Cavity QED with mesoscopic topological superconductors

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Detecting Majorana bound states with a microwave cavity ?

Outline

I) A short introduction on Majorana fermions

- The Kitaev model as a 1D p-wave superconductor
- Some realization

II) Detecting Majorana fermions with a microwave cavity

- What quantity does the cavity probe ?
- Beyond a low energy description
- Signatures of Majorana fermions

I) A short recap on Majorana fermions

Majorana fermions in condensed matter

Majorana fermions can occur as collective excitations in solids with unconventional SC pairing

Obey non-Abelian statistics: platform for fault-tolerant quantum computation

Ordinary fermions
2nd quantization

$$\boxed{\{c_i^+, c_j\} = \delta_{ij}}$$

→ Write in terms of Majorana fermions

$$c_i = (\gamma_{B,i} + i\gamma_{A,i})/\sqrt{2}$$

$$\boxed{\gamma_{\alpha,i} = \gamma_{\alpha,i}^\dagger} \quad \text{and} \quad \{\gamma_{\alpha,i}, \gamma_{\beta,j}\} = \delta_{\alpha,\beta}\delta_{ij}$$

Any fermionic Hamiltonian can be recast in terms of Majorana operators

→ **However**, very few can support solutions with **isolated localized** Majorana fermions

Where to look for Majorana fermions ?

Collective excitations in solids with unconventional (triplet) SC pairing: **why ?**

s-wave Bogoliubov qps: $\gamma_n^+ = \sum \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\downarrow} \right)$

p-wave Bogoliubov qps: $\gamma_n^+ = \sum_i \left(u_{ni} a_{i\uparrow}^+ + v_{ni} a_{i\uparrow} \right)$

zero energy:
- ABS states
- localized states

$$\gamma^+ = \sum_i u_{0i} a_{i\uparrow}^+ + u_{0i}^* a_{i\uparrow} = \gamma .$$

$$\gamma = \gamma^+ \Rightarrow \text{ its own antiparticle}$$

These zero-modes are Majorana fermions !

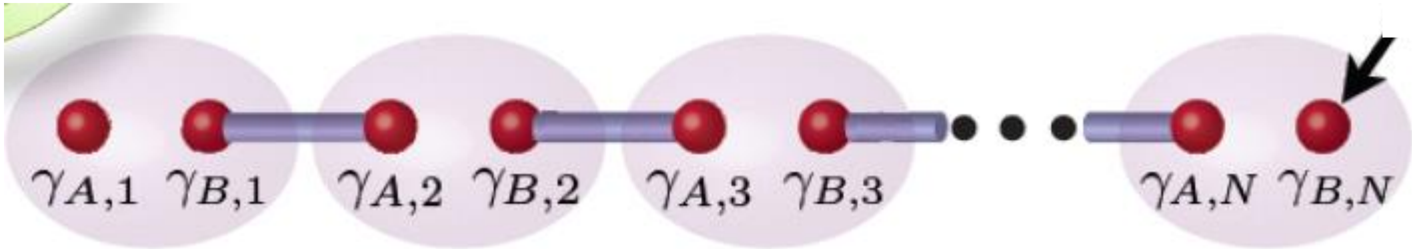
The Kitaev toy model: a 1D p-wave superconductor

$$H_{el} = -\mu \sum_{i=1}^{N} c_i^\dagger c_i - \frac{1}{2} \sum_{i=1}^{N-1} (\mathrm{t} c_i^\dagger c_{i+1} + \Delta c_i c_{i+1} + \mathrm{h.c.})$$

Introduce $\gamma_{A,i} = \gamma_{A,i}^\dagger \; , \; \gamma_{B,i} = \gamma_{B,i}^\dagger$ Majorana self-adjoint operators

$$\Rightarrow \qquad c_i = \frac{(\gamma_{B,i} + i\gamma_{A,i})}{\sqrt{2}}$$

$$\boxed{\mu = 0 \; , \; t = \Delta} \quad \Rightarrow \quad \boxed{H_{el} = -it \sum_{i=1}^{N-1} \gamma_{B,i} \gamma_{A,i+1}}$$



Unpaired end Majorana fermions!

Kitaev, 01’

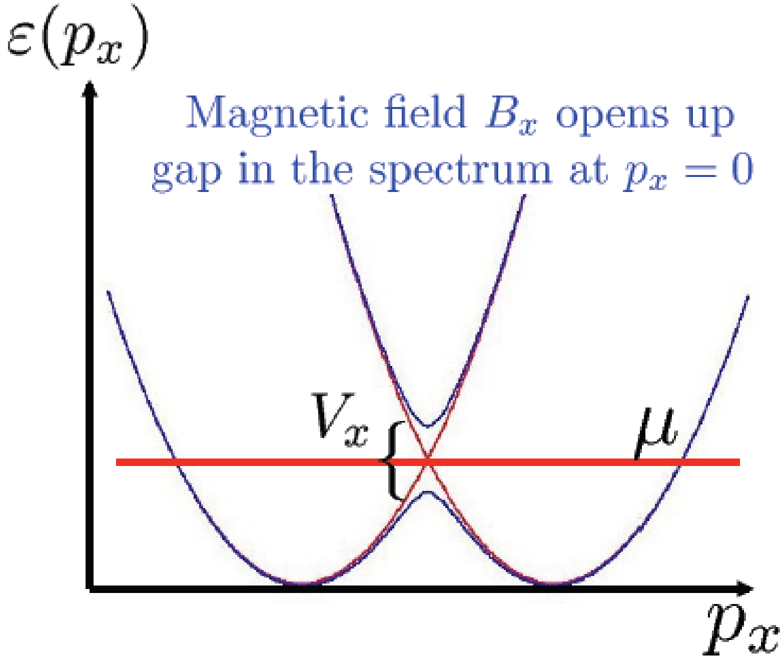
Band structure of a quantum wire with SO

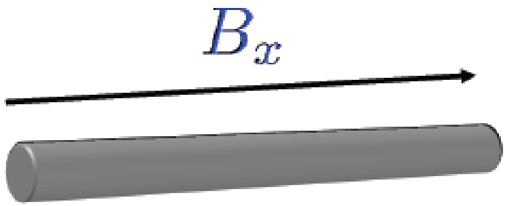
$$H_0 = \int_{-L}^{L} dx\, \psi^\dagger_\sigma(x)\left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x\right)_{\sigma\sigma'}\psi_{\sigma'}(x)$$

single channel nanowire

spin-orbit coupling

Zeeman splitting


Magnetic field B_x opens up gap in the spectrum at $p_x = 0$



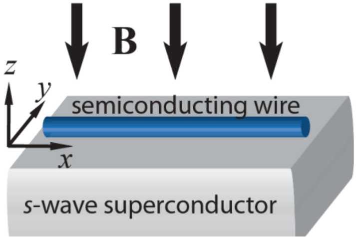
InAs, InSb nanowires

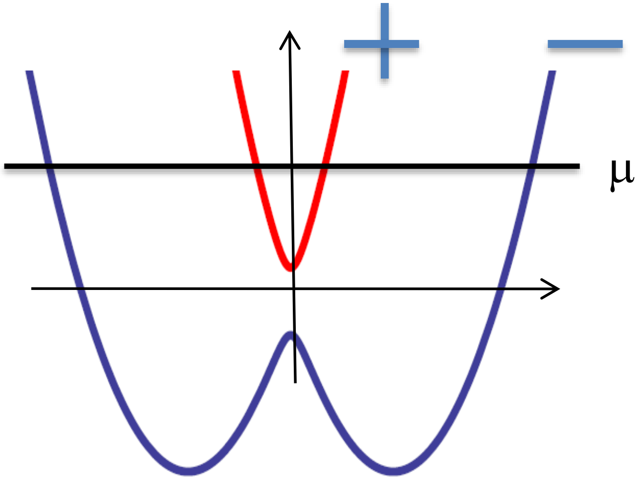
large spin-orbit ($\alpha \sim 0.1 eV\AA$)

large g-factor ($g \sim 10-50$)

good contacts with metals

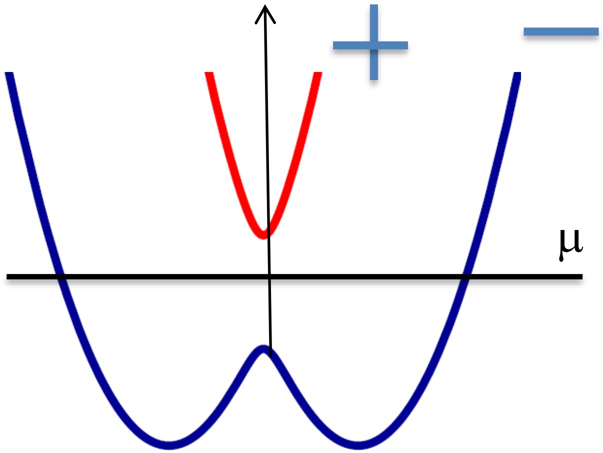
Quantum wire in proximity of a superconductor





