 1D donne : $-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2}+V(x)\phi(x)=E\phi(x)$

$$H\phi(x)=E\phi(x)$$

$$H=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)$$

b) Soit $\phi_0(x)=C_0e^{-ax^2}$

$$\frac{d\phi_0(x)}{dx}=-2axC_0e^{-ax^2} \qquad \frac{d^2\phi_0(x)}{dx^2}=-2aC_0e^{-ax^2}+4a^2x^2C_0e^{-ax^2}$$

$$-\frac{\hbar^2}{2m}\left[-2a\phi_0(x)+4a^2x^2\phi_0(x)\right]+\frac{1}{2}kx^2\phi_0(x)=E_0\phi_0(x)$$

$$-\frac{\hbar^2}{2m}\left(-2a\phi_0(x)\right)-\frac{\hbar^2}{2m}\left(4a^2x^2\phi_0(x)\right)+\frac{1}{2}kx^2\phi_0(x)=E_0\phi_0(x)$$

$$-\frac{\hbar^2}{2m}\left(-2a\phi_0(x)\right)-\frac{\hbar^2}{2m}\left(4a^2x^2\phi_0(x)\right)+\frac{1}{2}kx^2\phi_0(x)=E_0\phi_0(x)$$

$$-\frac{2\hbar^2a^2}{m}+\frac{1}{2}k=0$$

$$\Rightarrow a=\sqrt{\frac{km}{4\hbar^2}}=\frac{m\omega}{2\hbar}$$

$$-\frac{\hbar^2}{2m}\left[-2a\phi_0(x)+4a^2x^2\phi_0(x)\right]+\frac{1}{2}kx^2\phi_0(x)=E_0\phi_0(x)$$

$$E_0=\frac{\hbar^2a}{m}$$

$$E_0=\frac{\hbar^2}{m}\sqrt{\frac{km}{4\hbar^2}}=\frac{\hbar}{2}\sqrt{\frac{k}{m}}=\frac{1}{2}\hbar\omega$$

c) Soit $\phi_1(x) = C_1 x e^{-ax^2}$

$$-\frac{\hbar^2}{2m}\left[-6a\phi_1(x) + 4a^2x^2\phi_1(x)\right] + \frac{1}{2}kx^2\phi_1(x) = E_1\phi_1(x)$$

ϕ_1 solution de l'équation de S si : $-\frac{2\hbar^2 a^2}{m} + \frac{1}{2}k = 0$

$$\Rightarrow a = \sqrt{\frac{km}{4\hbar^2}} = \frac{m\omega}{2\hbar}$$

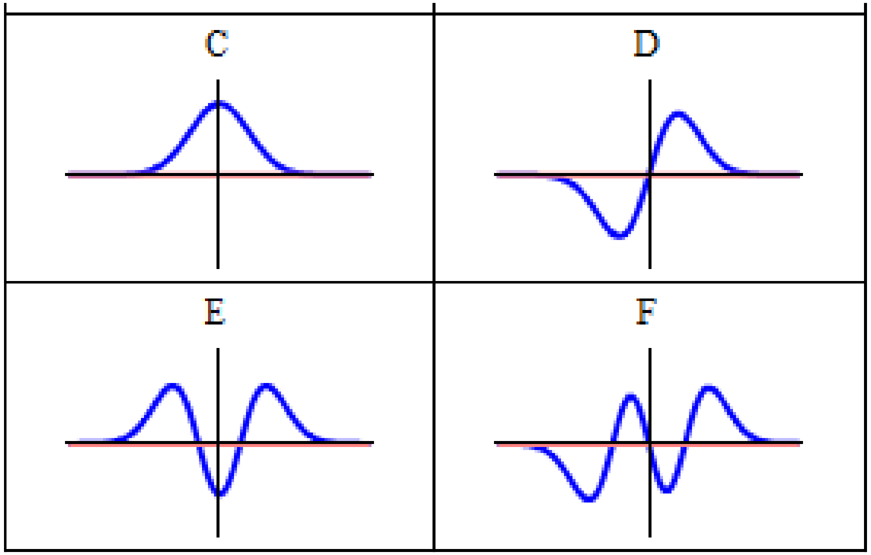
$$-\frac{\hbar^2}{2m}\left[-6a\phi_1(x)\right]=E_1\phi_1(x)$$

