

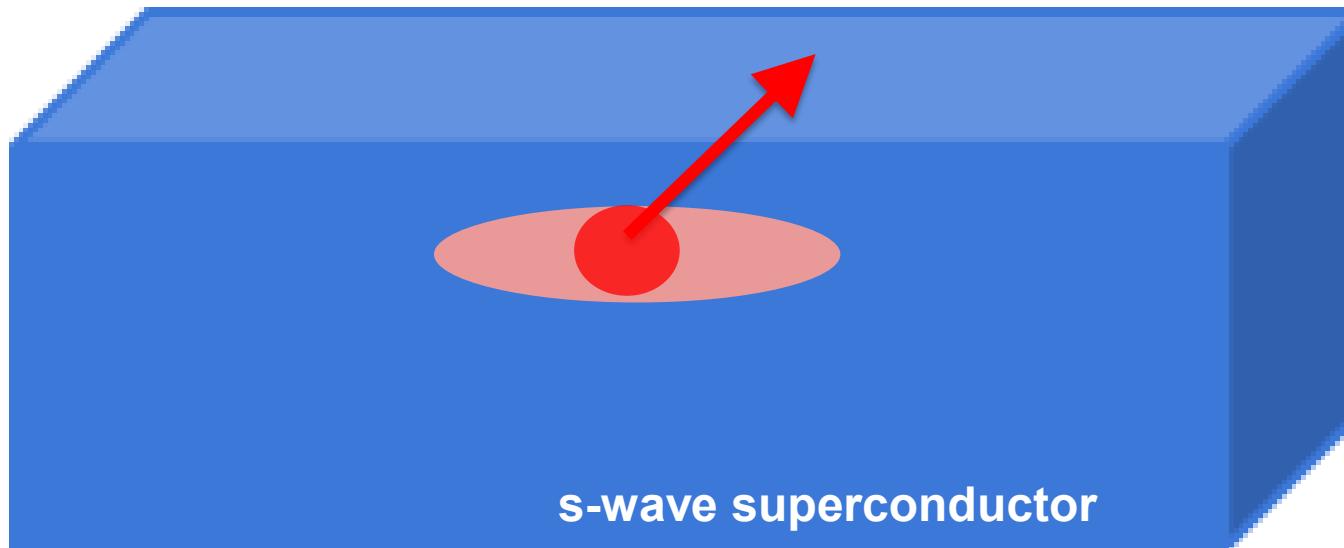
# Exotic physics induced by magnetic moments in a superconductor

Pascal Simon  
University Paris Saclay



# Why studying magnetic impurities in a superconductor ?

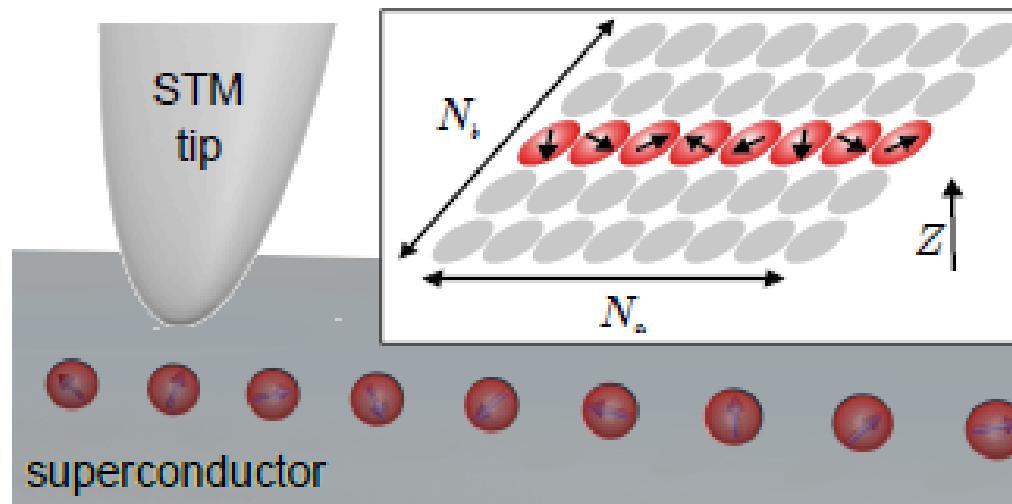
Atom with a magnetic moment:  
**Fe, Co, Mn**



**An old solved problem ?**

# Why studying magnetic impurities in a superconductor ?

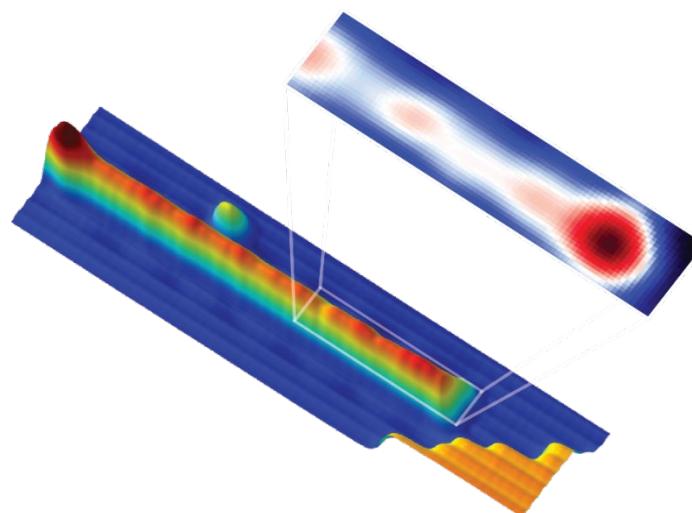
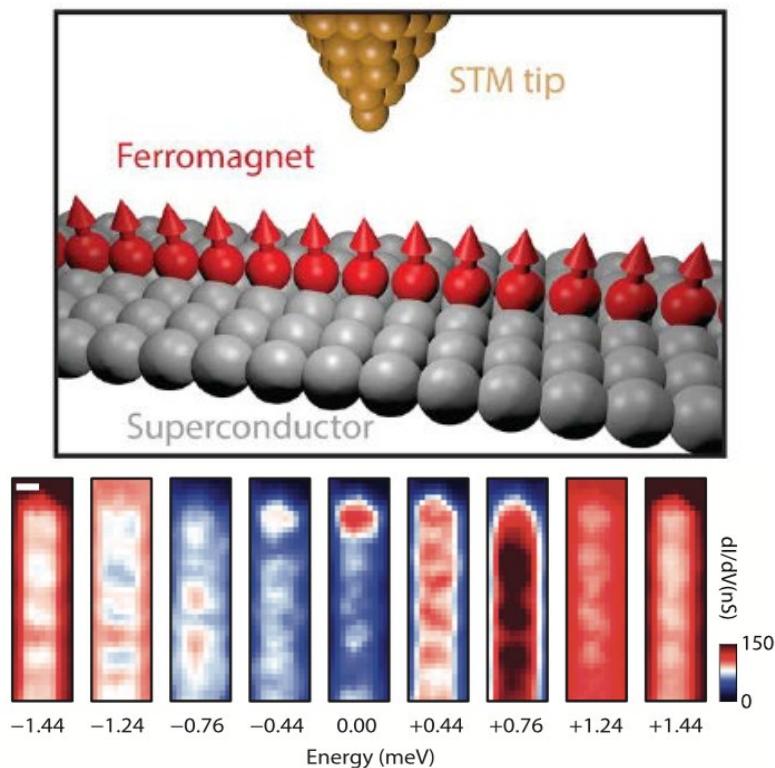
- Chains of magnetic atoms on a superconductor: a new platform for Majorana physics ?



Nadj-Perge, Drozdov, Bernevig, Yazdani, PRB (2013)

# Majorana end states in magnetic chains : Fe/Pb(110)

Zero-bias anomaly localized on  
the last atoms of the Fe chain,  
almost no extension into the Pb  
substrate



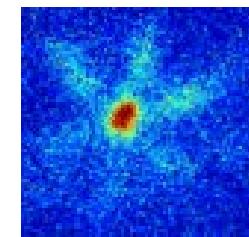
S. Nadj-Perge et al., Science **346**, 6209 (2014)  
B. E. Feldman et al., Nature Physics (2016)  
S. Jeon et al., Science (2017) (Princeton)

see also

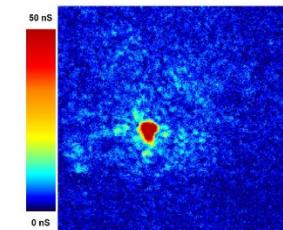
M. Ruby et al., PRL 2015 (Berlin)  
R. Pawlak et al., NPJ QI (2016) (Basel)  
H. Kim et al. Science Advances (2018) (Hamburg)

# Outline

I) Shiba states in a 2D superconductor:  
large real space extent



II) Odd-frequency pairing around a magnetic  
impurity in a superconductor



III) Shot noise tomography of bound states  
in (topological) superconductors

**I) Shiba state in a 2D  
superconductor:  
Large real space extent**

# A magnetic impurity in a superconductor



Bogoliubov-de Gennes Ham.  $\mathcal{H}_0 = \xi_p \tau_z + \Delta \tau_x$

Assumes  $\Delta$  constant  
(homogeneous)

$$\mathcal{H}_{\text{imp}} = -JS \cdot \sigma \delta(\mathbf{r})$$



$$\mathcal{H}_{\text{imp}} = -JS\sigma^z \delta(\vec{r})$$

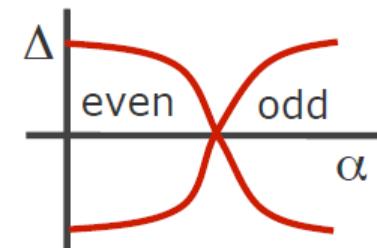
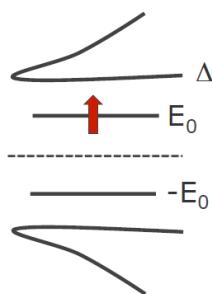
Like a local magnetic field

It creates an attractive potential

**Shiba  
bound state**

$$E_0 = \Delta \frac{1-\alpha^2}{1+\alpha^2}$$

$$\alpha = \pi v_0 JS$$



Yu Lu (1965), Shiba (1968),  
Rusinov (1969)

# Asymptotics of the Shiba wave-function

Convenient parametrization  $E = \Delta \cos(\delta^+ - \delta^-)$

$$\tan \delta^\pm = V\nu_0 \pm \pi\nu_0 JS$$

Rusinov  
(1969)

Potential scattering term

Asymptotics of the Shiba wave

In 3D

$$\psi_\pm(r) \approx \frac{1}{\sqrt{\mathcal{N}}} \frac{\sin(k_F r + \delta^\pm)}{k_F r} e^{-\Delta \sin(\delta^+ - \delta^-)r/\hbar v_F}$$

Rusinov  
(1969)

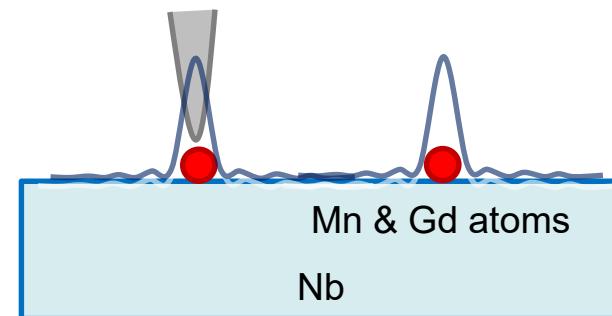
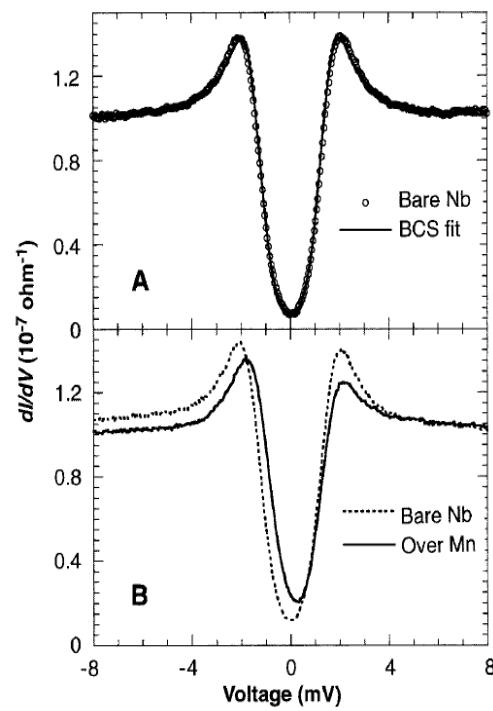
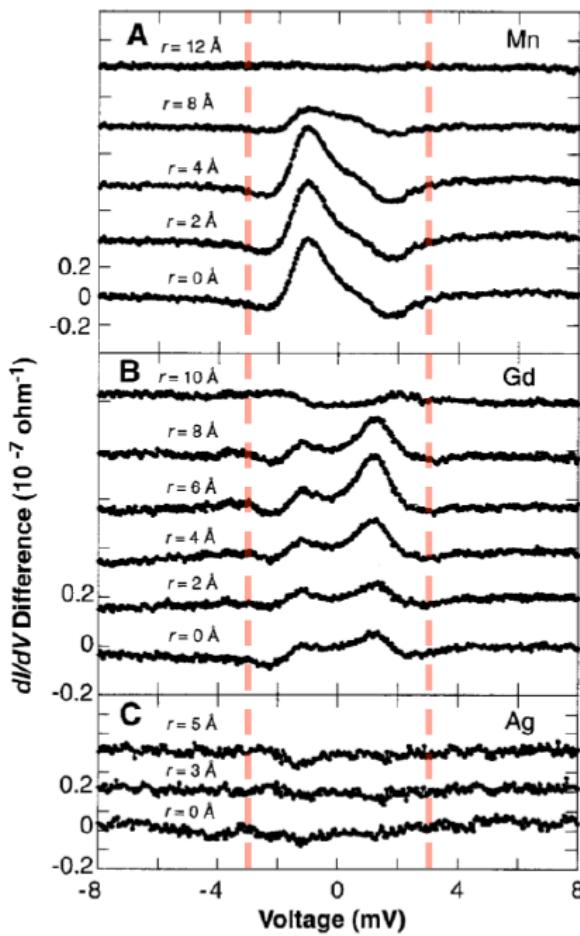
In 2D

$$\psi_\pm(r) \approx \frac{1}{\sqrt{\mathcal{N}}} \frac{\sin(k_F r - \frac{\pi}{4} + \delta^\pm)}{\sqrt{\pi k_F r}} e^{-\Delta \sin(\delta^+ - \delta^-)r/\hbar v_F}$$

Can we test these predictions experimentally ?