Turn on the B-field

Reduce μ



Effective Pairing:

+ +

− −

− +

Effective Pairing:

− −

TRIPLET

Relation to the Kitaev model

1D semiconducting quantum-wires

- Spin-orbit interaction : $\alpha_R k_F$
- Magnetic field Δ_Z
- s-wave superconductivity Δ_S

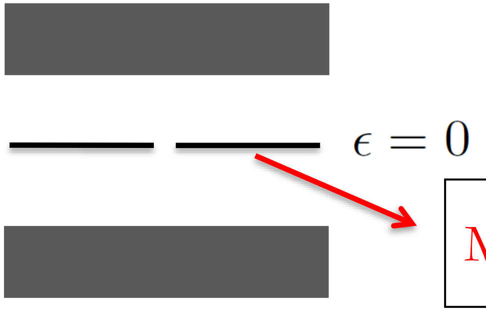
•Topological phase

$$\boxed{\Delta_Z > \sqrt{\Delta_S^2 + \mu^2}}$$

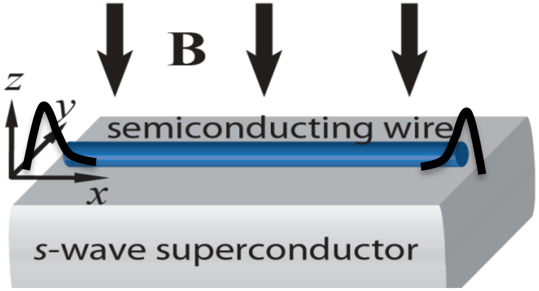
Sato et al., PRB (2009)

Sau et al., PRL 104, (2010)

Oreg, Refael, von Oppen, PRL 105, (2010)



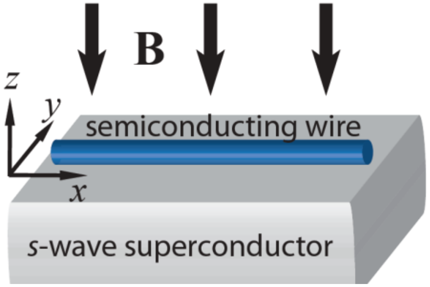
$$\boxed{\text{Majorana: } \gamma_0 = \gamma_0^\dagger}$$



In the large magnetic field limit, the Hamiltonian reduces to the Kitaev model

Proposed realizations for a 1D topological SC :

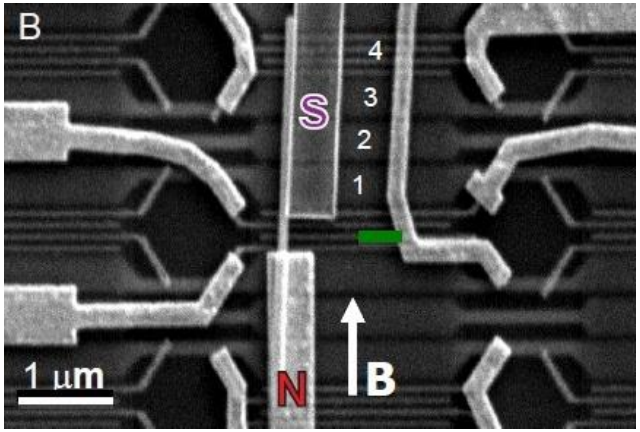
- 1D quantum semiconducting wires
  In proximity of a s-wave superconductor

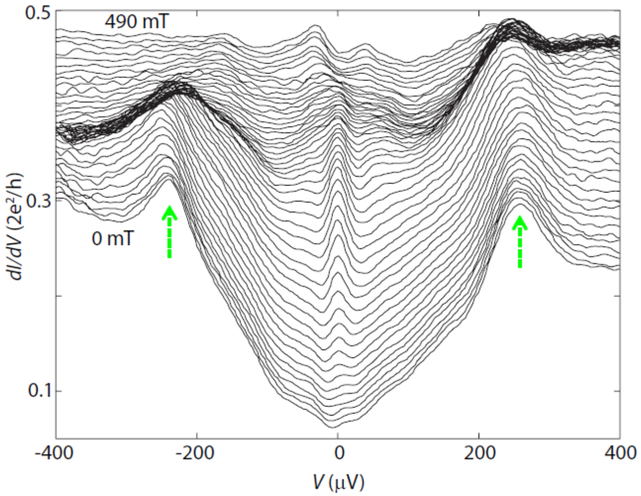


Lutchyn et al., PRL 104, (2010)

Oreg, Refael, von Oppen, PRL 105, (2010)

Possible experimental signatures





Mourik et al., Science 2012 (Kouwenhoven group)

See also results by Heiblum, Xu, Marcus, Rohkinson groups', etc.

Detecting Majorana bound state

$$\gamma = \gamma^{\dagger}$$

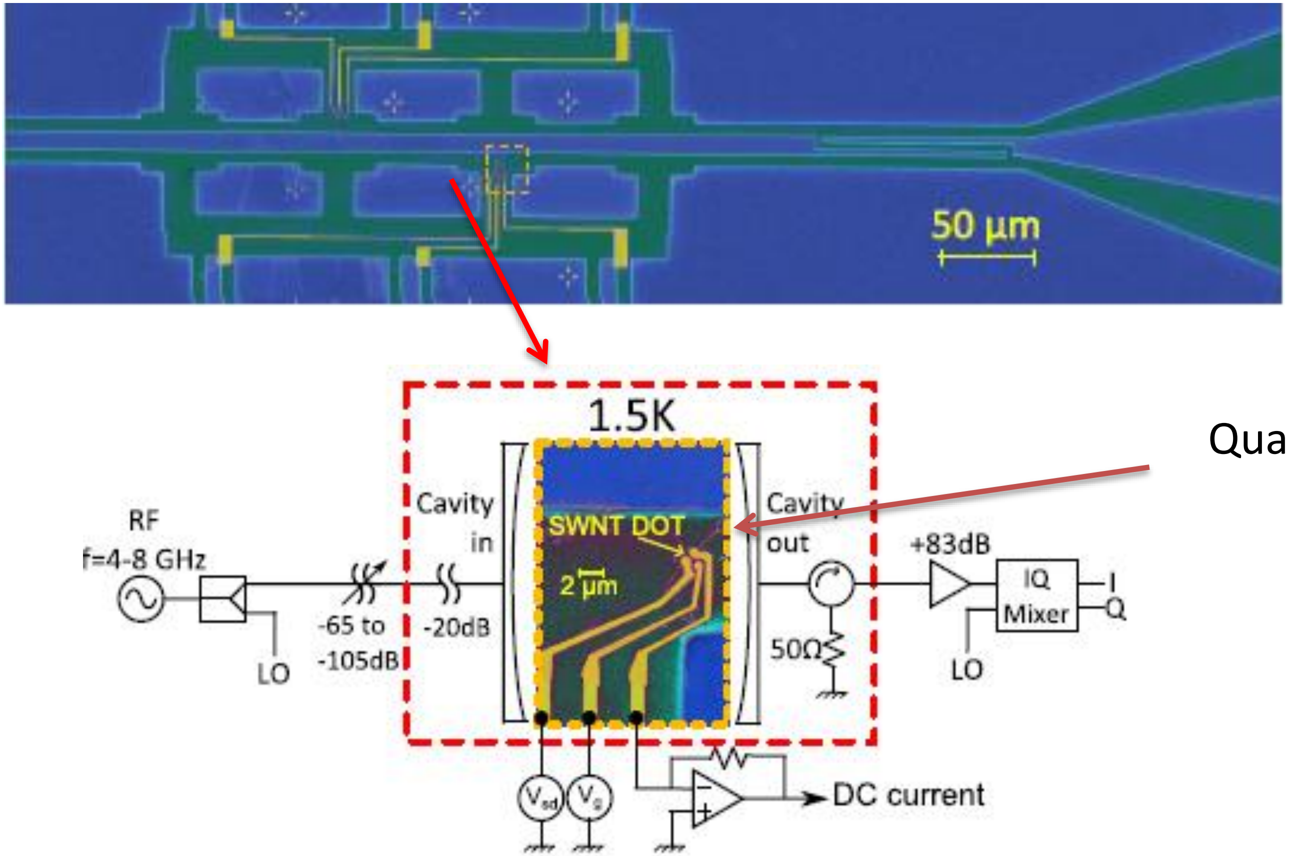
- Magnetic moment Zero
- Electric charge Zero
- Energy Zero

→ Most experimental works focused on transport quantities

Can we detect Majorana bound states using microwaves optical observables ?