$$E_1=\frac{3\hbar^2 a}{m}$$

$$\Rightarrow E_1=3\frac{\hbar^2}{m}\sqrt{\frac{km}{4\hbar^2}}=\frac{3}{2}\hbar\sqrt{\frac{k}{m}}=\frac{3}{2}\hbar\omega$$

$$\boxed{E_n=\left(n+\frac{1}{2}\right)\hbar\omega}$$

$$\phi_0 = C_0 e^{-ax^2} \qquad \phi_1 = C_1 x e^{-ax^2}$$



$$\phi_2 = C_2 (x^2 - 1) e^{-ax^2} \qquad \phi_3 = C_3 x^3 e^{-ax^2}$$

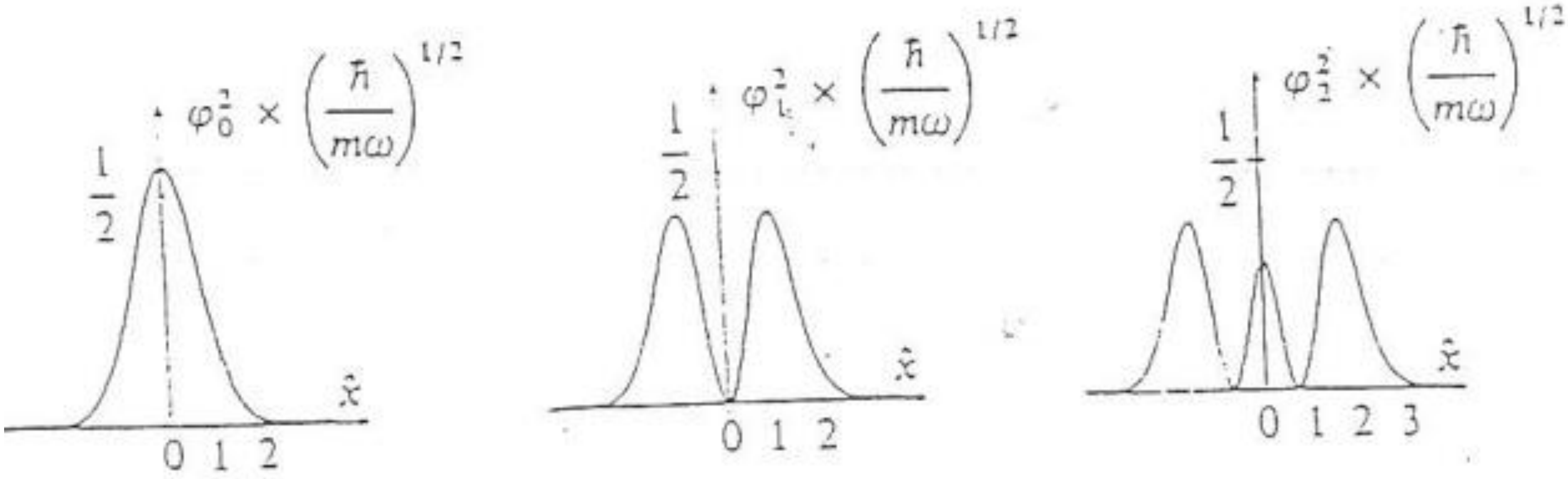


FIGURE 5

Densités de probabilité associées aux trois premiers niveaux de l'oscillateur harmonique.

d) On a démontré en cours que si $\Psi(x,0)=\sum C_n\varphi_n(x)$

$$\Psi(x,t)=\sum_n C_n(t)\varphi_n(x)$$ avec $$C_n(t)=C_n e^{-i\frac{E_n t}{\hbar}}$$

Propriété liée au postulat 6

$$\Psi(x,0)=\phi_0(x)+3\phi_1(x)$$

$$\Psi(x,t)=e^{-i\frac{E_0 t}{\hbar}}\phi_0(x)+3e^{-i\frac{E_1 t}{\hbar}}\phi_1(x)$$

$$\Psi(x,t)=e^{-i\frac{\omega t}{2}}\phi_0(x)+3e^{-i\frac{3\omega t}{2}}\phi_1(x)$$

$$\Psi(x,t)=e^{-i\frac{\omega t}{2}}\left(\phi_0(x)+3e^{-i\omega t}\phi_1(x)\right)$$