# Single magnetic impurities observed by STM

- Bound states for magnetic impurities (Mn & Gd) on Nb
- No bound states for non magnetic Ag adatoms on Nb



The wave function of the bound states is localized at less than 10 Å from the impurities

# Single magnetic impurities observed by STM: Angular momentum dependence

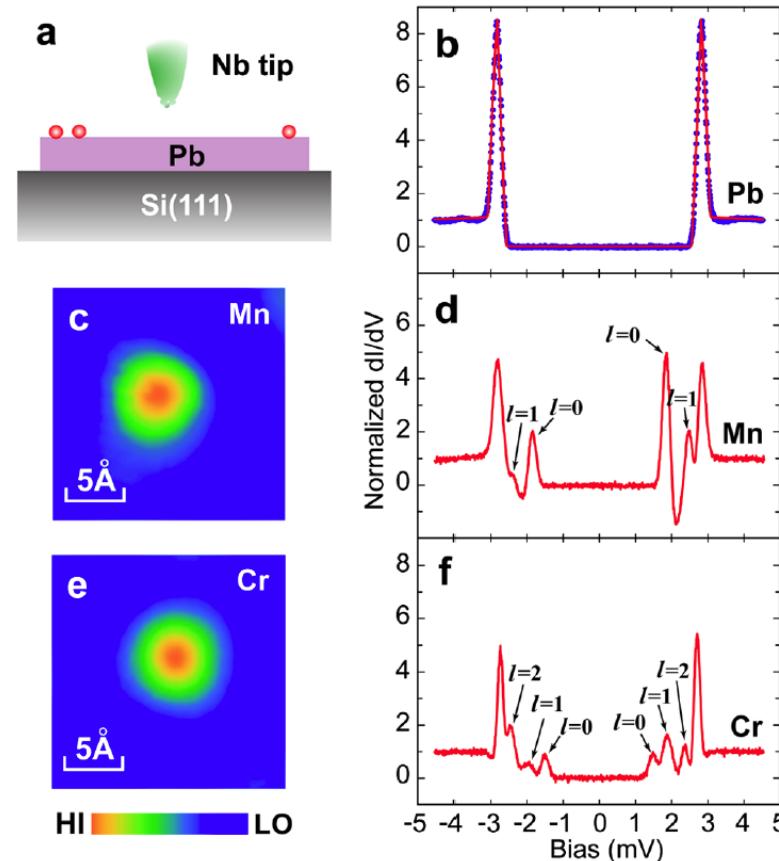
The number of Shiba peaks depends on the atom nature

$$Mn \rightarrow l = 0, 1$$

$$Cr \rightarrow l = 0, 1, 2$$

Every peak may correspond to a different angular momentum scattering channels for the superconducting electrons.

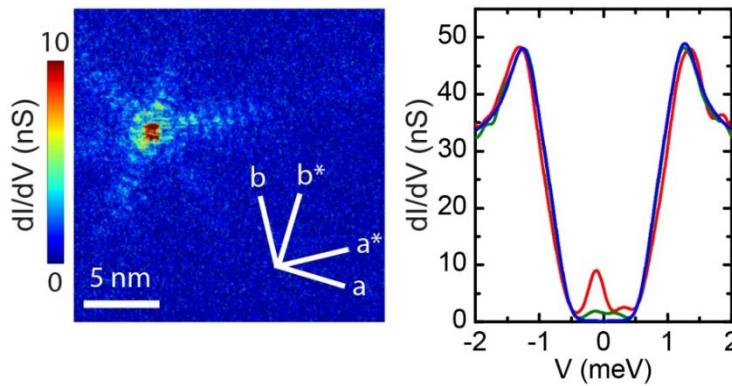
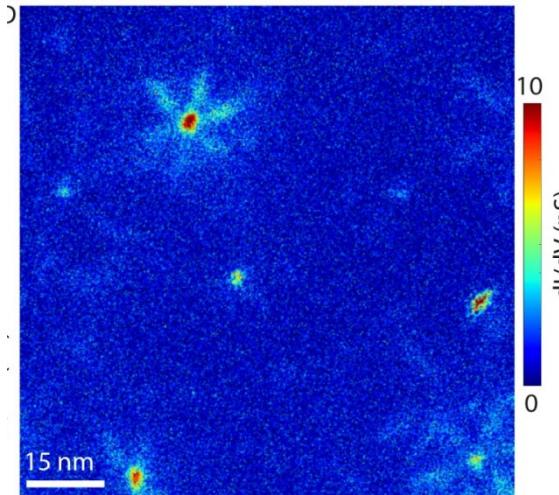
Extremely local effect of the impurities (a few Å)



Shuai-Hua Ji et al. *PRL* **100**, 226801 (2008)

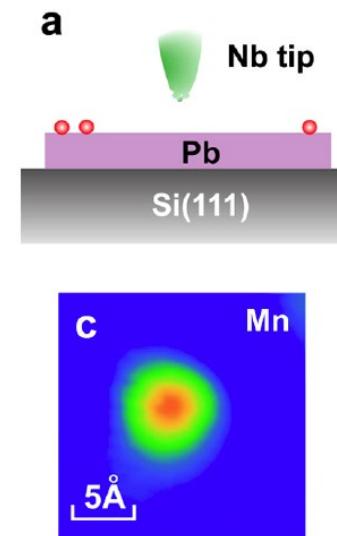
See also N. Hatter et al., *Nature Commun.* (2015)

# Observation of bound states around magnetic impurities in 2H-NbSe<sub>2</sub>



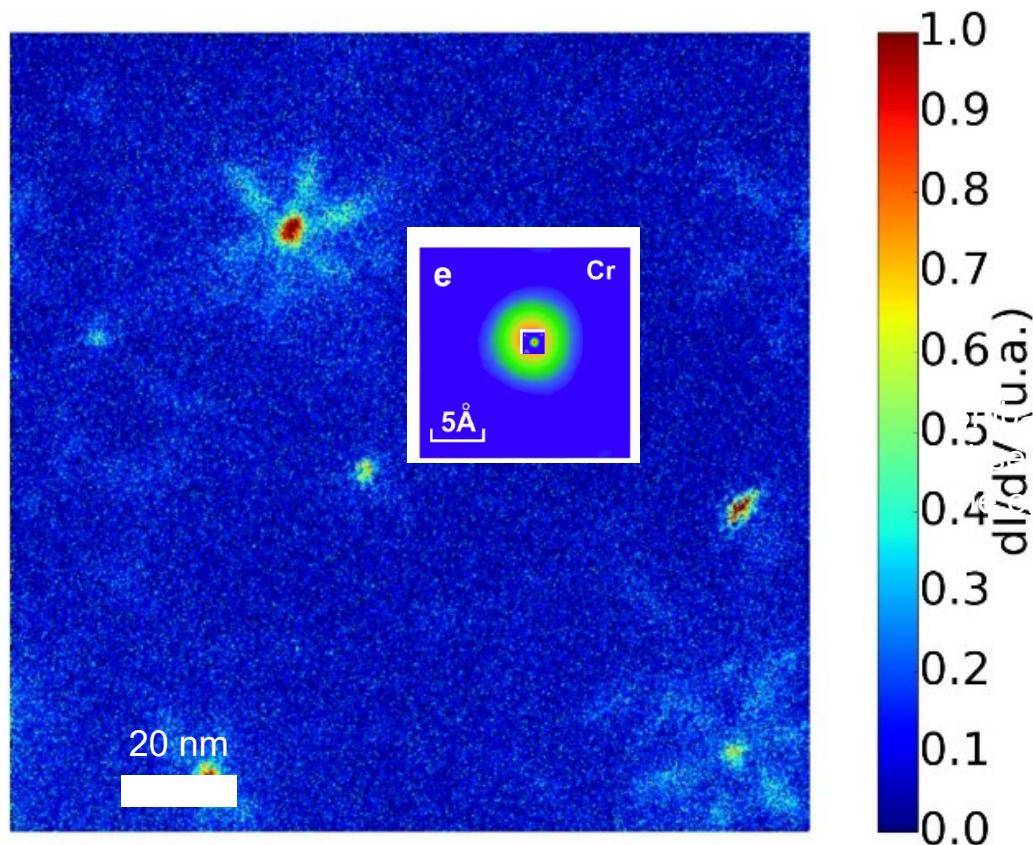
Nb used for the crystal growth contains magnetic impurities :

- 175 ppm of Fe
- 54 ppm of Cr
- 22 ppm of Mn



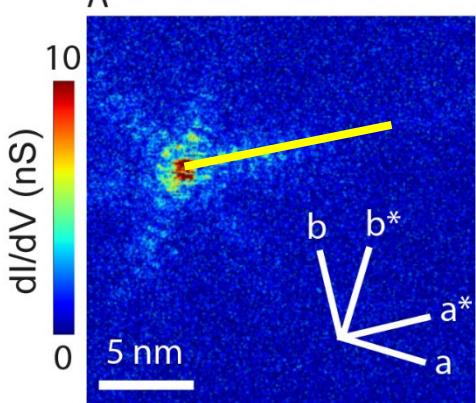
G. Ménard et al., Nature Physics 2015

# Observation of bound states around magnetic impurities in 2H-NbSe<sub>2</sub>

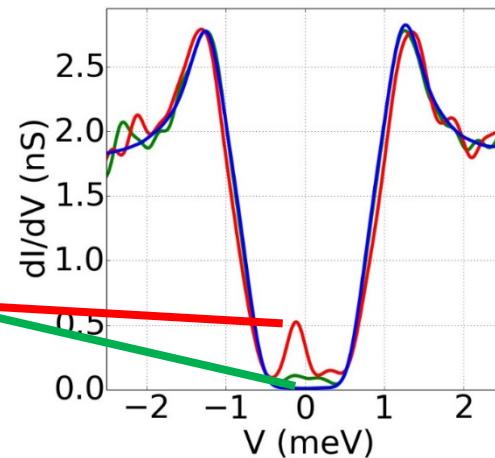
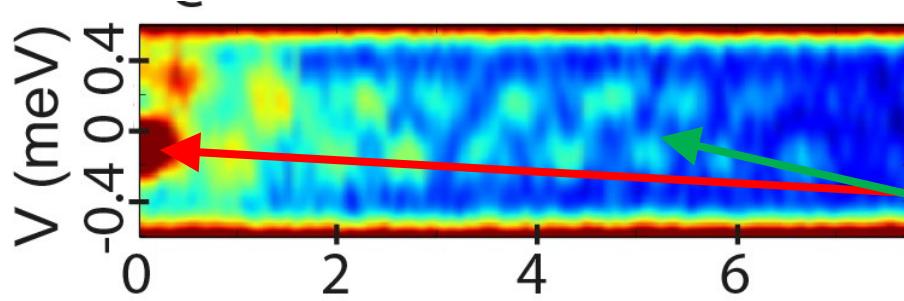