II) Detecting Majorana bound states with a microwave cavity

Coupling on-chip a quantum conductor and a microwave cavity



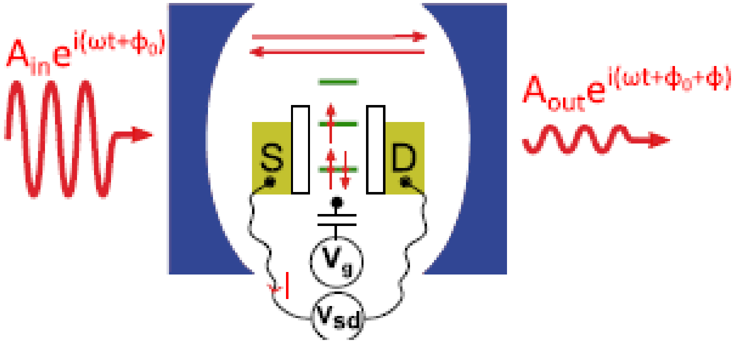
Coplanar wave guide

Quantum dot embedded In the cavity

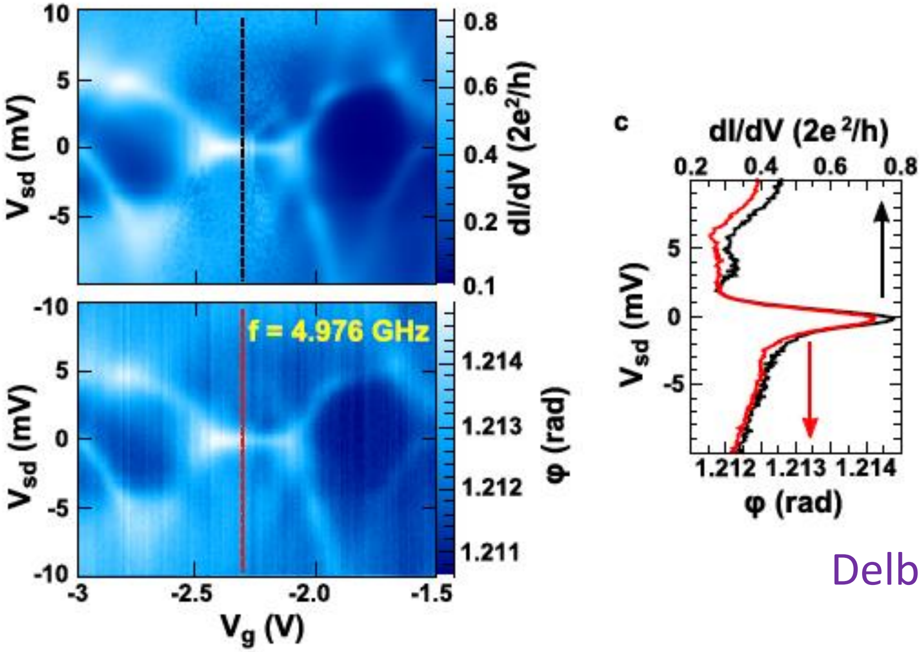
From Kontos' group in ENS Paris

Transport and optical meaurements

Schematics



Measure optical phase shift (reactive and dissipative parts)

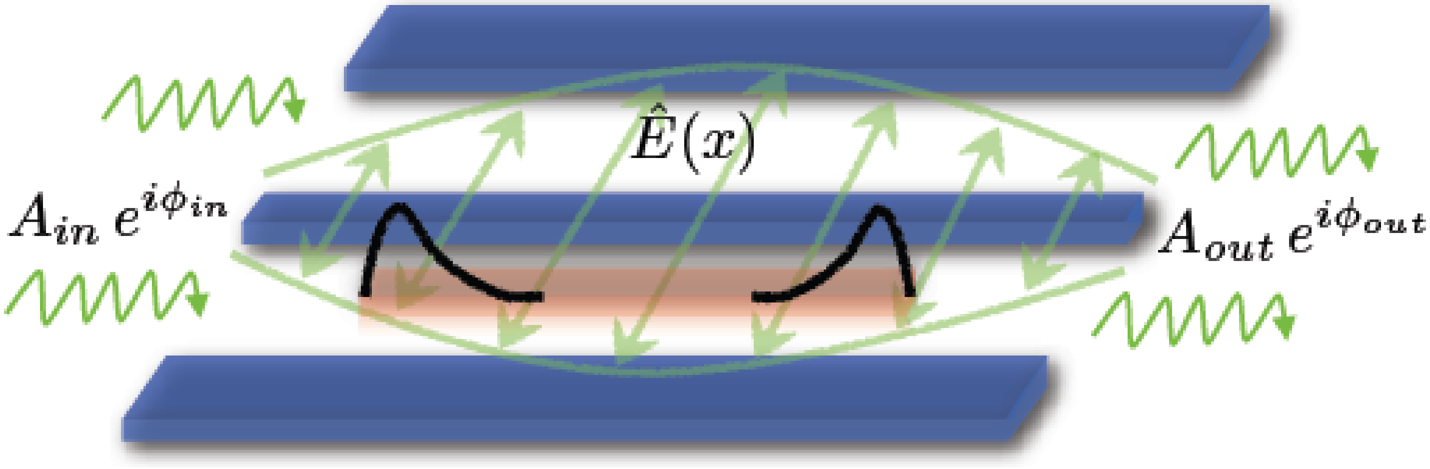


Delbecq et al., PRL 2011

Differential conductance and optical phase shift are alike !

Moreover optical probe is less-invasive !

What does the cavity probe ?



Optical transmission coefficient:

Input-output theory

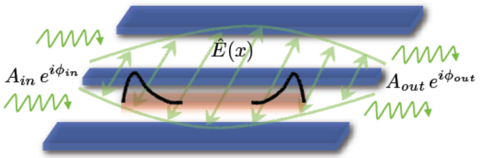
$$\boxed{\tau = Ae^{i\phi} = \frac{A_{out}}{A_{in}} e^{i(\phi_{out} - \phi_{in})}}$$

see e.g. Clerk et al., RMP 2010

What does the cavity probe ?

Optical transmission Coefficient:

$$\boxed{\tau = \frac{\kappa}{-i(\omega - \omega_c) + \kappa/2 - i\Pi(-\omega_c)}}$$



Cavity frequency

Escape rate of the cavity
=cavity quality factor

Proportional to the el. charge susceptibility

$$\boxed{\Pi(t) = -i\alpha^2\theta(t)\langle[n(t), n(0)]\rangle}$$

$$\Pi(\omega) = \Pi'(\omega) + i\Pi''(\omega)$$

For $\omega \approx \omega_c$ and in the large κ limit

$$\boxed{\frac{A_{in} - A_{out}}{A_{in}} \approx \frac{\Pi'(\omega)}{\kappa} \quad , \quad \phi_{out} - \phi_{in} \approx \frac{\Pi''(\omega)}{\kappa}}$$

Coupling Majorana bound states to a cavity ?

A low energy phenomenological Hamiltonian ?

$$\begin{aligned} H_{\text{eff}} &\approx -2i\gamma_A\gamma_B\left[\beta(a^\dagger + a) + \epsilon_M\right] + \hbar\omega_c a^\dagger a \\ &\approx \underbrace{(2n_M - 1)}_{\textbf{Parity}}\left[\beta(a^\dagger + a) + \epsilon_M\right] + \hbar\omega_c a^\dagger a \end{aligned}$$

$$c_M = (\gamma_B + i\gamma_A)/\sqrt{2}$$

Exponential overlap between the γ's

$$[H_{\text{eff}}, n_M] = 0 \quad \Longrightarrow \quad \boxed{\Pi_M(t) = -i\alpha^2\theta(t)[n_M(t), n_M(0)] = 0}$$

Expected (parity protected)!