Exercice III : Vibrations molécules CO

1) $V(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^2$

$V(r)$ tq $r > 0$

$r \to \infty \quad V(r) \to D_e$

$V(r = r_e) = 0$

$V(0) = D_e\left(1 - e^{ar_e}\right)^2$

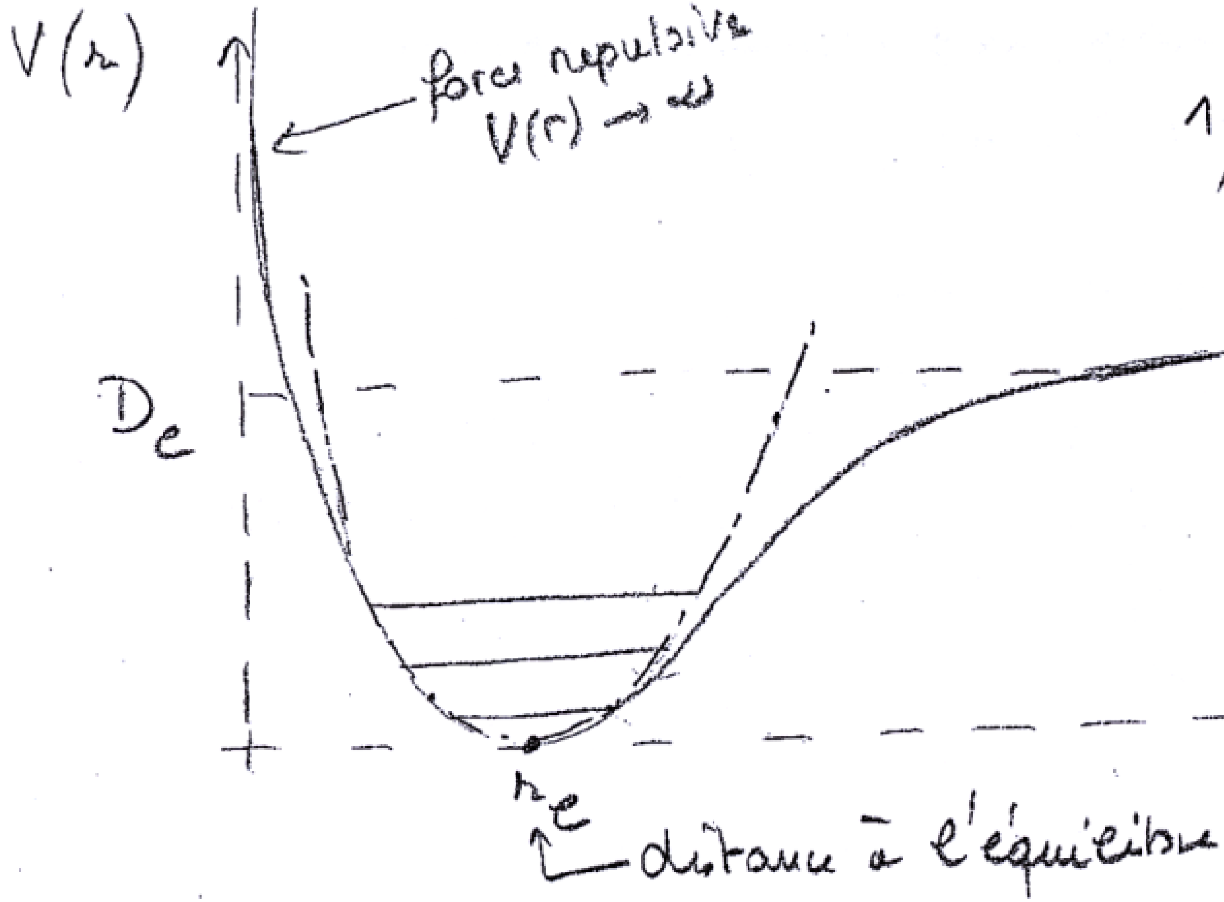
très grd $\gg D_e$

mais pas ∞

(défaut du modèle)

$$\frac{\partial V}{\partial r} = 2D_e\left(1 - e^{-a(r-r_e)}\right) a e^{-a(r-r_e)}$$

$= 0$ où $r = r_e$



1) V(r

$$D_e = V(r \to \infty)$$

$$V(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^2$$

2) $r \simeq r_e \quad r - r_e \ll 1$

$e^{\varepsilon} \simeq 1 + \varepsilon + \frac{\varepsilon^2}{2} \quad \varepsilon \ll 1 \qquad 1 - e^{\varepsilon} \simeq -\varepsilon$