G. Ménard et al., Nature Physics 2015

# Spatial oscillation of Shiba bound states Electron-hole asymmetry

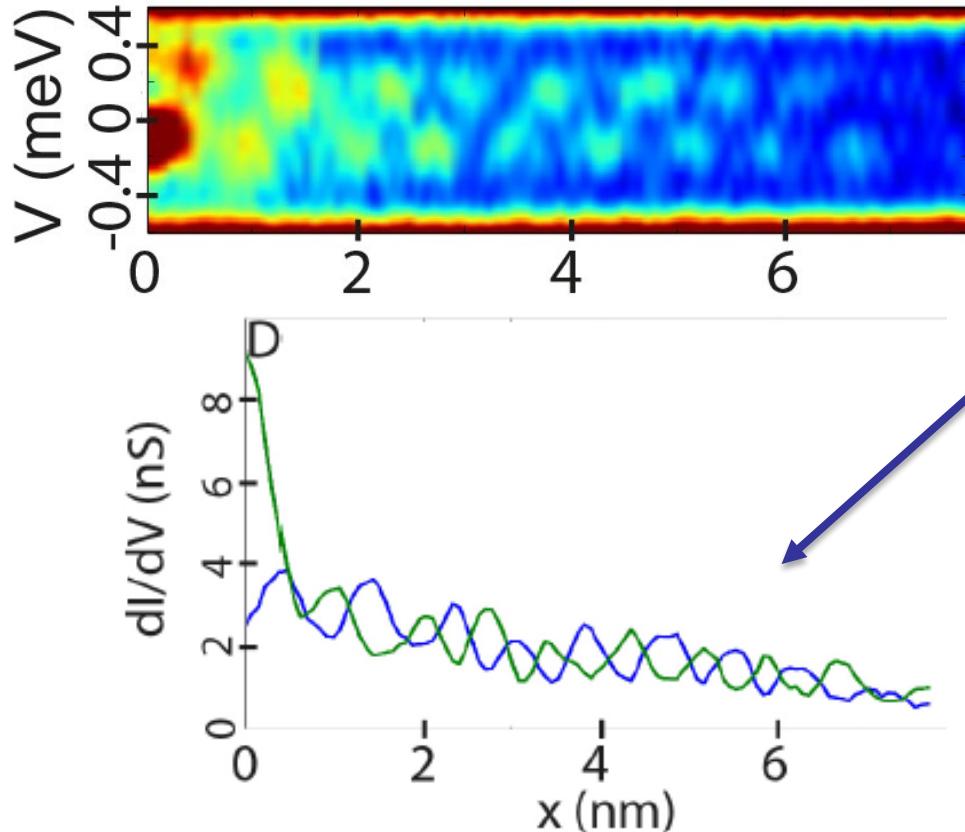


- Oscillations of the local density of states with a phase opposition between positive and negative energy states
- Decrease of the Shiba bound states on a size of the order of the coherence length  $\xi$



G. Ménard et al., Nature Physics 2015

# Spatial oscillations and electron-hole asymmetry



Good agreement with theoretical calculations for Shiba state in a **2D SC** in the asymptotic limit.

$$\psi_{\pm}(r) = \frac{1}{\sqrt{N\pi k_F r}} \sin\left(k_F r - \frac{\pi}{4} + \delta^{\pm}\right) e^{-\Delta \sin(\delta^+ - \delta^-) r / \hbar v_F}$$

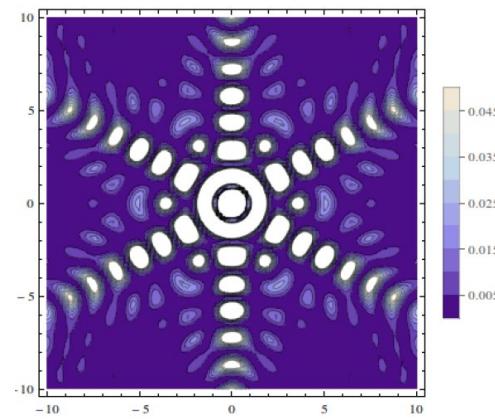
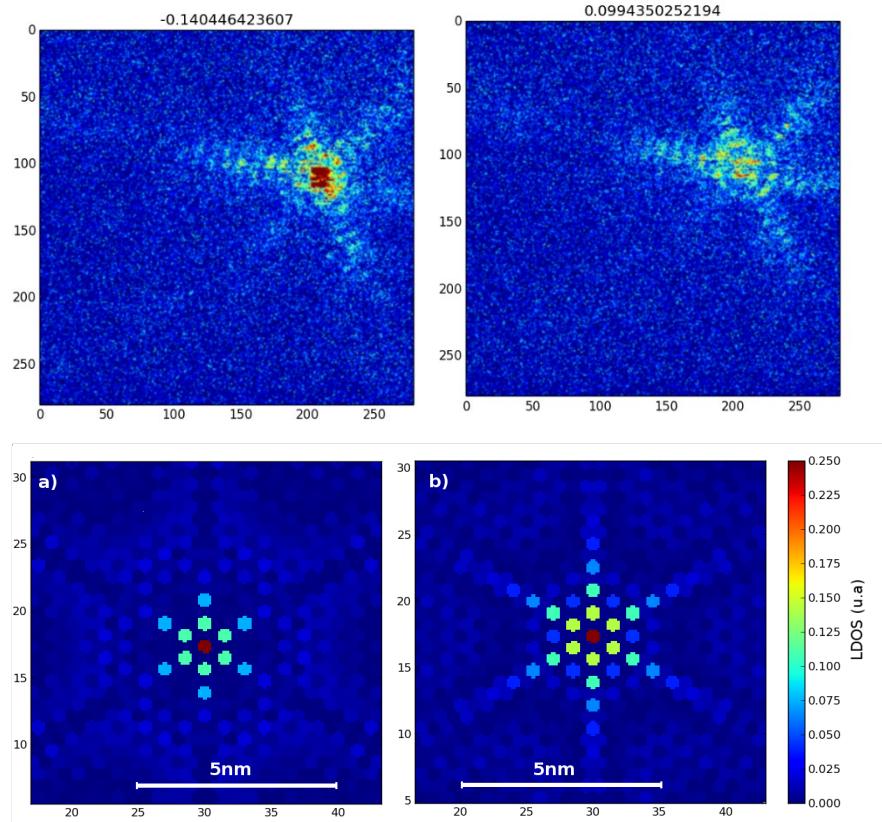
$$E = \Delta \cos(\delta^+ - \delta^-)$$

$$\tan \delta^{\pm} = (K\nu_0 \pm \nu_0 JS/2)$$

The Shiba peaks **position relatively to the gap** is directly related to the phase shift.

# Theoretical modelling

numerical resolution of the Shiba equation using a  
a Bogoliubov-de Gennes tight-binding model extracted from ARPES



Continuous asymptotic model

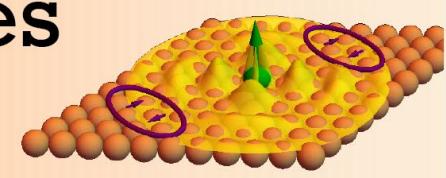
**The star shape comes  
from the anisotropy of  
the Fermi surface**

## Anisotropy of Yu-Shiba-Rusinov states

Mateo Uldemolins\*, Andrej Mesaros and Pascal Simon

Laboratoire de Physique des Solides, Université Paris-Saclay, CNRS, 91405, Orsay, France

\*mateo.uldemolins-nivela@universite-paris-saclay.fr



## **II) Odd-frequency pairing around a magnetic impurity**

# Even/odd-frequency pairing

Key quantity: the pairing correlation is defined by:

$$F_{\alpha,\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = -i \langle \mathcal{T} \psi_\alpha(\mathbf{r}_1 t_1) \psi_\beta(\mathbf{r}_2 t_2) \rangle$$

Under exchange of time:

Conventional even-frequency pairing:

$$F_{\alpha,\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = F_{\alpha,\beta}(\mathbf{r}_1 t_2, \mathbf{r}_2 t_1)$$

**odd-frequency pairing :**

$$F_{\alpha,\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2) = -F_{\alpha,\beta}(\mathbf{r}_1 t_2, \mathbf{r}_2 t_1)$$

# Symmetry of the pairing function

Orbital  $\otimes$  Spin  $\otimes$  Frequency = Odd

Several options

	Frequency (time)	Spin	Orbital	Total	
ESE	+ (even)	- (singlet)	+ (even)	-	BCS; cuprates
ETO	+ (even)	+ (triplet)	- (odd)	-	P-wave SC => Majoranas
OTE	- (odd)	+ (triplet)	+ (even)	-	
OSO	- (odd)	- (singlet)	- (odd)	-	

See e.g. Y. Tanaka, M. Sato, N. Nagaosa, J. Phys. Soc. Jpn. 81, 011013 (2012)  
J. Linder, A. V. Balatsky, Rev. Mod. Phys. 91, 045005 (2019)

# A long history....

Odd-frequency pairing was proposed

- First by Berezinskii in 74' (superfluid He)
- As a purely **intrinsic** electronic mechanism to generate bulk odd- $\omega$  spin triplet pairing
  - Belitz, Kirkpatrick, 91',92'
  - Balatsky, Abrahams, 91',92', etc.
- In the Kondo lattice and heavy fermions
  - Zachar, Emery, Kivelson, 96', Coleman, Miranda, Tsvelik, 97'; etc.
- In hybrid SF junctions, odd- $\omega$  pair amplitudes are induced in a ferromagnet in contact with a spin-singlet s-wave superconductor
  - Bergeret, Efetov, Volkov, 01'

See e.g.

Y. Tanaka, M. Sato, N. Nagaosa, J. Phys. Soc. Jpn. 81, 011013 (2012)

J. Linder, A. V. Balatsky, Rev. Mod. Phys. 91, 045005 (2019)

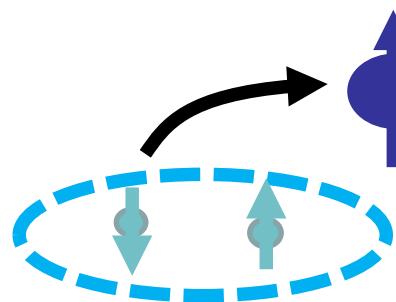
1D case: J. Cayao, C. Triola, A. M Black-Schaffer, The European Physical Journal Special Topics, 229, 545 (2020)

# Where to find it ?

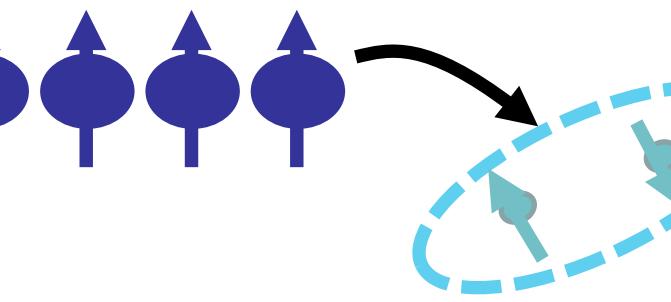
Actually, odd- $\omega$  pairing state is rather ubiquitous:

**Starting from conventional s-wave even- $\omega$  superconductivity,  
any system with broken spin rotational symmetry OR broken  
translation symmetry can induce odd- $\omega$  pairing states.**

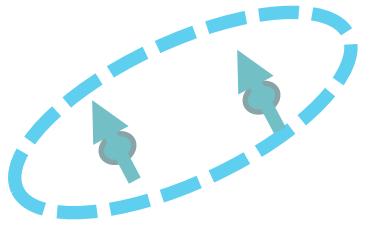
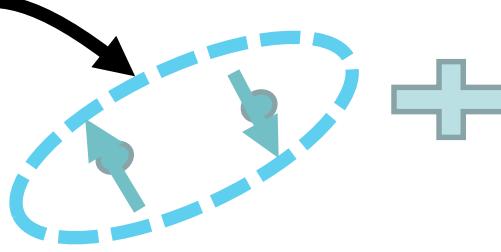
→ This involves inhomogeneous systems, S-F junctions, hybrid systems....