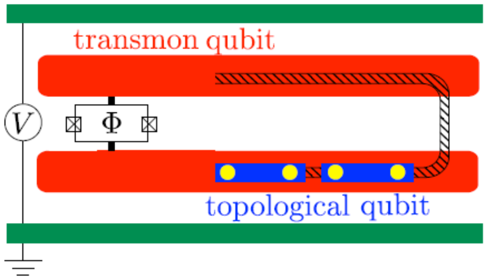
⇒ Effective Hamiltonians work if we consider a Majorana **qubit** made out of two topological wires (**four** Majorana fermions).

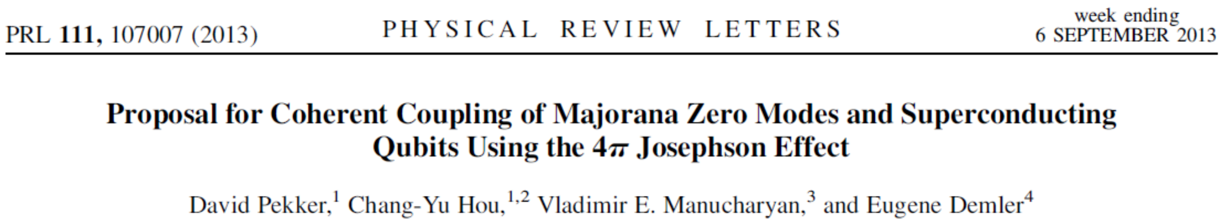
Majorana qubit and cavity QED

The top-transmon: a hybrid superconducting qubit for parity-protected quantum computation

F Hassler, A R Akhmerov and C W J Beenakker[1]
Instituut-Lorentz, Universiteit Leiden, PO Box 9506, 2300 RA Leiden,
The Netherlands
E-mail: beenakker@ilorentz.org

New Journal of Physics **13** (2011) 095004 (13pp)

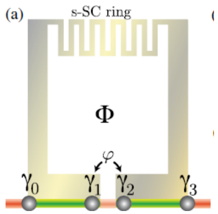


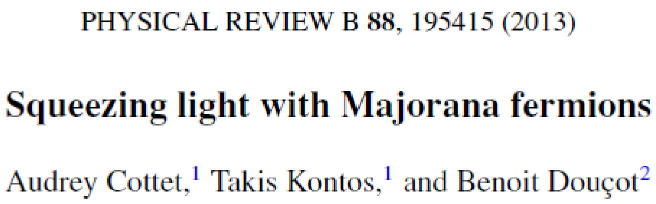


PRL **111,** 107007 (2013) PHYSICAL REVIEW LETTERS week ending 6 SEPTEMBER 2013

Proposal for Coherent Coupling of Majorana Zero Modes and Superconducting Qubits Using the 4π Josephson Effect

David Pekker,[1] Chang-Yu Hou,[1,2] Vladimir E. Manucharyan,[3] and Eugene Demler[4]

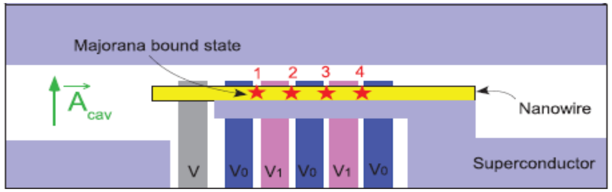


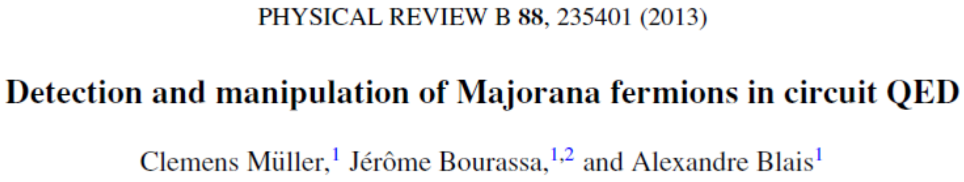


PHYSICAL REVIEW B **88**, 195415 (2013)

Squeezing light with Majorana fermions

Audrey Cottet,[1] Takis Kontos,[1] and Benoit Douçot[2]

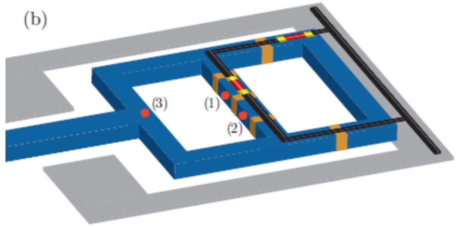




PHYSICAL REVIEW B **88**, 235401 (2013)

Detection and manipulation of Majorana fermions in circuit QED

Clemens Müller,[1] Jérôme Bourassa,[1,2] and Alexandre Blais[1]

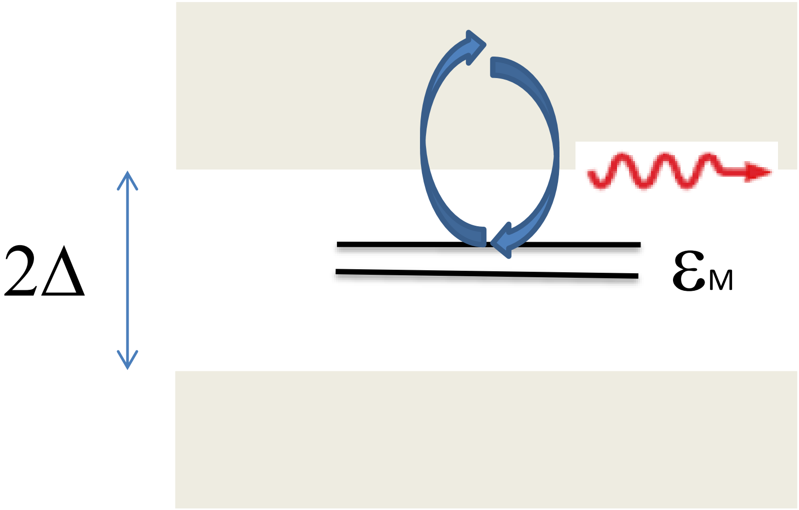


And many more ….

Beyond the effective low-energy Hamiltonian

- The effective Hamiltonian is not valid near the topological transition

- Does not take into account virtual excitations

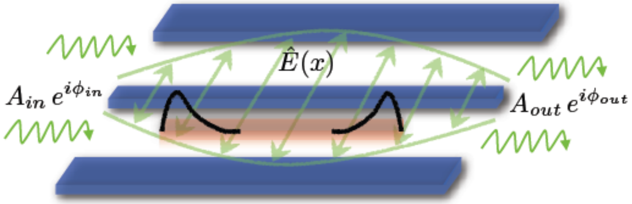


Non-zero matrix elements

→ Miss some important physics ?

Beyond the effective Hamiltonian: the whole wire

Model Hamiltonian



$$H_{sys} = H_{1D} + H_c + H_{ph}$$

$$H_{1D} = -\frac{t}{2}\sum_{i=1}^{N-1}(c_i^\dagger c_{i+1} + h.c.) - \mu\sum_{i=1}^{N} c_i^\dagger c_i \; -\frac{\Delta}{2}\sum_{i=1}^{N-1}(c_i c_{i+1} + h.c.)$$

Kitaev Hamiltonian

$$H_c = \alpha\sum_{i=1}^{N} c_i^\dagger c_i \; (a^\dagger + a)$$

$$H_{ph} = \omega_c a^\dagger a$$ Single mode cavity

Capacitive coupling
= Photonic Anderson-Holstein coupling

Remark: If Δ=0 ⇒ $\left[H_{sys}, \sum_i c_i^\dagger c_i\right] = 0$ ⇒ Π(t) = 0

Treatment

Strategy: write the Hamiltonian in the Bogoliubov basis

$$H_{el} = \sum_{m=1}^{N} \epsilon_m \left(\tilde{c}_m^\dagger \tilde{c}_m - \frac{1}{2} \right)$$

The electron-photon coupling reads:

$$H_{el-c} = \sum_{p,p'} \left[C^{(1)}_{pp'} \tilde{c}_p^\dagger \tilde{c}_{p'} - i C^{(2)}_{pp'} \tilde{c}_p^\dagger \tilde{c}_{p'}^\dagger + \mathrm{h.c.} \right] (a^\dagger + a)$$

$C^{(1,2)}_{pp'}$ are coefficients that depend on the transformation from the electronic to Bogoliubov basis.

$$C^{(1,2)}_{pp'} = \alpha \sum_{j=1}^{N} \vec{\psi}_p^\dagger(j) \tau_{z,y} \vec{\psi}_{p'}(j)$$

$$\boxed{\text{In general, all } C^{(1,2)}_{pp'} \neq 0, \text{ for } p \neq p'}$$

→

Coupling between all the levels via the cavity !