$V(r) \simeq D_e\, a^2 (r-r_e)^2 \equiv \frac{1}{2} k (r-r_e)^2 \qquad \boxed{k \equiv 2 D_e a^2}$

PARABOLE centrée sur $r=r_e$ / cte de force

$\omega = \sqrt{\frac{k}{\mu}} \qquad \frac{1}{\mu} = \frac{1}{m_O} + \frac{1}{m_C}$

3)

$\mu = \frac{m_O m_C}{m_O + m_C} = 1.138\, 10^{-26}$ kg $\qquad \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{1895}{1.138 \cdot 10^{-26}}} = 4.081\, 10^{14} s^{-1}$

$\hbar\omega = \frac{1.05 \cdot 10^{-34} \times 4.081 \cdot 10^{14}}{1.6 \cdot 10^{-19}} = 0.268\, eV$

$(n+\frac{1}{2})\hbar\omega \quad$ fondal $= 0.134 eV$ ($\hbar\omega/2$) ; 1er excité $= 0.402 eV$ ($3/2\, \hbar\omega$), 2e excit $= 0.670$

3e ét. ex. $= 0.937 eV$

$E_{liaison} = E(r \to \infty) - E_{fondal} = 11.23 - 0.134\, eV = 11.096\, eV$

4) $\Delta E = \hbar\omega = h\nu = h\frac{c}{\lambda} \longrightarrow \lambda = 4.66\ \mu m$ IR.

↑ ts les niv. sont équidistants → sp. vib = 1 seule raie .

Traitement de l’oscillateur harmonique avec les opérateurs création et annihilation

$$\mathcal{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2 = \frac{p^2{}_x}{2m} + \frac{1}{2}m\omega^2x^2$$

$$avec\ [\hat{x},\hat{p}] = i\hbar$$

$$\hat{X} = (\frac{m\omega}{\hbar})^{\frac{1}{2}}\ \hat{x}$$

$$[(\frac{m\omega}{\hbar})^{\frac{1}{2}}] = \frac{MT^{-1}}{ML^2T^{-1}})^{\frac{1}{2}} = L^{-1}$$

$$[\hat{X}] = 1$$

$$\hat{P} = (\frac{1}{m\omega\hbar})^{\frac{1}{2}}\ \hat{p}$$

$$\left[\frac{1}{m\omega\hbar}\right] = \frac{1}{MT^{-1}ML^2T^{-1}} = \frac{1}{M^2T^{-2}L^2}$$

$[p]=MLT^{-1}$ → $[p]\left(\frac{1}{m\omega\,\hbar}\right)^{1/2} = [\hat{P}] = 1$

$$\hat{X} = (\frac{m\omega}{\hbar})^{\frac{1}{2}}\ \hat{x}$$
$$\hat{P} = (\frac{1}{m\omega\hbar})^{\frac{1}{2}}\ \hat{p}$$

$$[\hat{X}, \hat{P}\ ] = \left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\left(\frac{1}{m\omega\,\hbar}\right)^{1/2}[\hat{x}, \hat{p}] = \frac{1}{\hbar}\,i\hbar = i$$

$$\hat{\mathcal{H}} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \left(\frac{m\omega\hbar}{2m}\right)\hat{P}^2 + \left(\frac{\hbar}{m\omega}\right)\frac{1}{2}m\omega^2\hat{X}^2$$

$$\hat{\mathcal{H}} = \frac{\hbar\omega}{2}(\hat{P}^2 + \hat{X}^2) = \hbar\omega\hat{H}$$ avec $\hat{H} = \frac{1}{2}(\hat{P}^2 + \hat{X}^2)$ et donc $[\hat{H}] = \frac{[\mathcal{H}]}{[\hbar\omega]} = 1$

2) Opérateurs création $\widehat{a^{+}}$ *et annihilation* $\hat{a}$

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{X} + i\hat{P}) \qquad \widehat{a^{+}} = \frac{1}{\sqrt{2}}(\hat{X} - i\hat{P})$$

$\hat{X}$ *et* $\hat{P}$ opérateurs hermitiques car associés à *une grandeur physique mesurable*

$$\hat{X} = \hat{X}^{+} \quad et\ \hat{P} = \hat{P}^{+}$$

$(\hat{a})^{+} = \frac{1}{\sqrt{2}}\left(\hat{X}^{+} - i\hat{P}^{+}\right) = \frac{1}{\sqrt{2}}\left(\hat{X} - i\hat{P}\right) = \widehat{a^{+}} \neq \hat{a}$