Even- $\omega$  singlet  
pairing

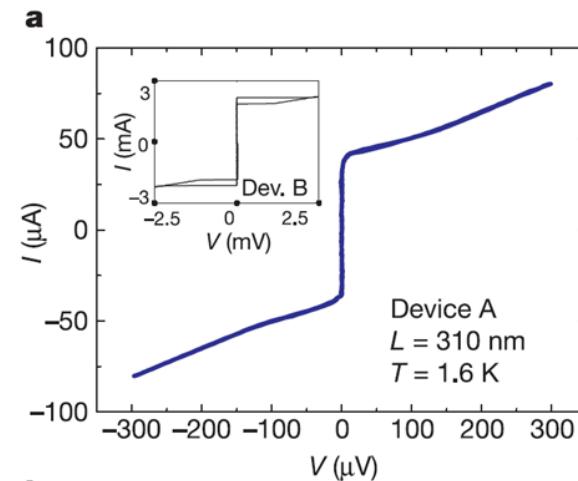
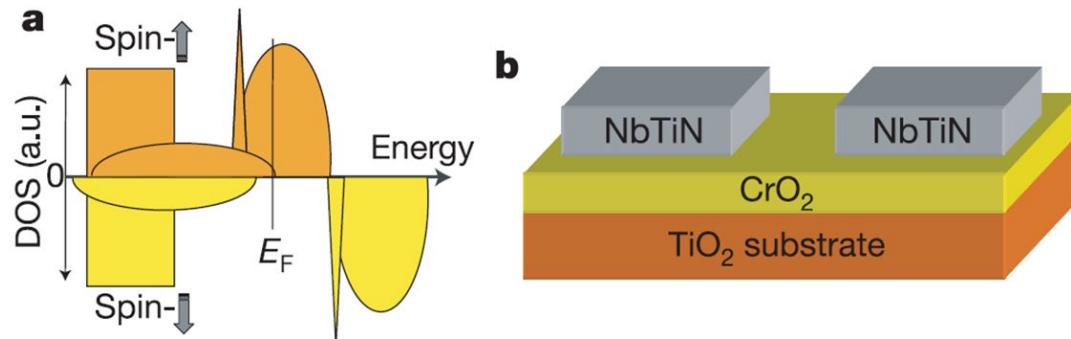


Even- $\omega$  singlet  
pairing



Odd- $\omega$  triplet  
pairing

# Odd-w pairing in ferromagnet-superconductor hybrid systems



Long range proximity effect in a S-F-S junction as a direct manifestation of odd triplet s-wave pairing

R. S. Keizer et al., Nature 439, 825 (2006)

# Odd-Frequency Pairing on the impurity site

$$\begin{aligned}\hat{G}^R(t, t') &= -i\theta(t - t') \langle \Psi(\mathbf{0}, t) \Psi^\dagger(\mathbf{0}, t') \rangle \\ &= \begin{bmatrix} G_{\uparrow}^R(t - t') & F_{\uparrow,\downarrow}^R(t - t') \\ -F_{\downarrow,\uparrow}^R(t - t')^* & -G_{\downarrow}^R(t - t')^* \end{bmatrix}\end{aligned}$$

**Retarded Green function  
in Nambu space**

**Dyson equation :**  $[\hat{G}^R]^{-1}(\omega) = [g^R(\omega)]^{-1} - \Sigma$

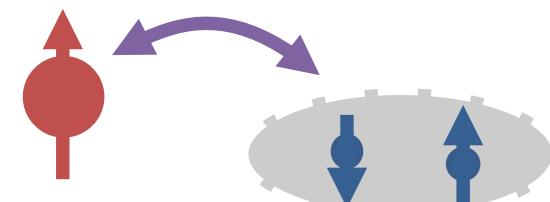
$$g^R(\omega) = \begin{bmatrix} g_{\uparrow}^R(\omega) & f_{\uparrow,\downarrow}^R(\omega) \\ f_{\downarrow,\uparrow}^R(\omega) & -(g_{\downarrow}^R(-\omega))^* \end{bmatrix} \quad \hat{\Sigma} = \begin{bmatrix} V - J - i\Gamma & 0 \\ 0 & -(V + J) - i\Gamma \end{bmatrix}$$

**Width of the bound state**

**Two possibilities:**

1.  $F_{\uparrow,\downarrow}^R(\omega) = F_{\uparrow,\downarrow}^{R*}(-\omega)$  Even  $\omega$ ; spin singlet,

2.  $F_{\uparrow,\downarrow}^R(\omega) = -F_{\uparrow,\downarrow}^{R*}(-\omega)$  Odd  $\omega$ ; spin triplet.



# Odd-Frequency Pairing on the impurity site

Local density of states on the impurity:

$$\rho_{\text{even/odd}}(\omega) = [\rho(\omega) \pm \rho(-\omega)]/2$$

$$|\omega| < \Delta$$

$$\rho_{\text{even/odd}}(\omega) = C_{e/o}(E_0) \times \Im F_{\text{odd/even}}^R(\omega)$$

**General proportionality relation between  
LDOS and the odd- $\omega$  pairing fuction**

# Odd-Frequency Pairing on the impurity site

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$$\rho_{\text{even/odd}}(\omega) = [\rho(\omega) \pm \rho(-\omega)]/2$$

$$|\omega| < \Delta$$

$$\rho_{\text{even/odd}}(\omega) = C_{e/o}(E_0) \times \Im F_{\text{odd/even}}^R(\omega)$$

Shiba bound state:

$$E_0 = \Delta \frac{1 - \alpha^2 + \beta^2}{\sqrt{(1 - \alpha^2 + \beta^2)^2 + 4\alpha^2}} \quad \begin{aligned} \alpha &= \pi\nu_0 J \\ \beta &= \pi\nu_0 V \end{aligned}$$



$$\begin{aligned} C_e(E_0) &= -\frac{2}{\Delta} [E_0 + \pi J \nu_0 \sqrt{\Delta^2 - E_0^2}] \\ &= -\frac{2}{\pi} \frac{1 + \beta^2 + \alpha^2}{\sqrt{(1 - \alpha^2 + \beta^2)^2 + 4\alpha^2}} \end{aligned}$$

# Concrete protocol to extract the odd- $\omega$ pairing

Assuming a constant DOS in the normal regime

$$\hat{G}(\omega) = \frac{1}{\omega - E_0 + i\Lambda} \begin{bmatrix} u^2 & uv \\ uv & v^2 \end{bmatrix} \quad u^2, v^2 = 2\pi\alpha\nu_0\Delta \frac{1 + (\alpha \pm \beta)^2}{((1 - \alpha^2 + \beta^2)^2 + 4\alpha^2)^{3/2}}$$

→  $\rho(\omega) = \frac{\Lambda u^2/\pi}{(\omega - E_0)^2 + \Lambda^2} + \frac{\Lambda v^2/\pi}{(\omega + E_0)^2 + \Lambda^2}$

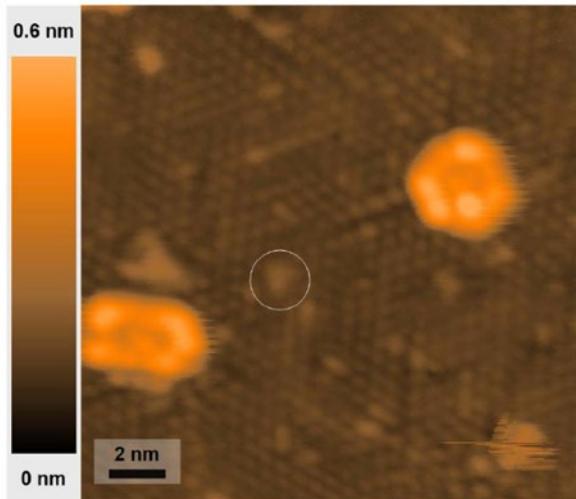
M. Ruby et al., Phys. Rev. Lett. 115, 087001(2015)

→  $C_e(E_0) = -\frac{u^2 + v^2}{\pi uv}$

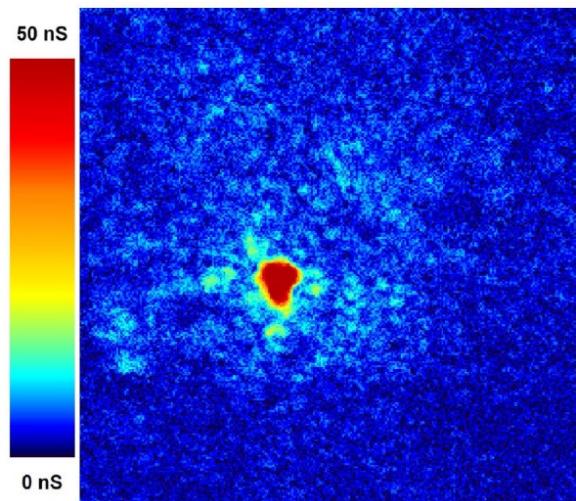
$E_0, u, v$  **funtions of  $J, V, \Delta \dots$**  → Extracted from the measured deconvoluted LDoS

$$\Im F_{odd}^R(\omega) = \rho_{even}(\omega)/C_e(E_0)$$

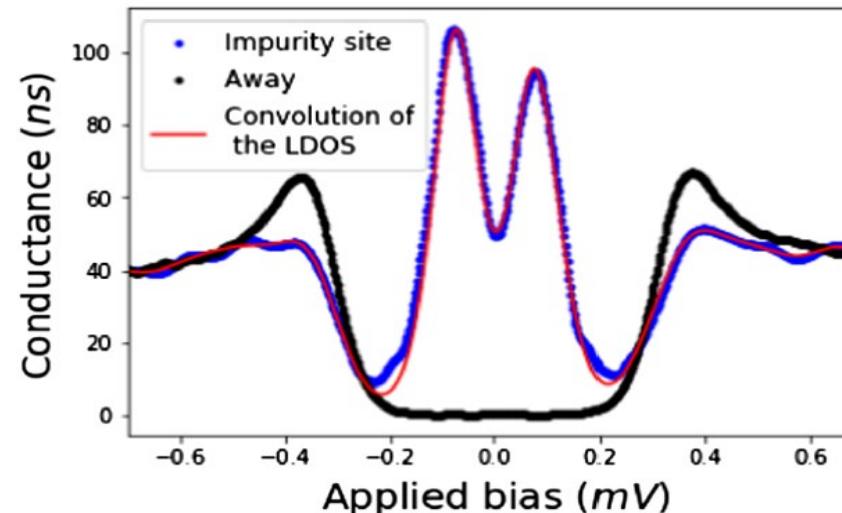
# Odd-Frequency Pairing on the impurity site



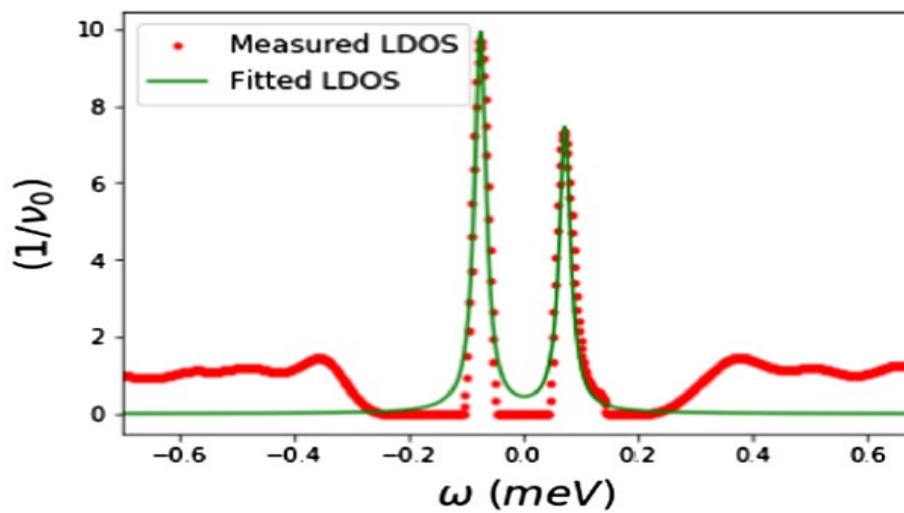
Pb/Si(111) monolayer



Conductance map at  $E_F$

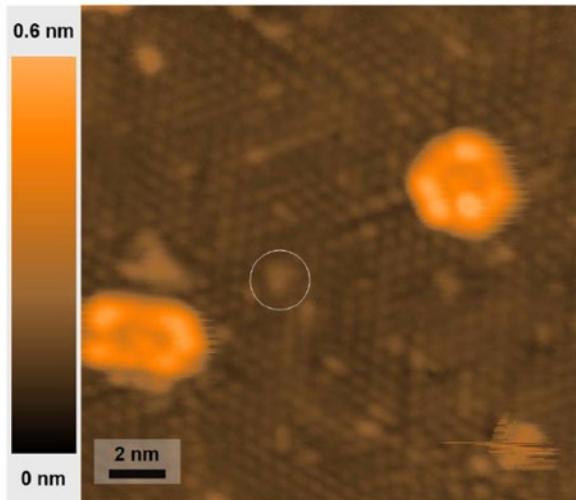


Conductance on top of the impurity

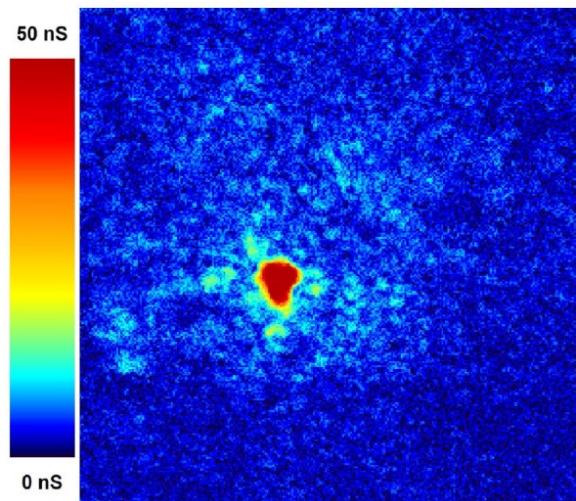


Deconvoluted LDOS on top of the impurity

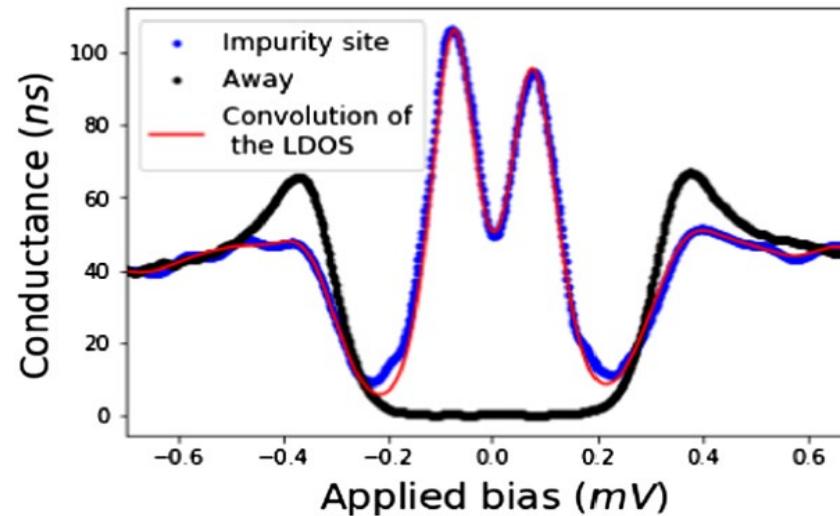
# Odd-Frequency Pairing on the impurity site