M. Trif, Y. Tserkovnyak, PRL 2013

A. Cottet, T. Kontos, B. Douçot, PRB 2015

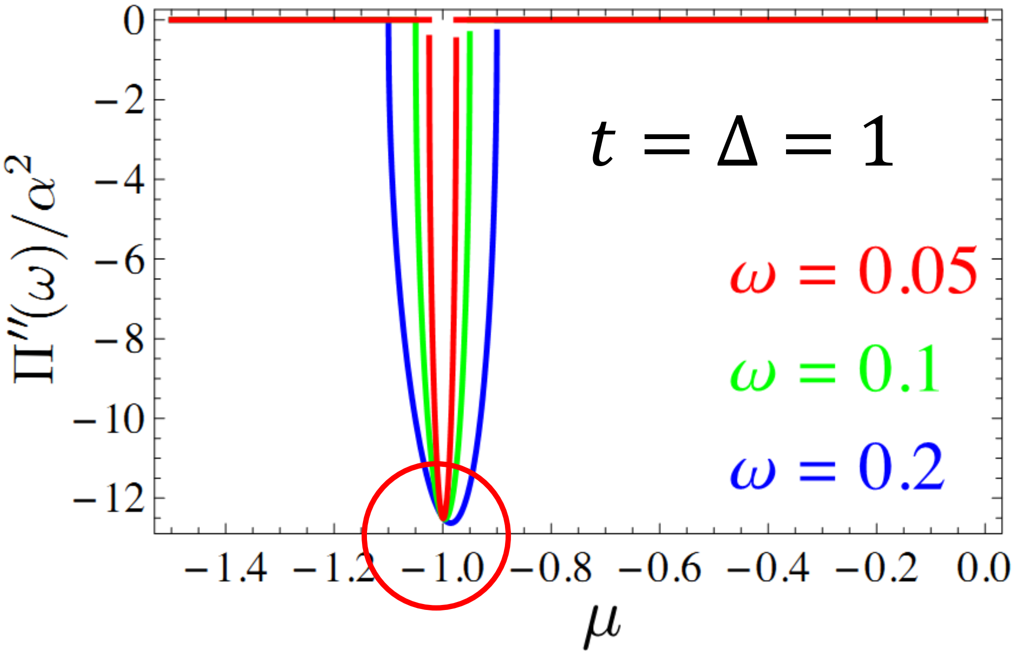
Topological phase transition

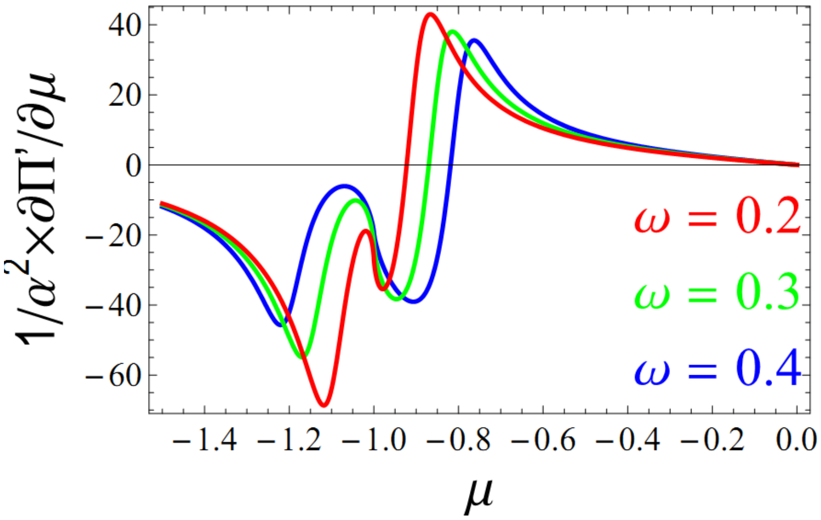
$$\Pi(\omega) = \Pi'(\omega) + i\Pi''(\omega)$$

Periodic b.c.

$$\boxed{\Pi(\omega) = -\alpha^2 \sum_{k>0;p=\pm} \frac{(\Delta \sin k)^2}{E_k^2} \frac{p}{\omega + 2pE_k + i\eta}}$$