$\left(\widehat{a^{+}}\right)^{+} = \frac{1}{\sqrt{2}}\left(\hat{X}^{+} + i\hat{P}^{+}\right) = \frac{1}{\sqrt{2}}\left(\hat{X} + i\hat{P}\right) = \hat{a} \neq \widehat{a^{+}}$

$\hat{a}\ et \widehat{a^{+}}$ sont adjoints l'un l'autre et pas hermitiques. Ce ne sont pas des observables.

2-2) $\hat{a}\,\widehat{a^{+}} = \frac{1}{2}\left(\hat{X} + i\hat{P}\right)\left(\hat{X} - i\hat{P}\right) = \frac{1}{2}\left(\hat{X}^{2} - i\hat{X}\hat{P} + i\hat{P}\hat{X} + \hat{P}^{2}\right)$

$\hat{a}\,\widehat{a^{+}} = \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} - i\left[\hat{X}, \hat{P}\right]\right) = \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} + 1\right) = \hat{H} + \frac{1}{2}$

$\widehat{a^{+}}\hat{a} = \frac{1}{2}\left(\hat{X} - i\hat{P}\right)\left(\hat{X} + i\hat{P}\right) = \frac{1}{2}\left(\hat{X}^{2} + i\hat{X}\hat{P} - i\hat{P}\hat{X} + \hat{P}^{2}\right)$

$\widehat{a^{+}}\hat{a} = \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} + i\left[\hat{X}, \hat{P}\right]\right) = \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} - 1\right) = \hat{H} - \frac{1}{2}$

$[\hat{a}, \widehat{a^{+}}] = \hat{a}\widehat{a^{+}} - \widehat{a^{+}}\hat{a} = \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} + 1\right) - \frac{1}{2}\left(\hat{X}^{2} + \hat{P}^{2} - 1\right) = 1$

$[\hat{a}, \widehat{a^{+}}] = 1$

$\widehat{a^+}\hat{a} = \hat{N}$
$\hat{a}\widehat{a^+} = \hat{N}$+1

3) Opérateur $\hat{N}$=$\widehat{a^+}\hat{a}$ hermitique?

<u>1ère méthode</u> : $\hat{N}^+ = (\widehat{a^+}\hat{a})^+$= $\hat{a}^+\widehat{a^+}^+ = \widehat{a^+}\hat{a} = \hat{N}$

<u>2ème méthode</u> : $\hat{H} = \widehat{a^+}\hat{a} + \frac{1}{2} = \hat{N} + 1/2$
$\hat{H}$ hermitique donc $\hat{N}$ hermitique

3-2)

$[\hat{N}, \hat{a}] = -\widehat{a}$?
$[\hat{N}, \widehat{a^+}] = \widehat{a^+}$?

$$[\hat{N}, \hat{a}] = [\widehat{a^+}\hat{a}, \widehat{a}] = \widehat{a^+}\hat{a}\widehat{a} - \hat{a}\widehat{a^+}\hat{a} = (\widehat{a^+}\hat{a} - \hat{a}\widehat{a^+})\hat{a} = [\widehat{a^+}, \hat{a}]\hat{a} = -\hat{a}$$

$$[\hat{N}, \widehat{a^+}] = [\widehat{a^+}\hat{a}, \widehat{a^+}] = \widehat{a^+}\hat{a}\widehat{a^+} - \widehat{a^+}\widehat{a^+}\widehat{a} = \widehat{a^+}(\hat{a}\widehat{a^+} - \widehat{a^+}\hat{a}) = \widehat{a^+}$$

$$[\hat{N},\hat{a}] = -\hat{a}$$
$$[\hat{N},\widehat{a^+}] = \widehat{a^+}$$

4) Etats propres de $\hat{N}$

4-1) Si $\hat{N}|\varphi_\alpha> = \alpha|\varphi_\alpha> \ alors\ \alpha \geq 0?$

$$<\varphi_\alpha|\hat{N}|\varphi_\alpha> =<\varphi_\alpha|\widehat{a^+}\hat{a}|\varphi_\alpha> =<\varphi_\alpha|\alpha|\varphi_\alpha> = \alpha$$