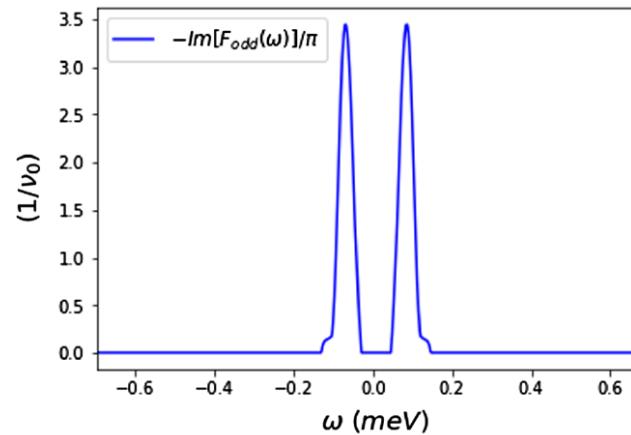
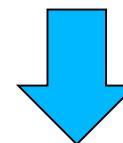
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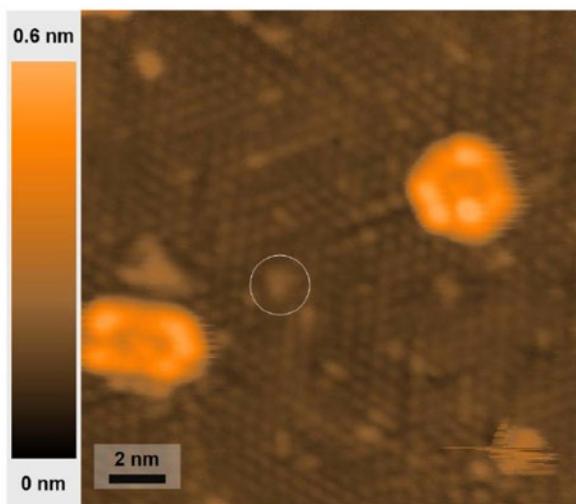


Conductance on top of the impurity

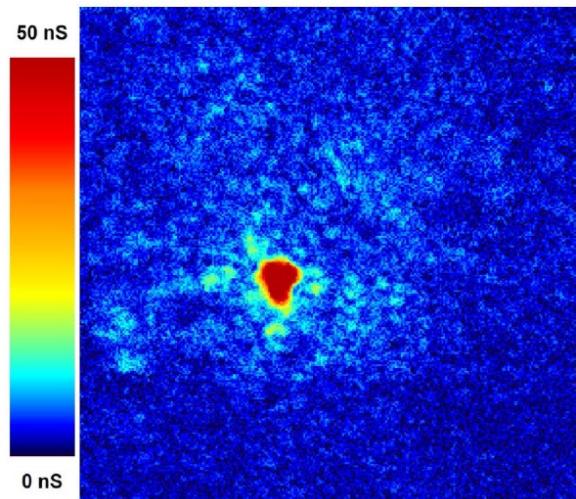


Odd-triplet correlations on top of the impurity

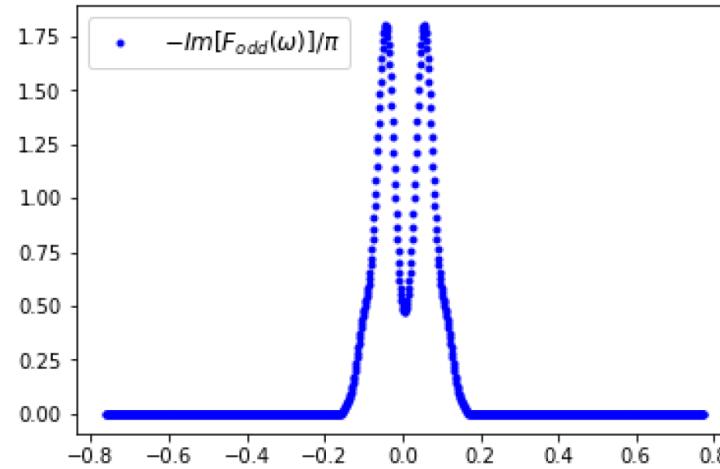
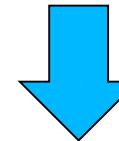
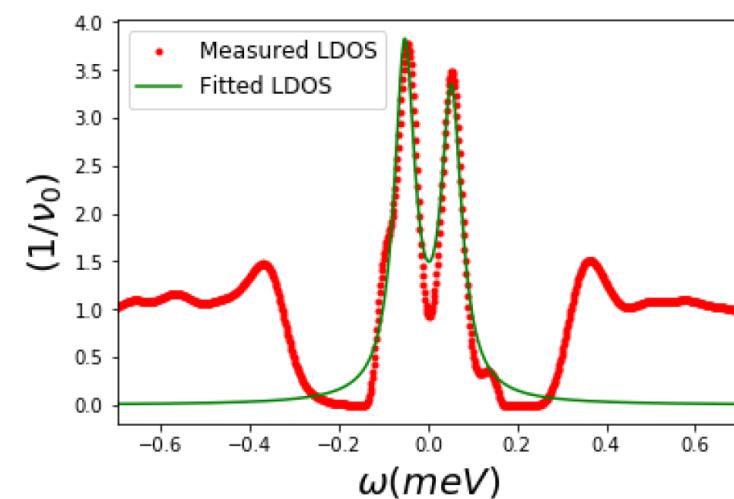
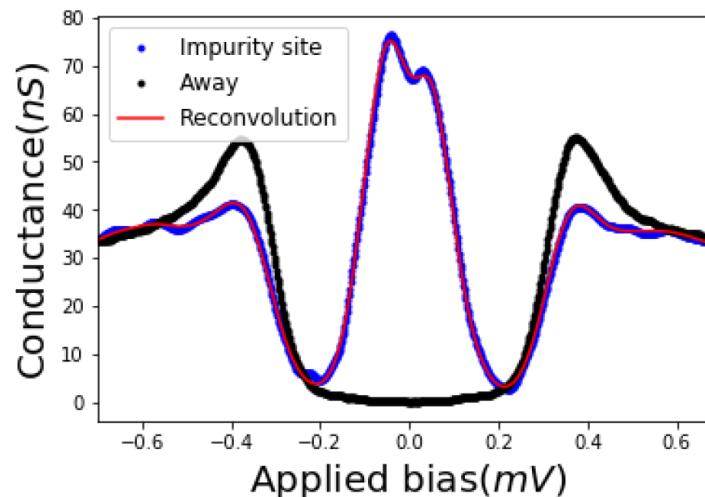
# Another magnetic impurity



Pb/Si(111) monolayer



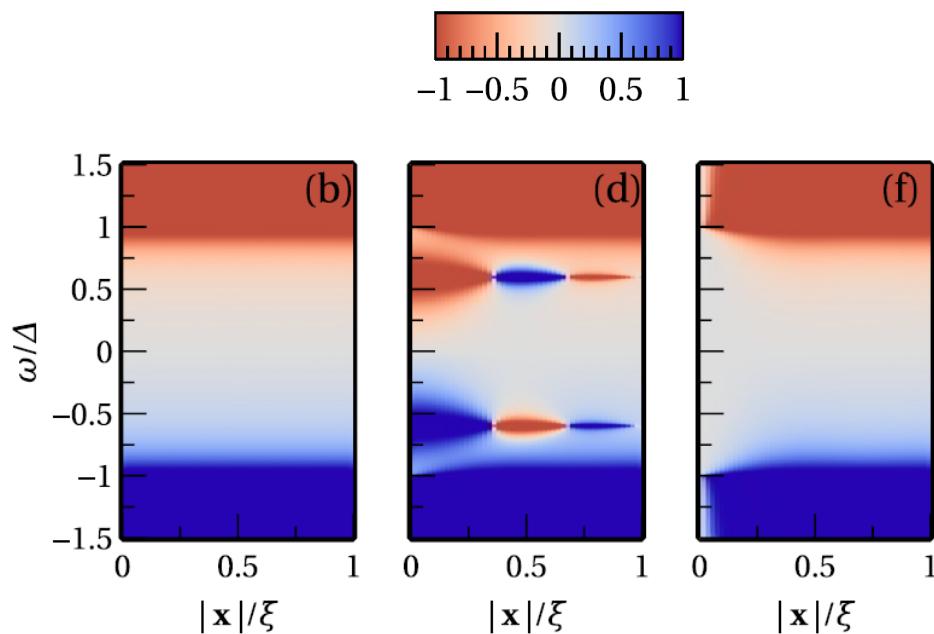
Conductance map at  $E_F$



A macroscopic fraction of the LDOS !

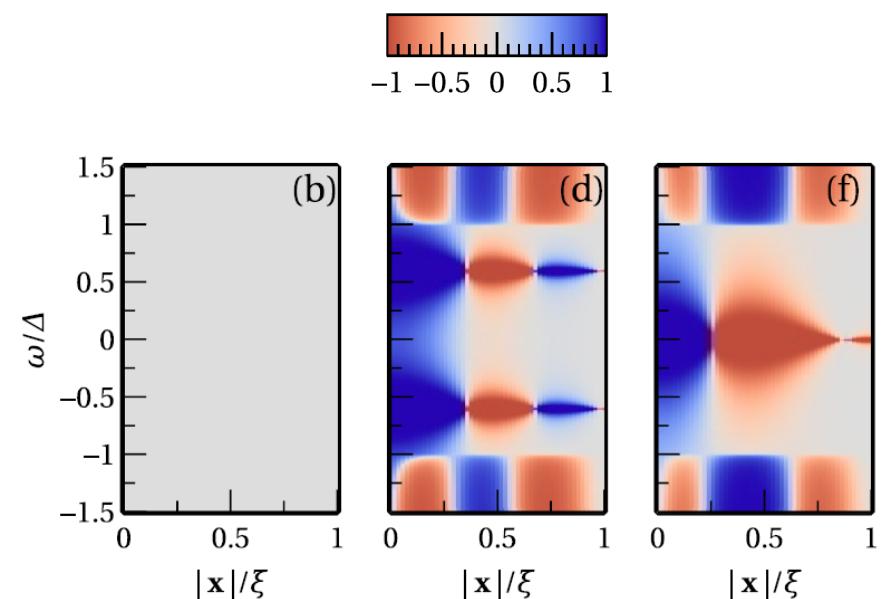
V. Perrin et al., Phys. Rev. Lett. 125, 117003 (2020)