$$E_k = \sqrt{(-\mathrm{t}\cos k - \mu)^2 + (\Delta \sin k)^2}$$

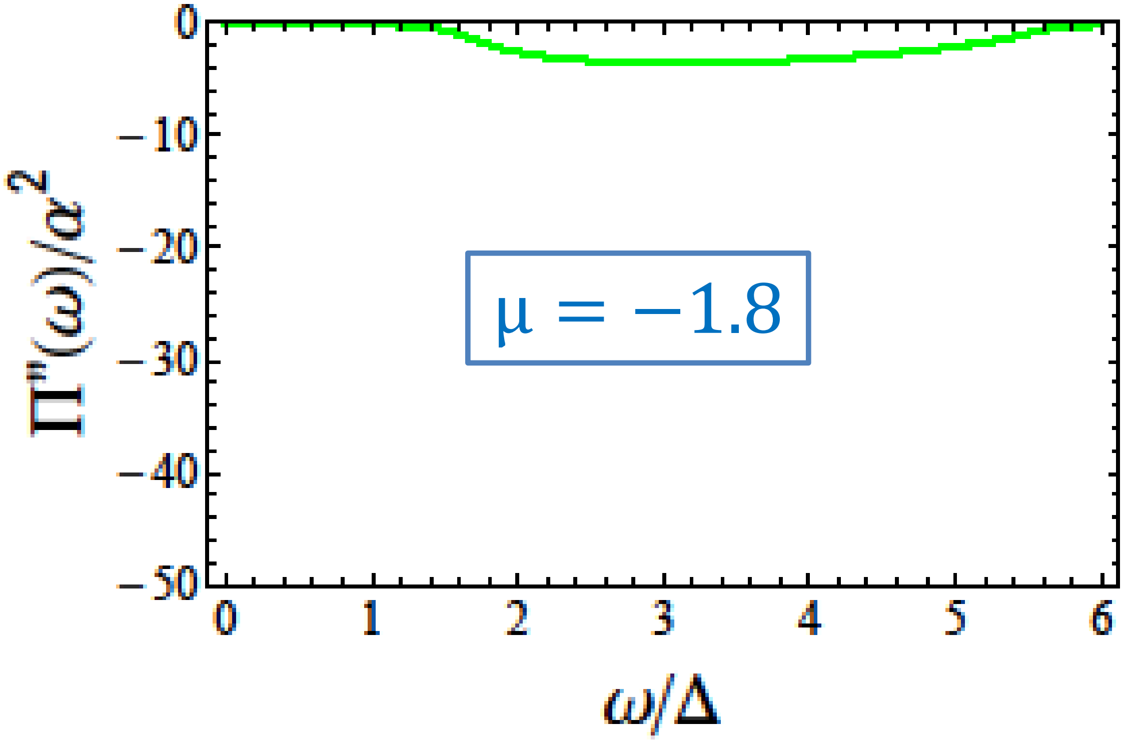




O. Dmytruk, M. Trif, PS, arXiv:1502.03082

Results for the charge susceptibility

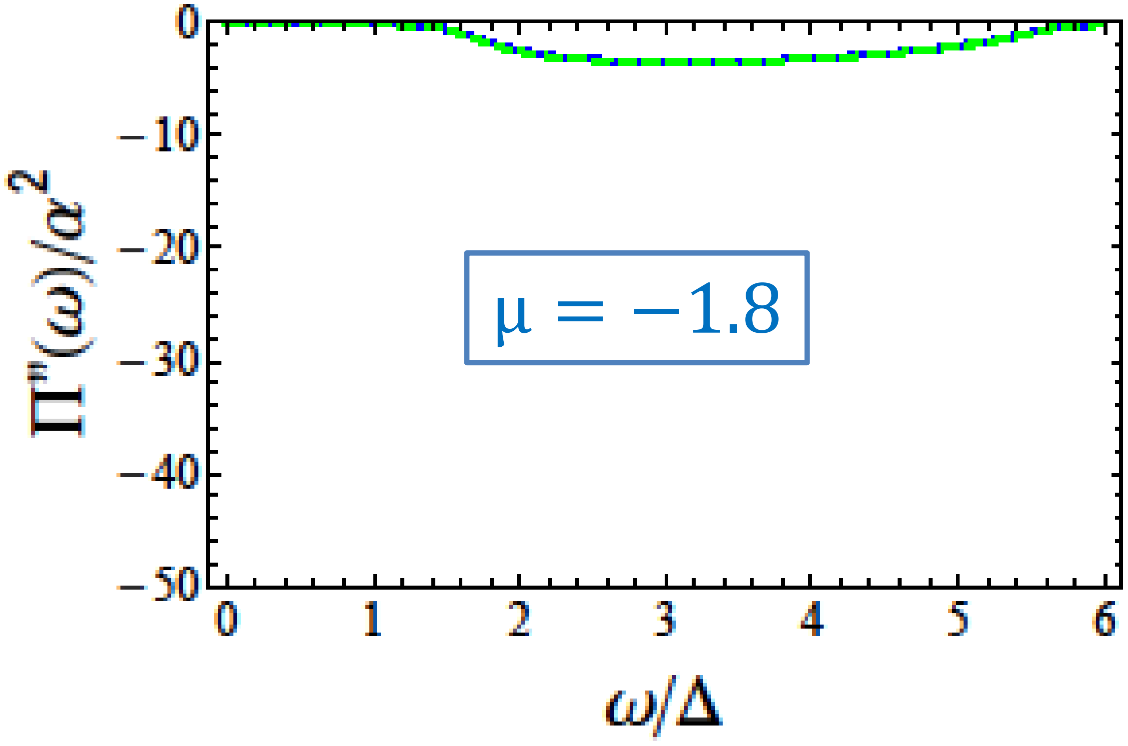
Non topological region (periodic boundary conditions)



O. Dmytruk, M. Trif, PS, arXiv:1502.03082

Results for the charge susceptibility

Non topological region (Open boundary conditions)



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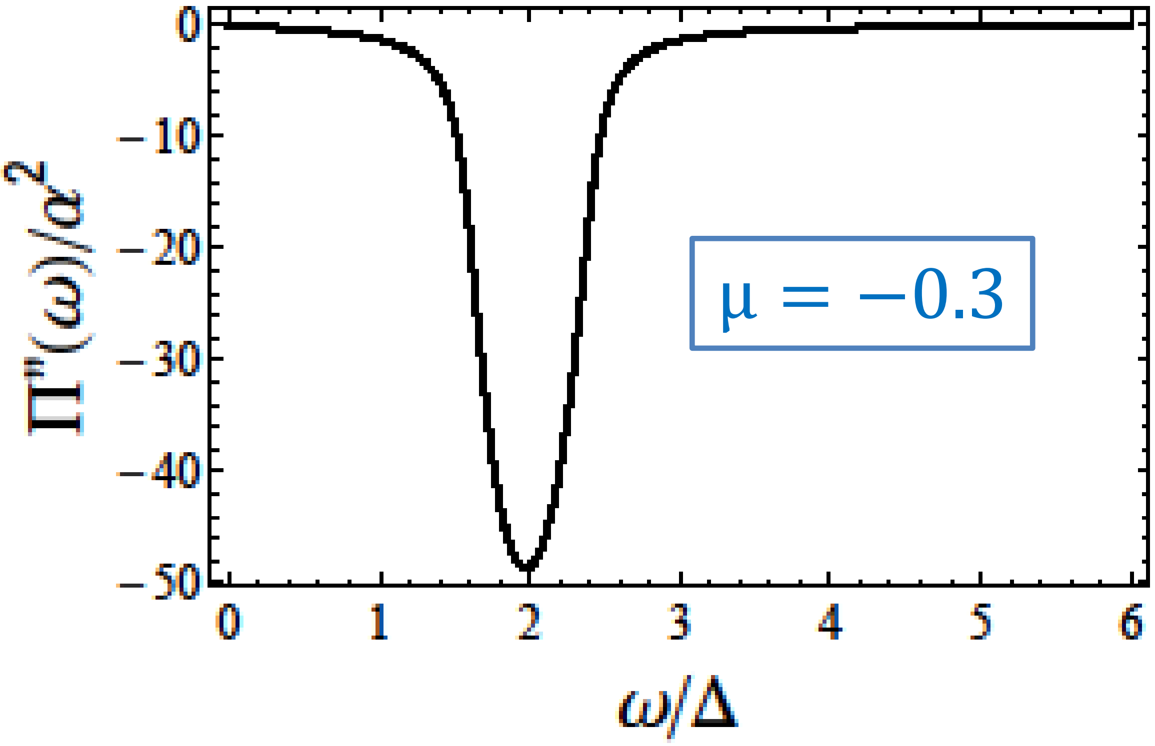

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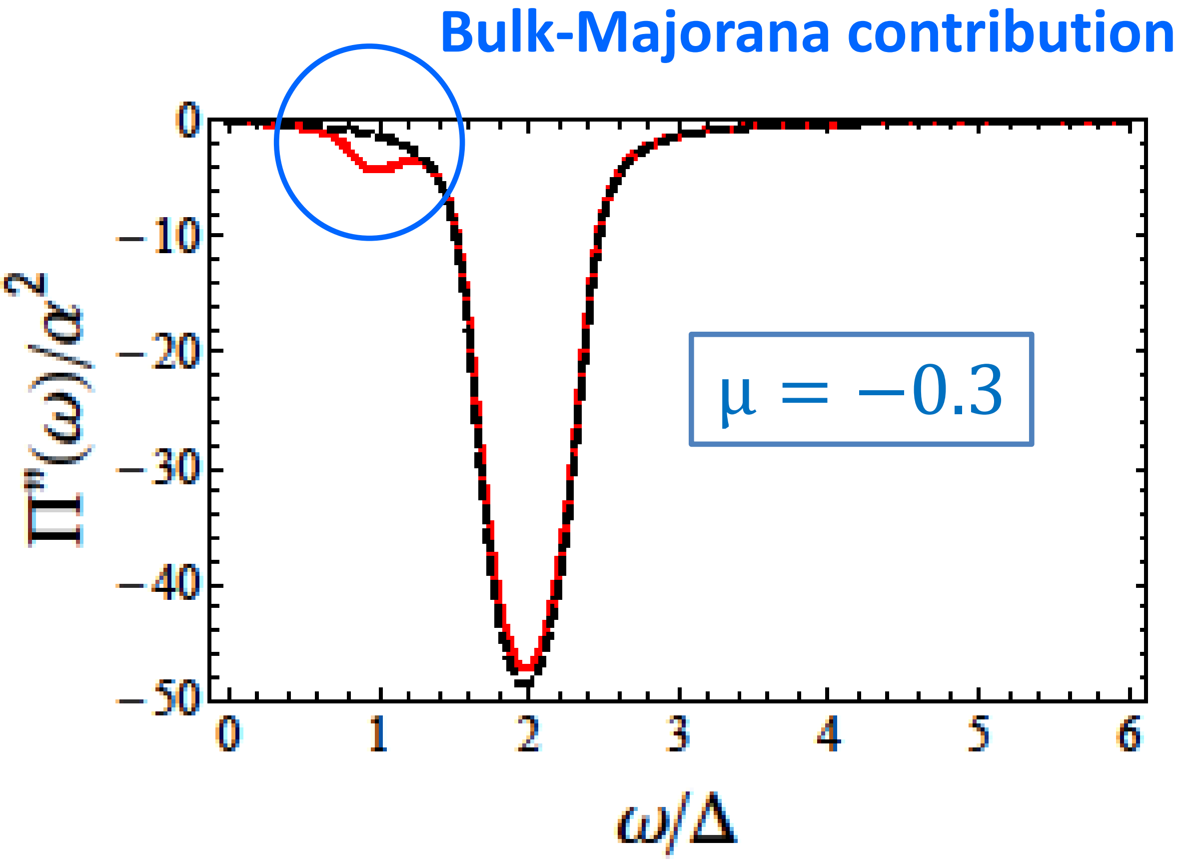
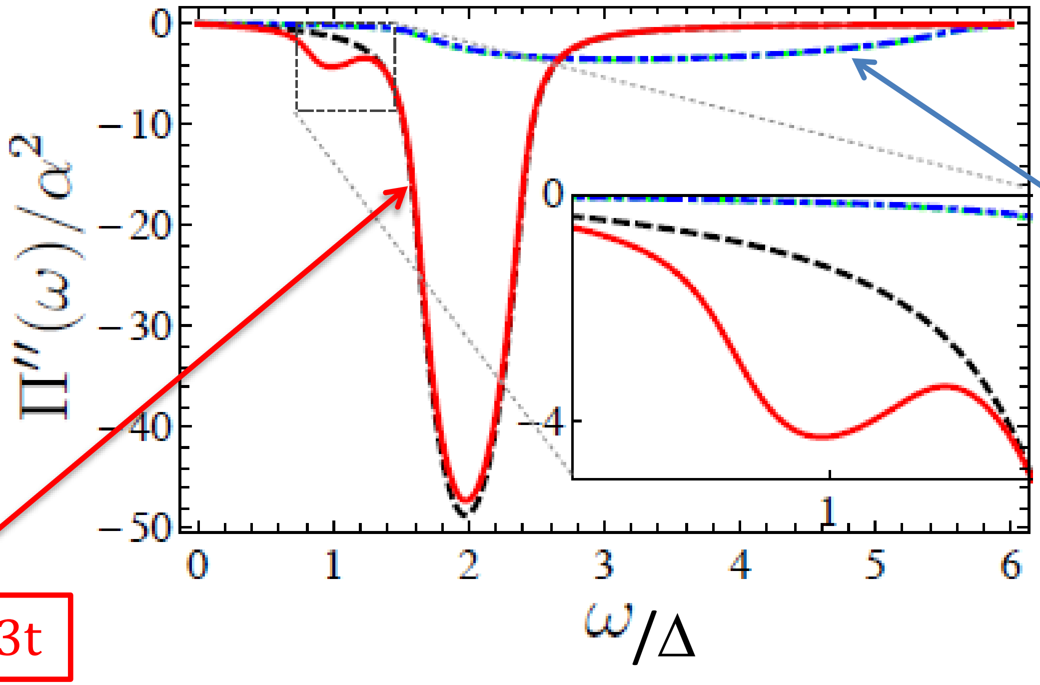