Si $|\varphi_\alpha{}'> = \hat{a}|\varphi_\alpha> \ alors <\varphi_\alpha{}'| = <\varphi_\alpha|\widehat{a^+}$

$$<\varphi_\alpha{}'|\varphi_\alpha'> =<\varphi_\alpha|\widehat{a^+}\hat{a}|\varphi_\alpha> = \alpha>=0$$

4-2) $\hat{N}|\varphi_\alpha> = \alpha|\varphi_\alpha>$

$$\hat{N}\hat{a}|\varphi_\alpha>= \hat{a}\hat{N}|\varphi_\alpha> -\hat{a}|\varphi_\alpha>=\alpha\hat{a}|\varphi_\alpha> -\hat{a}|\varphi_\alpha>=(\alpha-1)\hat{a}|\varphi_\alpha>$$

$\hat{a}|\varphi_\alpha>$ est vect. prop. de $\hat{N}$ associé à la val. prop. $\alpha-1$

$$\hat{N}\widehat{a^+}|\varphi_\alpha>= \widehat{a^+}\hat{N}|\varphi_\alpha> +\widehat{a^+}|\varphi_\alpha>=\alpha\widehat{a^+}|\varphi_\alpha> +\widehat{a^+}|\varphi_\alpha>=(\alpha+1)\widehat{a^+}|\varphi_\alpha>$$

$\widehat{a^+}|\varphi_\alpha>$ est vect. prop. de $\hat{N}$ associé à la val. prop. $\alpha+1$

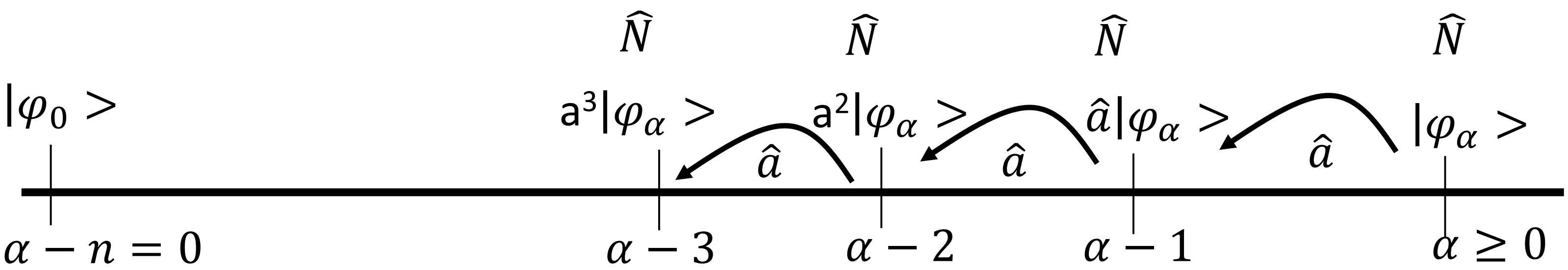
$\hat{a}|\varphi_0> = 0$

Montrer que α est un entier?

$\hat{a}|\varphi_0> = 0$

$|\varphi_\alpha>$ vect. prop. de $\hat{N}$ avec val. propr. α

$\hat{a}|\varphi_\alpha>$ vect. prop. de $\hat{N}$ avec val. propr. $\alpha - 1$



$\hat{N}|\varphi_\alpha> = n|\varphi_\alpha>$ avec n entier

$$\hat{\mathcal{H}}\,\left|\varphi_n > = \hbar\omega\hat{H}\,\right|\varphi_n > = \hbar\omega(\hat{N} + \frac{1}{2})|\varphi_n > = E_n|\varphi_n >$$

$E_n=(n+1/2)\,\hbar\omega$

Energie minimum ou fondamental :

$$E_0 = \frac{\hbar\omega}{2}\,??$$

$$|\hat{\mathcal{H}}|\varphi_0 > = E_0/\varphi_0>$$

$$\hbar\omega\,|\hat{H}|\varphi_0 > = E_0/\varphi_0>$$

$$\hbar\omega\,(\hat{N} + \frac{1}{2})/\varphi_0 > = E_0/\varphi_0>$$

$$\hbar\omega\,(\frac{1}{2})/\varphi_0 > = E_0/\varphi_0> \; car\; \hat{N}/\varphi_0 > = 0$$

$$\boxed{E_0 = \frac{1}{2}\hbar\omega}$$

Signification physique de l’opérateur création et annihilation:
ces opérateurs font passer dans un état d'énergie sup ou inf par création ou annihilation d'un quantum d'énergie (similaire à la création ou annihilation d’un boson)