# Space dependence of odd- $\omega$ pairing



## Even-singlet correlations

- b) No impurity
- d) Impurity with  $E_0 = 0.6 D$
- c) Impurity with  $E_0 = 0$



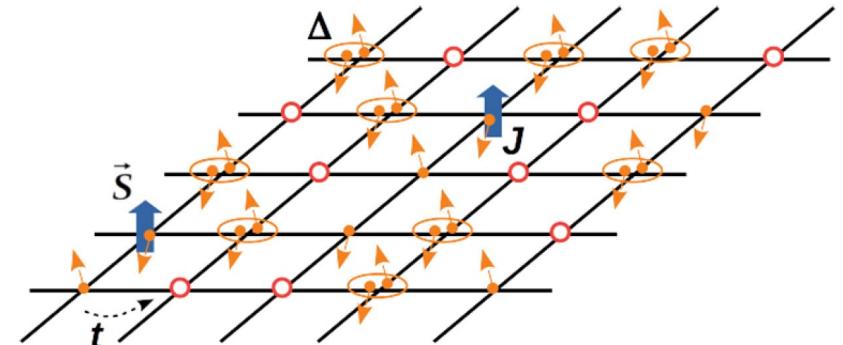
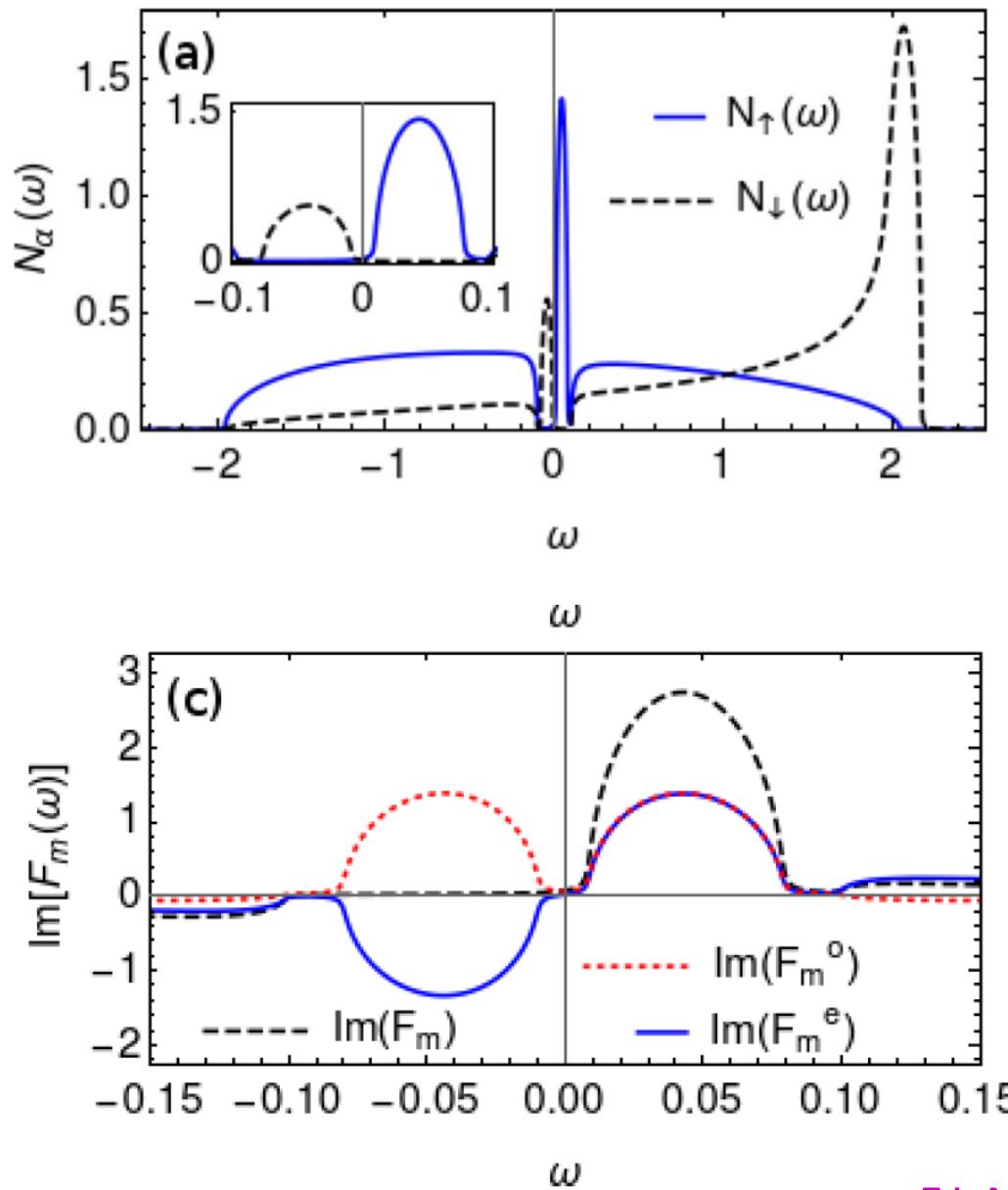
## Odd-triplet correlations

- b) No impurity
- d) Impurity with  $E_0 = 0.6 D$
- c) Impurity with  $E_0 = 0$

Odd-frequency superconductivity near a magnetic impurity in a conventional superconductor,

D. Kuzmanovski, R. S. Souto and A. V. Balatsky, Phys. Rev. B 101, 094505 (2020)

# A dilute magnetic s-wave superconductor

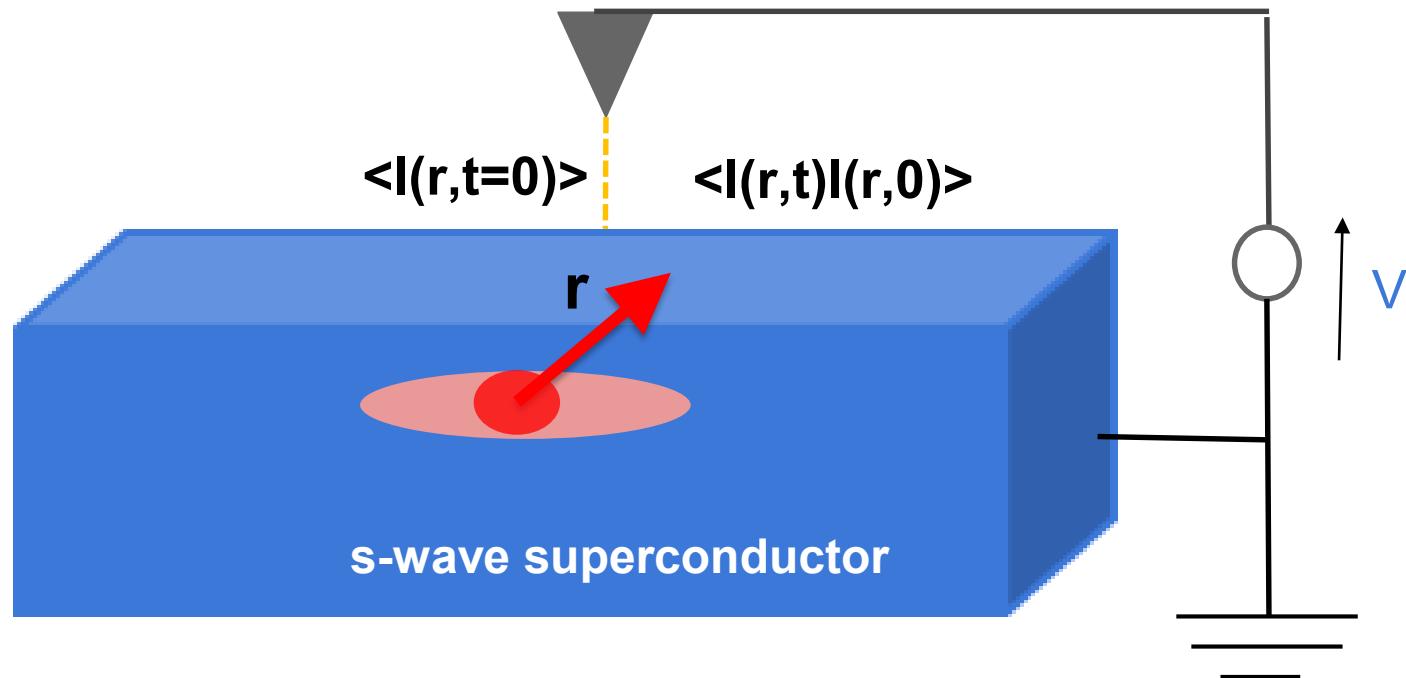


$$\text{Im}[F_m^o(\omega)] = a_F \left[ \frac{N_{\uparrow}(\omega)}{a_{\uparrow}} + \frac{N_{\downarrow}(\omega)}{a_{\downarrow}} \right].$$

Proportionality relations valid here too!

### **III) Shot noise tomography of bound states in (topological) superconductors**

# STM noise tomography



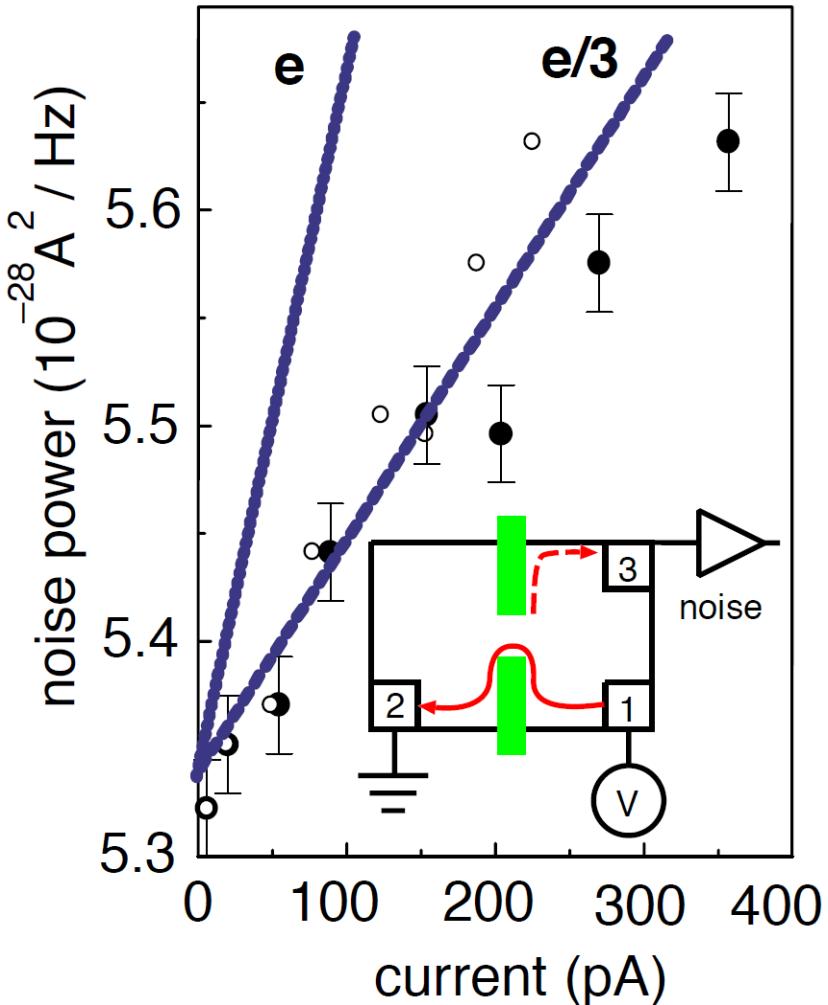
**Atomically resolved noise :**  $S(\mathbf{r}) = \int dt \langle \delta \hat{I}(t, \mathbf{r}) \delta \hat{I}(0, \mathbf{r}) \rangle$

**where**  $\delta \hat{I}(t, \mathbf{r}) = \hat{I}(t, \mathbf{r}) - \langle \hat{I}(t, \mathbf{r}) \rangle$

**Atomically resolved  
Fano factor :**

$$F(\mathbf{r}) = \frac{S(\mathbf{r})}{2e|\langle \hat{I}(\mathbf{r}) \rangle|}$$

# Fano factor: a measure of the effective transferred charge



**Evidence of the fractional charge in the fractional quantum Hall effect**

L. Saminadayar et al., Phys. Rev. Lett. 79, 2526 (1997)

R. De Picciotto et al., Nature 399, 238 (1999)

# Application to YSR state

## A simple theoretical model

### Assumptions :

- Retain **only YSR contributions** to the transport observables
- Local tunneling of quasiparticles from tip to sample.
- Neglect direct injection of quasiparticles into the impurity's orbitals.



Expected to be relevant to describe STM experiments probing the tail of the YSR.

# Application to YSR state

## Energy of the YSR bound state

$$E_0 = \frac{1 - \alpha^2 + \beta^2}{\sqrt{(1 - \alpha^2 + \beta^2)^2 + 4\alpha^2}}$$

$\alpha = \pi\nu_0 J$  → Magnetic exchange

$\beta = \pi\nu_0 U$  → scalar potential

$$g_{SS}^r(\omega) \sim \frac{1}{\omega - E_0 + i\Lambda/2} \begin{bmatrix} u^2 & uv \\ uv & v^2 \end{bmatrix}$$

$$u^2, v^2 = 2\pi\alpha\nu_0\Delta \frac{1 + (\alpha \pm \beta)^2}{((1 - \alpha^2 + \beta^2)^2 + 4\alpha^2)^{3/2}}$$

$\tau = \hbar/\Lambda$     Intrinsic YSR lifetime due to dissipation

(4 parameters and T= temperature)

# Current and shot-noise in YSR state

Exact expressions :

$$I = \frac{e}{h} \int d\omega 2 \left[ \begin{aligned} & \frac{\Gamma_e \Gamma_h}{(\omega - E_0)^2 + \Gamma_t^2/4} [f(\omega^-) - f(\omega^+)] \\ & + \frac{\Gamma_e \Lambda}{(\omega - E_0)^2 + \Gamma_t^2/4} [f(\omega^-) - f(\omega)] \\ & + \frac{\Gamma_h \Lambda}{(\omega - E_0)^2 + \Gamma_t^2/4} [f(\omega) - f(\omega^+)]. \end{aligned} \right]$$

Andreev Current

Quasiparticle Current  
(incoherent)

$$\omega^\pm = \omega \pm eV$$

$$S = \frac{2e^2}{h} \int d\omega \left[ \begin{aligned} & \frac{4\Gamma_e \Gamma_h \{(\omega - E_0)^2 + (\Gamma_e - \Gamma_h)^2/4\}}{\{(\omega - E_0)^2 + \Gamma_t^2/4\}^2} [f(\omega^-)f(-\omega^+) + f(\omega^+)f(-\omega^-)] \\ & + \frac{\Gamma_e \Lambda \{(\omega - E_0)^2 + (3\Gamma_h - \Gamma_e)^2/4\}}{\{(\omega - E_0)^2 + \Gamma_t^2/4\}^2} [f(\omega^-)f(-\omega) + f(-\omega^-)f(\omega)] \\ & + \frac{\Gamma_h \Lambda \{(\omega - E_0)^2 + (3\Gamma_e - \Gamma_h)^2/4\}}{\{(\omega - E_0)^2 + \Gamma_t^2/4\}^2} [f(\omega)f(-\omega^+) + f(-\omega)f(\omega^+)] \\ & + \frac{\Gamma_e^2 (\Lambda + 2\Gamma_h)^2 f(\omega^-)f(-\omega^-) + \Gamma_h^2 (\Lambda + 2\Gamma_h)^2 f(\omega^+)f(-\omega^+)}{((\omega - E_0)^2 + \Gamma_t^2/4)^2} \\ & + \frac{\Lambda^2 (\Gamma_e - \Gamma_h)^2 f(\omega)f(-\omega)}{((\omega - E_0)^2 + \Gamma_t^2/4)^2}. \end{aligned} \right]$$