O. Dmytruk, M. Trif, PS, arXiv:1502.03082

Frequency dependence of the charge susceptibility

$$\Pi(\omega) = \Pi_{BB}(\omega) + \Pi_{BM}(\omega) + \underset{\overset{\|}{0}}{\Pi_{MM}(\omega)}$$



μ = −1.8t

μ = −0.3t

O. Dmytruk, M. Trif, PS, arXiv:1502.03082

A closer look at the Bulk-Majorana term

$$\Pi_{BM}(\omega) = \sum_{p \neq M} \left(\frac{1}{\epsilon_p + \omega + i\eta} + \frac{1}{\epsilon_p - \omega - i\eta} \right)$$

$$\times \left[|C^{(1)}_{Mp}|^2 (n_M - n_p) - |C^{(2)}_{Mp}|^2 (n_M - 1 + n_p) \right]$$

n_p = bulk occupation

n_M = Majorana state occupation

➡ The bulk-Majorana contribution depends on the Majorana parity!

$$C^{(1)}_{Mp} = \sum_j [u^j_M \; u^j_p - v^j_M \; v^j_p]$$

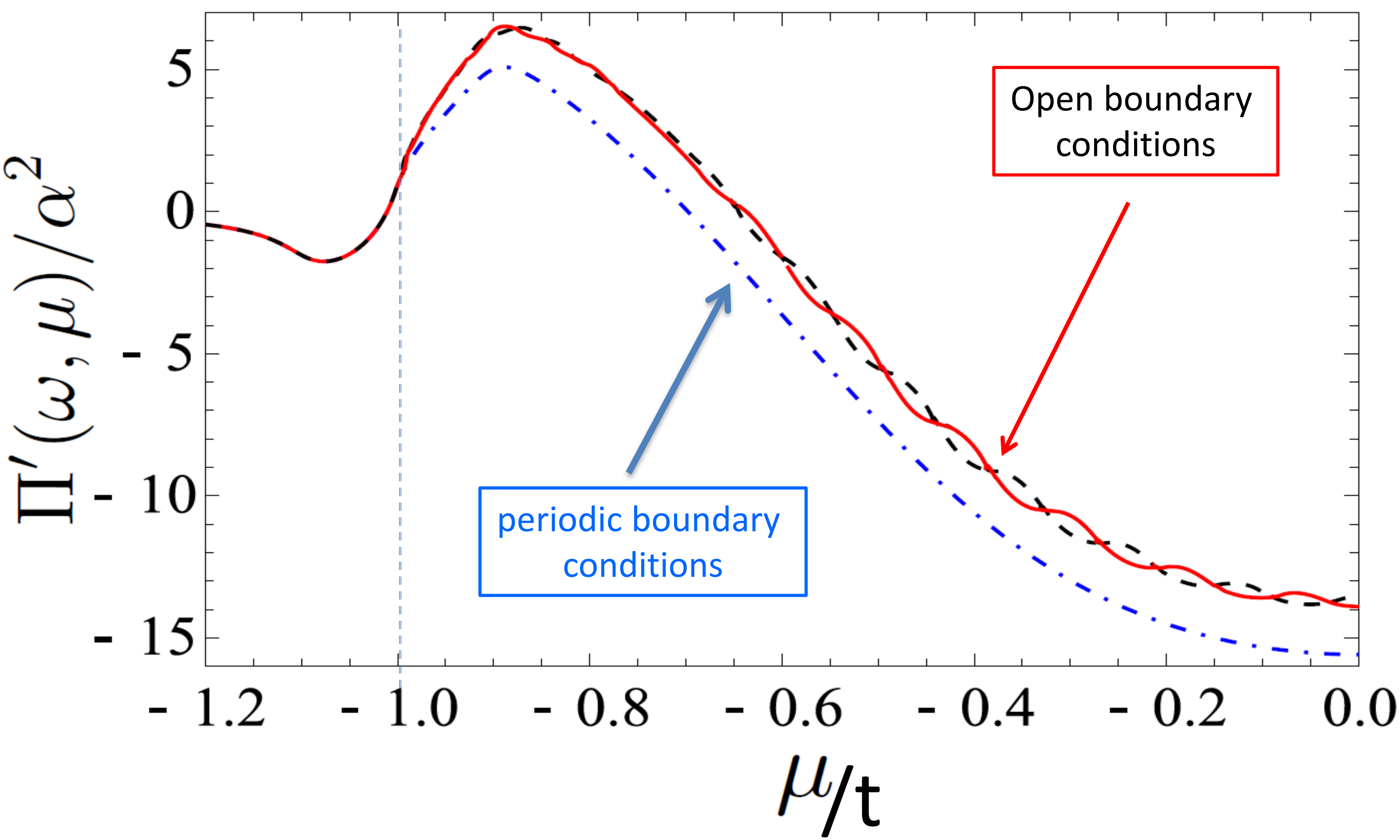
VS

$$C^{(2)}_{Mp} = \sum_j [v^j_M \; u^j_p - u^j_M v^j_p]$$

$$\epsilon_M = 0 \Rightarrow u^j_M = v^j_M \Rightarrow C^{(1)}_{Mp} = C^{(2)}_{Mp}$$

$$\epsilon_M \neq 0 \Rightarrow u^j_M \neq v^j_M \Rightarrow C^{(1)}_{Mp} \neq C^{(2)}_{Mp}$$