4-5) $\widehat{a^+}|\varphi_n>$ en fonction $|\varphi_{n+1}>$?

$$\hat{N}|\varphi_n> = n|\varphi_n>$$
$$\hat{N}|\varphi_{n+1}> = (n+1)|\varphi_{n+1}>$$
$$\hat{N}\widehat{a^+}|\varphi_n> = (n+1)\,\widehat{a^+}|\varphi_n>$$

$\widehat{a^+}|\varphi_n>$ est vect. prop. de $\hat{N}$ associé à la val. prop. *n+1*
$|\varphi_{n+1}>$ est vect. prop. de $\hat{N}$ associé à la val. prop. *n+1*
Donc $|\varphi_{n+1}>$ et $|\varphi_n>$ sont vect. prop. de N avec la même val. prop. non dégénéré :

$$\widehat{a^+}|\varphi_n> = c\,|\varphi_{n+1}>$$

$$\left|\widehat{a^+}|\varphi_n>\right|^2 = <\varphi_n|\underbrace{\hat{a}\widehat{a^+}}_{\widehat{N}+1}|\varphi_n> = c^2 <\varphi_{n+1}/\varphi_{n+1}> = <\varphi_n/\varphi_n> + n<\varphi_n/\varphi_n>$$

$$|c|^2 = n+1$$

$$\widehat{a^+}|\varphi_n> = \sqrt{n+1}\,|\varphi_{n+1}>$$

$\hat{a}|\varphi_n >$ en fonction $|\varphi_{n-1} >$???

$\hat{N}|\varphi_n > = n|\varphi_n >$

$\hat{N}|\varphi_{n-1} > = (n-1)|\varphi_{n-1} >$

$\hat{N}\hat{a}|\varphi_n > = (n-1)\ \hat{a}|\varphi_n >$

$\hat{a}|\varphi_n >$=B $|\varphi_{n-1} >$

$< \varphi_n|\underbrace{\widehat{a^+}\hat{a}}_{\hat{N}}|\varphi_n > = |\hat{a}|\varphi_n >|^2 = B^2 < \varphi_{n-1}/\varphi_{n-1}>$

n=B^2

$\hat{a}|\varphi_n > = \sqrt{n}\ |\varphi_{n-1} >$

4-6) $\widehat{a^+}\ |\varphi_0 >\ \ tq\ \ \hat{N}|\varphi_0 > = 0$

$\hat{a}|\varphi_0 > = 0$

$\widehat{a^+}\ |\varphi_n > = \sqrt{n+1}\ |\varphi_{n+1} >$

$\widehat{a^+}\ |\varphi_{n-1} > = \sqrt{n}\ |\varphi_n >$

$\frac{1}{\sqrt{n}}\widehat{a^+}|\varphi_{n-1} > = |\varphi_n >$

$\frac{1}{\sqrt{n}}\widehat{a^+}\frac{1}{\sqrt{n-1}}\widehat{a^+}|\varphi_{n-2} > = |\varphi_n >$

$\frac{1}{\sqrt{n!}}(\widehat{a^+})^n|\varphi_0 > = |\varphi_n >$

6) Fonctions d'onde

6-1) $|\varphi_n>$ $\varphi_n(x)$

$<x|\varphi_n> = \varphi_n(x)$

$\hat{a}\left|\varphi_0\right> = |0>$ *on devrait* écrire $\left|\varphi_0^i\right>$ *degr*é *de d*é*g*é*n*é*rescence*

$\hat{H}|\varphi_n{}^i> = E_n\varphi_n{}^i>$ *avec* $E_n = (n+1/2)\hbar\omega$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\hat{X} + i\hat{P}\right) = \frac{1}{\sqrt{2}}\left(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\hat{x} + \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\hat{P}\right)\right)$$

$$\frac{1}{\sqrt{2}}\left(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\hat{x}|\varphi_0> \quad + \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\hat{p}|\varphi_0>\right)\right)=|0>$$ (car $\hat{a}\left|\varphi_0\right> = |0>$)

$$\frac{1}{\sqrt{2}}\left(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}<x|\hat{x}|\varphi_0> + \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\right)<x|\hat{p}|\varphi_0>\right) =<x/0>=0$$