Andreev-like contribution

Quasiparticle-like contribution

Thermal contributions

$$\Gamma_e = u^2 \Gamma \quad \Gamma_h = v^2 \Gamma \quad \Gamma_t = \Gamma_e + \Gamma_h + \Lambda$$

# Current and shot-noise in YSR state

Simplifications:  $E_0 \gg k_B T \gg u^2 \Gamma, v^2 \Gamma$

$$I \simeq \frac{e}{\hbar} \left\{ \frac{2\Gamma_e \Gamma_h}{\Gamma_t} [f(E_0^-) - f(E_0^+)] + \frac{\Gamma_e \Lambda}{\Gamma_t} [f(E_0^-)] - \frac{\Gamma_h \Lambda}{\Gamma_t} [f(E_0^+)] \right\}$$

$$\begin{aligned} S \simeq & \frac{e^2}{\hbar} \left\{ 4\Gamma_e \Gamma_h \frac{(\Gamma_e - \Gamma_h)^2 + \Gamma_t^2}{\Gamma_t^3} [f(E_0^-)f(-E_0^+) + f(E_0^+)f(-E_0^-)] \right. \\ & + \Gamma_e \Lambda \frac{(3\Gamma_h - \Gamma_e + \Lambda)^2 + \Gamma_t^2}{\Gamma_t^3} [f(E_0^-)] + \Gamma_h \Lambda \frac{(3\Gamma_e - \Gamma_h + \Lambda)^2 + \Gamma_t^2}{\Gamma_t^3} [f(E_0^+)] \\ & \left. + 4 \frac{\Gamma_e^2 (\Lambda + 2\Gamma_h)^2 f(E_0^-) f(-E_0^-)}{\Gamma_t^3} + 4 \frac{\Gamma_h^2 (\Lambda + 2\Gamma_h)^2 f(E_0^+) f(-E_0^+)}{\Gamma_t^3} \right\}. \end{aligned}$$

- 
- Extract  $E_0, T$
  - Extract  $\Gamma \pi \nu_0$  from the normal conductance
  - Extract  $u^2, v^2$  from the height of the peaks

$\Lambda$  only left parameter



NEXT TALK  
By Freek Massee

**Can we use shot noise  
tomography as a signature  
of Majorana zero modes ?**

# Spectral identification of Majorana fermions

BdG Formalism

$$\mathcal{H} = \frac{1}{2} \sum_{j,j'} \psi_j^\dagger H_{BdG}^{j,j'} \psi_{j'}$$

$$\psi_j = (c_{j\uparrow}, c_{j\downarrow}, c_{j\downarrow}^\dagger, -c_{j\uparrow}^\dagger)^T$$

Redundant description : P-H “symmetry”

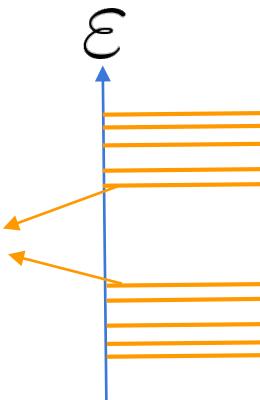
$$CH_{BdG}C^{-1} = -H_{BdG}$$

$$C = \sigma_y \tau_y \mathcal{K}$$

Symmetric spectrum around 0

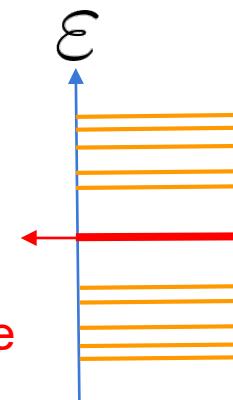
1 fermionic excitation = 2 eigenstates  
with eigenvalues  $\pm \varepsilon$

Spectrum of a 1D topological SC  
in **semi-infinite** geometry



Trivial case

1 fermionic  
excitation



Topological case

Single  
eigenstate !  
Majorana state

$$C\phi_M = \phi_M$$



**SELF-CONJUGATE**

# Expected signatures in local transport

PRL 103, 237001 (2009)

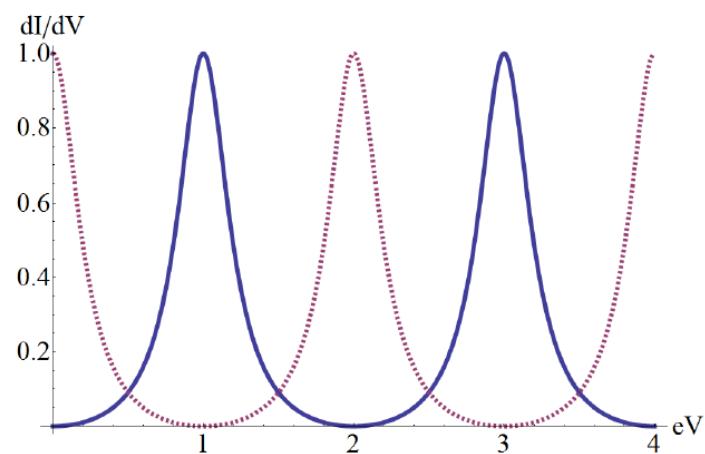
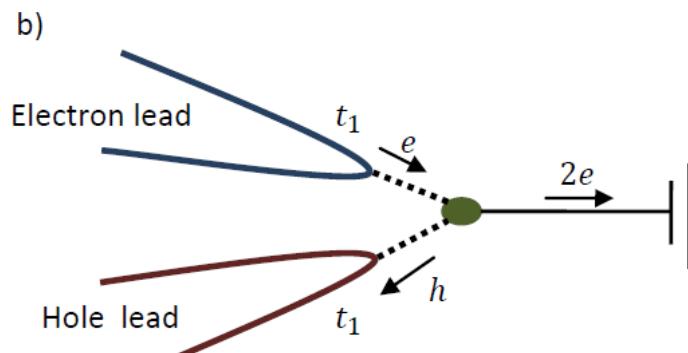
PHYSICAL REVIEW LETTERS

week ending  
4 DECEMBER 2009

## Majorana Fermion Induced Resonant Andreev Reflection

K. T. Law,<sup>1,2</sup> Patrick A. Lee,<sup>2</sup> and T. K. Ng<sup>3</sup>

We describe experimental signatures of Majorana fermion edge states, which form at the interface between a superconductor and the surface of a topological insulator. If a lead couples to the Majorana fermions through electron tunneling, the Majorana fermions induce *resonant Andreev reflections* from the lead to the grounded superconductor. The linear tunneling conductance is  $0$  ( $2e^2/h$ ) if there is an even (odd) number of vortices in the superconductor. Similar resonance occurs for tunneling into the zero mode in the vortex core. We also study the current and noise of a two-lead device.



- A perfect zero-bias quantized peak
- Fano factor  $F=1$  for a single-contact lead

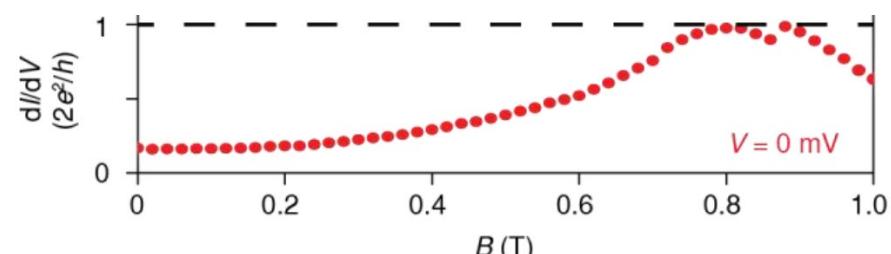
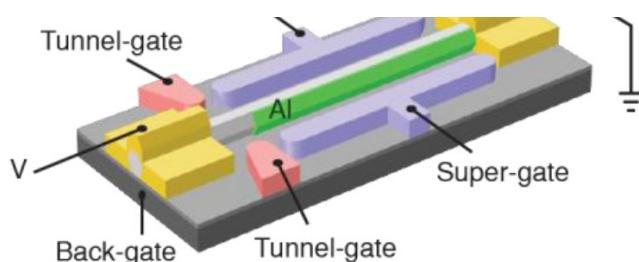
# Experimental evidences

## Quantized Majorana Conductance

Hao Zhang<sup>1\*</sup>, Chun-Xiao Liu<sup>2\*</sup>, Sasa Gazibegovic<sup>3\*</sup>, Di Xu<sup>1</sup>, John A. Logan<sup>4</sup>, Guanzhong Wang<sup>1</sup>, Nick van Loo<sup>1</sup>, Jouri D.S. Bommer<sup>1</sup>, Michiel W.A. de Moor<sup>1</sup>, Diana Car<sup>3</sup>, Roy L. M. Op het Veld<sup>3</sup>, Petrus J. van Veldhoven<sup>3</sup>, Sebastian Koelling<sup>3</sup>, Marcel A. Verheijen<sup>3,7</sup>, Mihir Pendharkar<sup>5</sup>, Daniel J. Pennachio<sup>4</sup>, Borzoyeh Shojaei<sup>4,6</sup>, Joon Sue Lee<sup>6</sup>, Chris J. Palmstrøm<sup>4,5,6</sup>,

EDAM ID: 300-0-21-DK-1-18

**RETRACTED  
ARTICLE**



# Experimental evidences

## Large zero-bias peaks in InSb-Al hybrid semiconductor-superconductor nanowire devices

Hao Zhang\*,<sup>1, 2, 3</sup> Michiel W.A. de Moor\*,<sup>1, 2</sup> Jouri D.S. Bommer\*,<sup>1, 2</sup> Di Xu,<sup>1, 2</sup> Guanzhong Wang,<sup>1, 2</sup> Nick van Loo,<sup>1, 2</sup> Chun-Xiao Liu,<sup>1, 2, 4</sup> Sasa Gazibegovic,<sup>5</sup> John A. Logan,<sup>6</sup> Diana Car,<sup>5</sup> Roy L. M. Op het Veld,<sup>5</sup> Petrus J. van Veldhoven,<sup>5</sup> Sebastian Koelling<sup>a</sup>,<sup>5</sup> Marcel A. Verheijen,<sup>5</sup> Mihir Pendharkar,<sup>7</sup> Daniel J. Pennachio,<sup>6</sup> Borzoyeh Shojaei,<sup>6, 8</sup> Joon Sue Lee<sup>b</sup>,<sup>8</sup> Chris J. Palmstrøm,<sup>6, 7, 8</sup> Erik P.A.M. Bakkers,<sup>5</sup> S. Das Sarma,<sup>4</sup> Leo P. Kouwenhoven<sup>1, 2, 9†</sup>

Corrected version, arXiv:2101.11456

### MAIN CONCLUSION:

Our work, presented here, along with the recent theoretical developments involving quasi-Majoranas and disorder, should serve as a strong cautionary message for all Majorana experiments in all platforms, clearly emphasizing that the experimental observations of zero-bias conductance peaks, no matter how compelling, should be considered only **as necessary and by no means sufficient conditions for the existence of topological MZMs**.