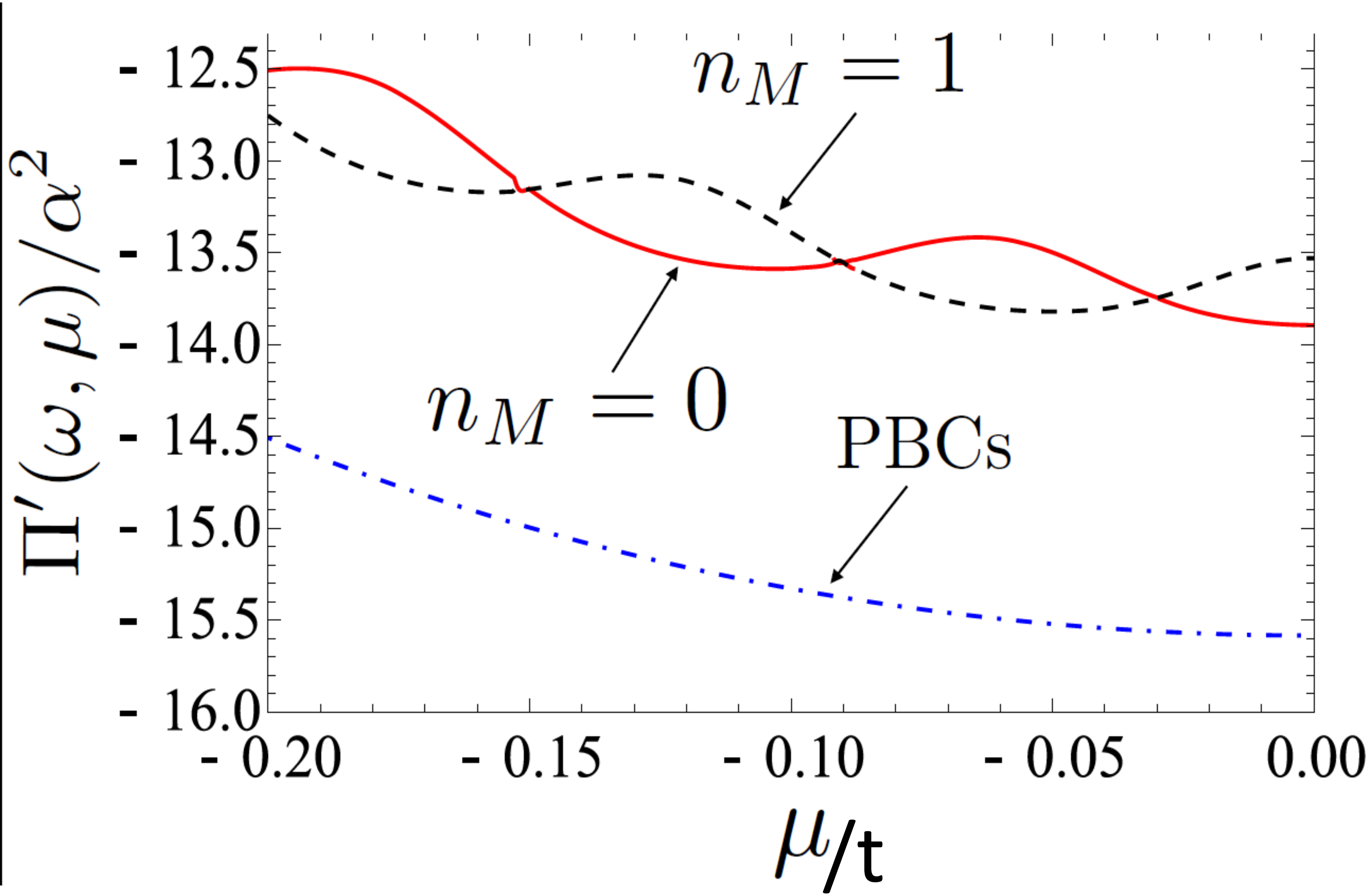
Cavity as a parity sensor



Open boundary conditions

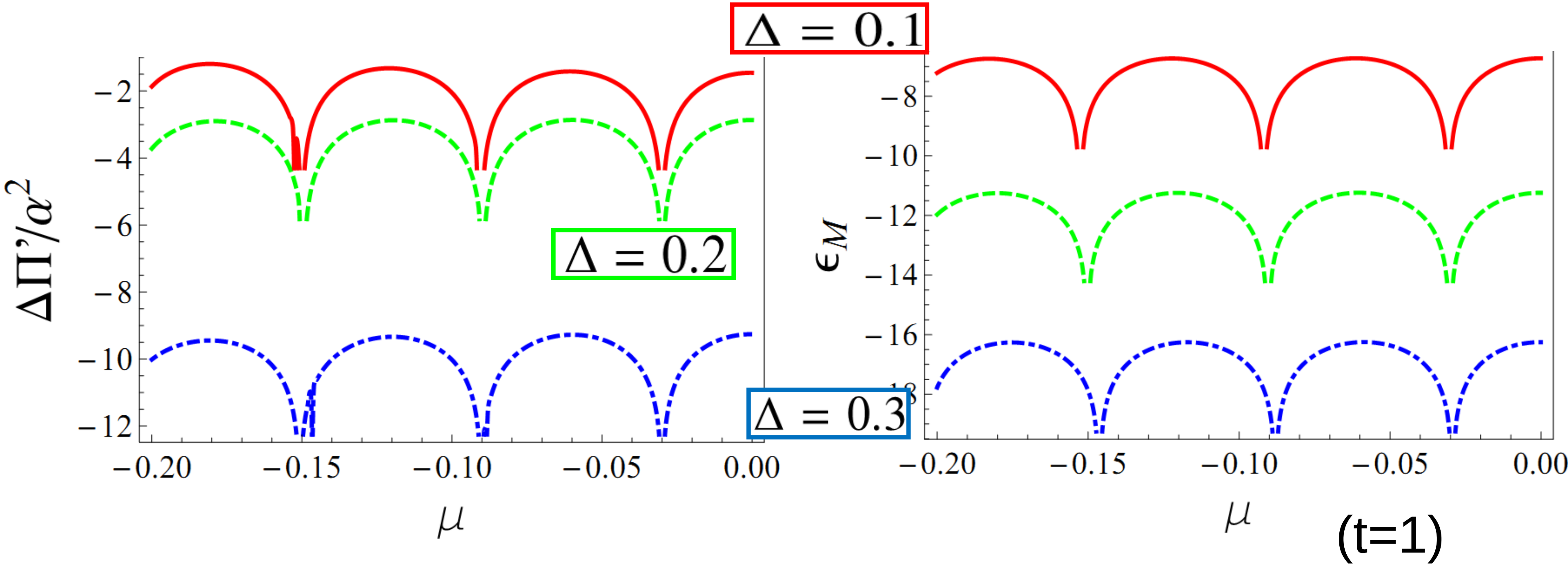
periodic boundary conditions

Cavity as a parity sensor



O. Dmytruk, M. Trif, PS, arXiv:1502.03082

Detection of the Majorana state parity



(t=1)

$$\Delta\Pi = 2\left|(\Pi^{+}_{BM} - \Pi^{-}_{BM})/(\Pi^{+}_{BM} + \Pi^{-}_{BM})\right|$$

$$\epsilon_M \sim \exp\left(-N/\xi\right)\left|\cos\left(k_F N\right)\right|$$

The cavity response is in one-to-one correspondence with the Majorana energy splitting.

Extension to more realistic wire model

$$H_{el} = \sum_{m=1}^{N} \sum_{b} \epsilon_{m,b} \left(\tilde{c}^{\dagger}_{m,b} \tilde{c}_{m,b} - \frac{1}{2} \right)$$

Band index

$$H_{el-c} = \sum_{p,p'} \sum_{b,b'} \left[C^{(1)}_{pb,p'b'} \tilde{c}^{\dagger}_{p,b} \tilde{c}_{p'b'} - i C^{(2)}_{pb,p'b'} \tilde{c}^{\dagger}_{p,b} \tilde{c}^{\dagger}_{p'b'} + h.c. \right] (a + a^{\dagger})$$

$$\boxed{\begin{aligned} \Pi_{BM}(\omega) = & \sum_{p \neq M} \sum_{b} \left(\frac{1}{\epsilon_{p,b} + \omega + i\eta} + \frac{1}{\epsilon_{p,b} - \omega - i\eta} \right) \\ & \times \left[|C^{(1)}_{M,pb}|^2 (n_M - n_p) - |C^{(2)}_{M,pb}|^2 (n_M - 1 + n_p) \right] , \end{aligned}}$$

$$C^{(1/2)}_{M,pb} = f(B, \lambda_{SO}, \Delta, \mu)$$

Conclusion

Cavity QED is an extremely versatile tool that allows to detect:

- the topological phase transition
- the occurrence of Majorana fermions
- the parity of the Majorana fermionic state

Special thanks to

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