$\hat{x}|x> = x|x>$

$<x\left|\hat{x}\right. =<x|x$

$$\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}x\varphi_0(x) \ + \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\right) * \left(-i\hbar\frac{\delta\varphi_0(x)}{\delta x}\right)=0$$

$$\left(\frac{m\omega}{\hbar}\right)x\varphi_0(x) \ + \left(\frac{\hbar}{\sqrt{\hbar m\omega}}\left(\frac{m\omega}{\hbar}\right)^{1/2}\left(\frac{\delta\varphi_0(x)}{\delta x}\right)\right)=0$$

$$\left(\frac{m\omega}{\hbar}\right)x\varphi_0(x) = -\frac{\delta\varphi_0(x)}{\delta x}$$

$$-\left(\frac{m\omega}{\hbar}\right)xdx = \frac{\delta\varphi_0(x)}{\varphi_0(x)}$$

$$Ln(\varphi_0(x)) = -\left(\frac{m\omega}{\hbar}\right)\frac{x^2}{2} + cste$$

$$\varphi_0(x) = A\, e^{-\frac{1}{2}\left(\frac{m\omega}{\hbar}\right)x^2}$$

Fonctions d'onde $|\varphi_1>$? avec $\widehat{a+}\,|\varphi_0> = |\varphi_1>$)

$$\widehat{a+} = \frac{1}{\sqrt{2}}(\hat{X} - i\hat{P}) = \frac{1}{\sqrt{2}}(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\hat{x} - \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\hat{P}\right))$$

$$\frac{1}{\sqrt{2}}(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}\hat{x}|\varphi_0> \quad - \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\hat{p}|\varphi_0>\right)) = |\varphi_1>$$

$$\frac{1}{\sqrt{2}}\left(\left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}} < x|\hat{x}|\varphi_0> - \left(\frac{\mathrm{i}}{\sqrt{\hbar m\omega}}\right) < x|\hat{p}|\varphi_0>\right) = \varphi_1(x)$$

$$\varphi_1(x) = \left(\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right)^{1/4} x e^{-\frac{1}{2}\frac{m\omega x^2}{\hbar}}$$

Fonctions d'onde = $|\varphi_2>$

$$\hat{a}\left|\varphi_0>=0 \qquad \varphi_0=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{1}{2}\frac{m\omega x^2}{\hbar}}\right.$$

$$\widehat{a^+}\left|\varphi_0>=|\varphi_1> \quad -\!\!\rightarrow\ \varphi_1=\left(\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right)^{1/4} x e^{-\frac{1}{2}\frac{m\omega x^2}{\hbar}}\right.$$

$$\widehat{a^+}\left|\varphi_1>=|\varphi_2> \quad -\!\!\rightarrow\ \varphi_2=\left(\frac{m\omega}{4\pi\hbar}\right)^{1/4}\left(\frac{2m\omega}{\hbar}x^2-1\right)e^{-\frac{1}{2}\frac{m\omega x^2}{\hbar}}\right.$$

$\widehat{a^+}\left|\varphi_n >\right.$ en fonction $|\varphi_{n+1} >???$

$\hat{N}\widehat{a^+}\left|\varphi_n > = (n+1)\widehat{a^+}|\varphi_n >\right.$

$\hat{N}\left|\varphi_{n+1} > = (n+1)|\varphi_{n+1} >\right.$

$\widehat{a^+}\left|\varphi_n > = c_n\, |\varphi_{n+1} >\right.$

$$\left|\left|\widehat{a^+}|\varphi_n >\right|^2 = < \varphi_n \left|\hat{a}\widehat{a^+}\, |\varphi_n >\right.\right.$$

$$\left[\hat{a}, \widehat{a^+}\right] = 1 = \hat{a}\widehat{a^+} - \widehat{a^+}\hat{a}$$

$$\hat{a}\widehat{a^+} = \widehat{a^+}\hat{a} + 1 = 1 + \hat{N}$$

$$\left|\left|\widehat{a^+}|\varphi_n >\right|^2 = < \varphi_n \left|1 + \hat{N}\, |\varphi_n > = 1 + n = |c_n|^2\right.\right.$$

$$\boxed{\widehat{a^+}|\varphi_n >= \sqrt{n+1}|\varphi_{n+1} >}$$