# Shot-noise signatures ?

PRL 98, 237002 (2007)

PHYSICAL REVIEW LETTERS

week ending  
8 JUNE 2007

## Observing Majorana bound States in *p*-Wave Superconductors Using Noise Measurements in Tunneling Experiments

C. J. Bolech<sup>1,2</sup> and Eugene Demler<sup>1</sup>

$$F_{\alpha\beta} \equiv \lim_{V/T \rightarrow \infty} \frac{S_{\alpha\beta}(\omega = 0)}{e(I_\alpha + I_\beta)} = \delta_{\alpha\beta} \quad \alpha, \beta = \{L, R\} = \pm 1$$

PRL 114, 166406 (2015)

PHYSICAL REVIEW LETTERS

week ending  
24 APRIL 2015

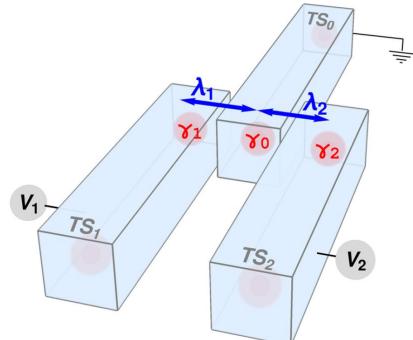
## Signatures of Majorana Zero Modes in Spin-Resolved Current Correlations

Arbel Haim,<sup>1</sup> Erez Berg,<sup>1</sup> Felix von Oppen,<sup>2</sup> and Yuval Oreg<sup>1</sup>

PHYSICAL REVIEW LETTERS 122, 097003 (2019)

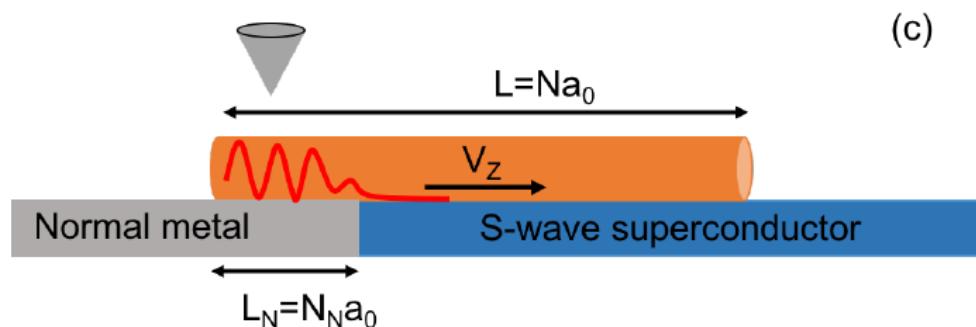
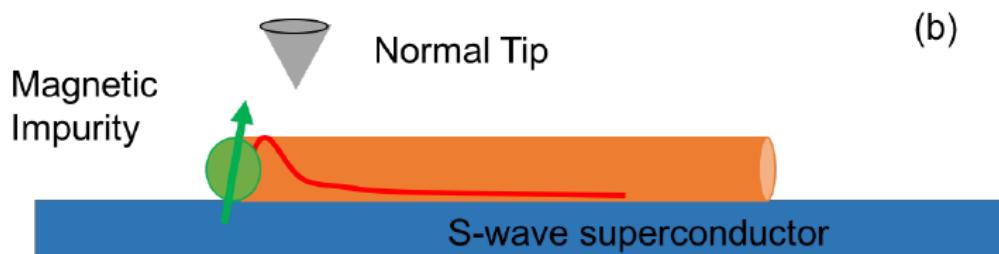
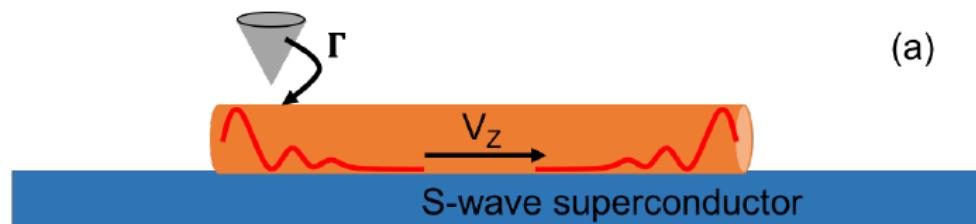
## Giant Shot Noise from Majorana Zero Modes in Topological Trijunctions

T. Jonckheere,<sup>1</sup> J. Rech,<sup>1</sup> A. Zazunov,<sup>2</sup> R. Egger,<sup>2</sup> A. Levy Yeyati,<sup>3</sup> and T. Martin<sup>1</sup>



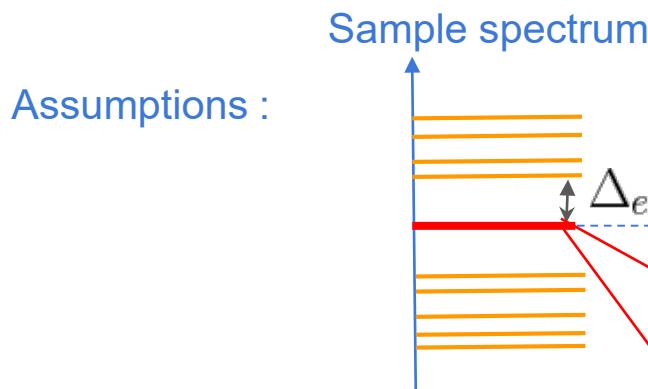
# Shot-noise tomography for zero-energy states

**Advantage: test the full spatial extent of the bound state wave function (non-local)**



# Low-energy analytical results

## Fano factor tomography of zero-energy fermions : Low-energy model



Low-energy model :  $\psi_S(j, t) \sim \phi_+(j)\gamma_0 + \phi_-(j)\gamma_0^\dagger$

$$\phi_+(j) = (u_\uparrow(j), u_\downarrow(j), v_\downarrow(j), -v_\downarrow(j))^T \quad \phi_-(j) = C\phi_+(j))$$

Simple analytical limit :  $eV \gg \Gamma_j$

$$\Gamma \equiv 2\pi\nu_T t^2$$

$$\Gamma \sum_{\sigma} (u_{\sigma}(j)^2 + v_{\sigma}(j)^2) \gg \Lambda$$

$\Gamma_j$

Tip induced linewidth

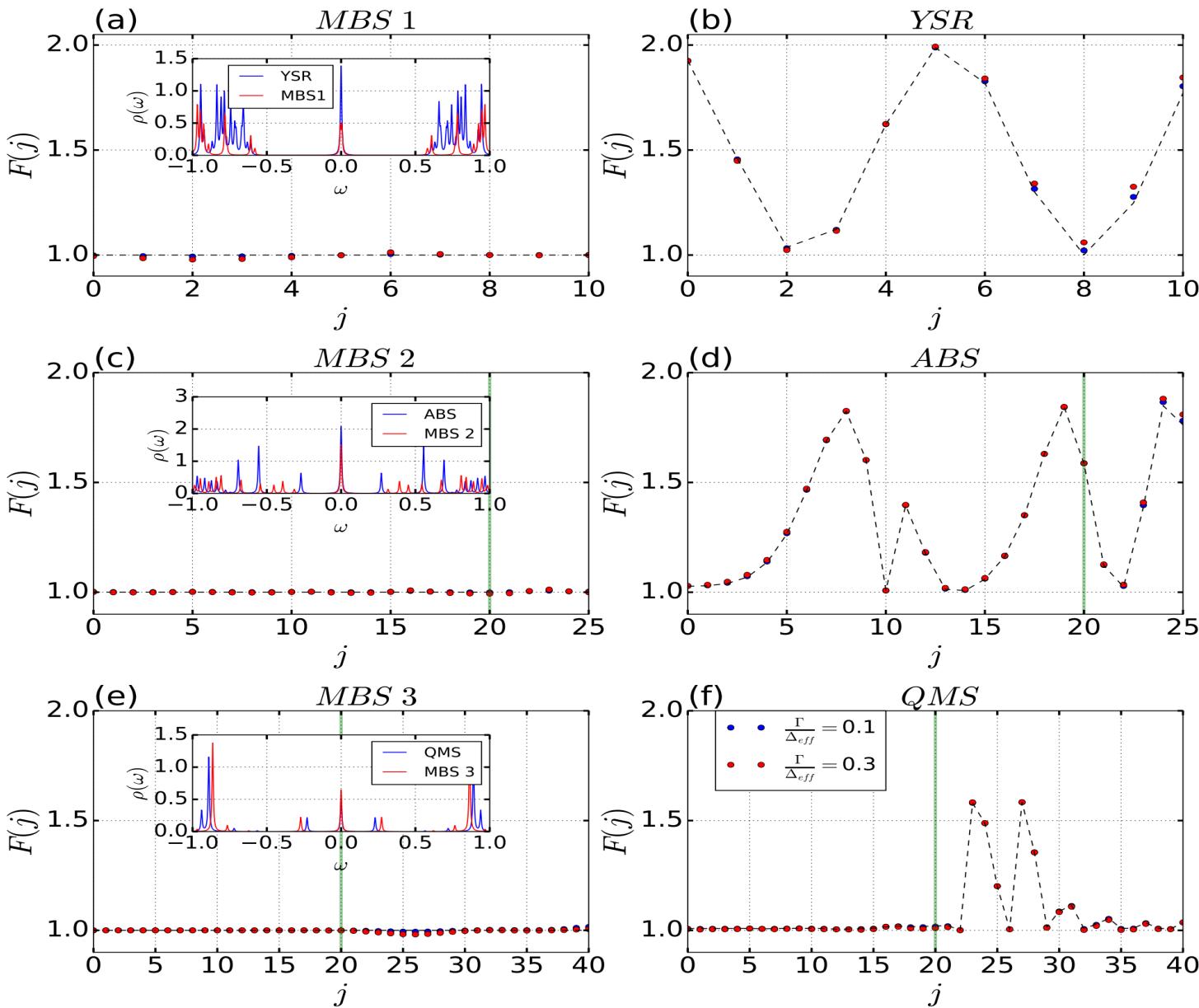
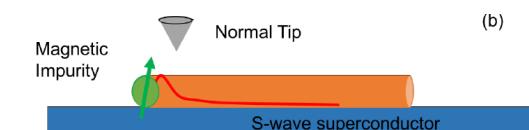
coherent  
transport  
regime

dissipative  
linewidth

Local BCS charge , vanishes for  
isolated Majorana !

$$F(j) \simeq 1 + \left( \frac{\sum_{\sigma} (u_{\sigma}^2 - v_{\sigma}^2)}{\sum_{\sigma} (u_{\sigma}^2 + v_{\sigma}^2)} \right)^2 = 1 + \delta_{\text{ph}}^2(j)$$

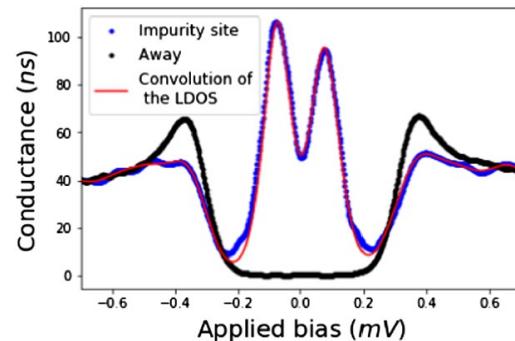
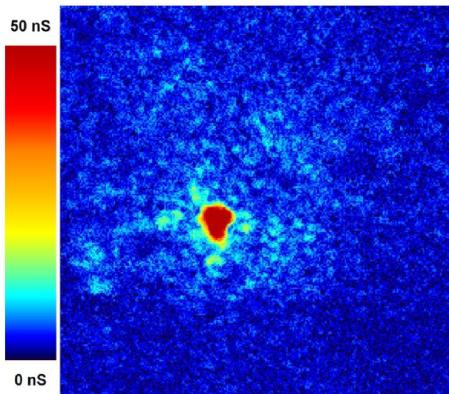
# Results from tight-binding simulations



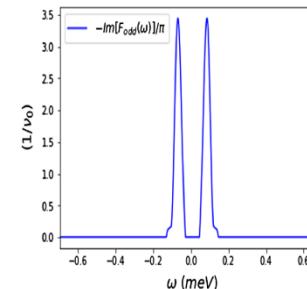
1. Key distinct features differentiating MBS from trivial ZBP
2. Quantitative agreement between low-energy effective theory (dashed line) and exact numerics (dots)
3. Sensitivity to Majorana overlap through BCS charge

# Conclusions

Odd-w frequency triplet pairing around a magnetic impurity



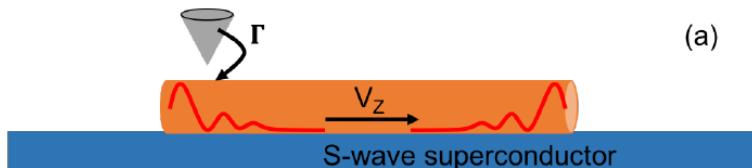
Conductance on top of  
the impurity



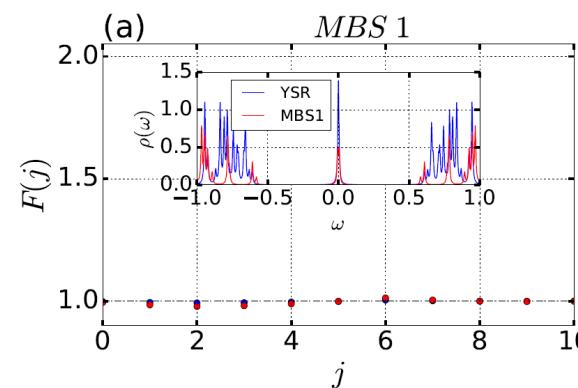
Odd-triplet correlations  
on top of the impurity

V. Perrin et al., Phys. Rev. Lett. 125, 117003 (2020)

A uniform Fano factor  $F=1$  as a signature of Majorana zero mode



(a)



V. Perrin et al., Phys. Rev. B 104, L121406 (2021)

# Main Collaborators

## Institut des NanoSciences de Paris CNRS & Sorbonne University

- **Gerbold Ménard**
- Christophe Brun
- François Debontridder
- Tristan Cren



## ESPCI

- Dimitri Roditchev



## LPS Theory CNRS & University Paris Saclay

- **Vivien Perrin**
- **Flavio Santos**
- Marcello Civelli



## LPS, CNRS & University Paris Saclay

- **U. Thupakula**
- F. Massee
- M. Aprili
- A. Palacio-Morales



## IMN

- Laurent Cario





Thanks for your attention !