Chiral edge states in topological insulators and superconductors

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Outline

0) Preliminaries of (topological) band theory

I) Integer quantum Hall effect

II) The anomalous quantum Hall effect

III) A brief incursion into 2D topological insulators

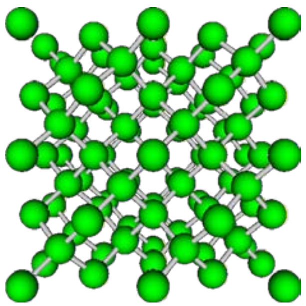
IV) 2D chiral topological superconductors

V) How about 3D ?

Preliminaries of (topological) band theory

Elements of Traditional Band Theory

Non-interacting electrons moving in a perfectly periodic array of atoms



- Electron Hamiltonian commutes with lattice translations

$$[\mathrm{H}, \mathrm{T}\,(\mathbf{R})] = 0$$

$\boldsymbol{R} = n_x \boldsymbol{a}_x + n_y \boldsymbol{a}_y + n_z \boldsymbol{a}_z,\ n_\alpha$ is an integer

Lattice translation Symmetry

$$\mathrm{T}(\boldsymbol{R})|\psi_{\boldsymbol{k}}\rangle = e^{i\boldsymbol{k}\cdot\boldsymbol{R}}|\psi_{\boldsymbol{k}}\rangle$$

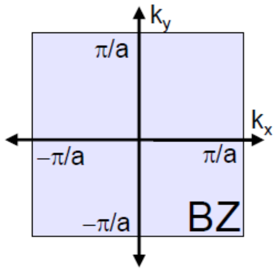
Crystal momentum ***k*** *is conserved*

- The wave vector k is defined modulo the reciprocal lattice vector (reciprocal lattice is the Fourier transform of the real-space lattice)

$$\boldsymbol{k} \sim \boldsymbol{k}\ mod\ \boldsymbol{G}$$

- The wave-vector ***k*** “lives” on a d-dimensional torus

$\boldsymbol{k} \in \mathbb{T}^d$ $(1D: -\pi/a \le \boldsymbol{k} \le \pi/a$, with the end points “glued”)



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Elements of Traditional Band Theory

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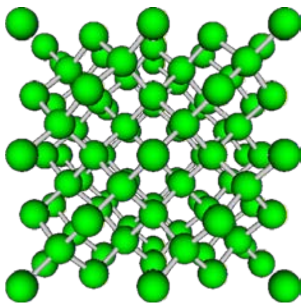


- Electron Hamiltonian commutes with lattice translations

$$[\mathrm{H}, \mathrm{T}\,(\mathbf{R})] = 0$$

$$\boldsymbol{R} = n_x \boldsymbol{a}_x + n_y \boldsymbol{a}_y + n_z \boldsymbol{a}_z, \;\; n_\alpha \text{ is an integer}$$

Lattice translation Symmetry

$$\mathrm{T}(\boldsymbol{R})|\psi_{\boldsymbol{k}}\rangle = e^{i\boldsymbol{k}\cdot\boldsymbol{R}}|\psi_{\boldsymbol{k}}\rangle$$

Crystal momentum ***k** is conserved*

Bloch thm:

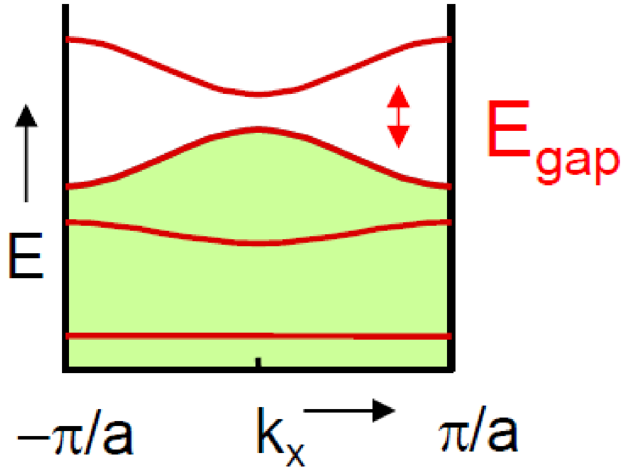
$$|\psi_{\boldsymbol{k}}\rangle = e^{i\boldsymbol{k}\cdot\boldsymbol{R}}|u_{\boldsymbol{k}}\rangle$$

of period of **a**

Bloch Hamiltonian:

$$H(\mathbf{k}) = e^{-i\mathbf{k}\cdot\mathbf{r}}\mathrm{H}e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$H(\mathbf{k})|u_n(\mathbf{k})\rangle = E_n(\mathbf{k})|u_n(\mathbf{k})\rangle$$

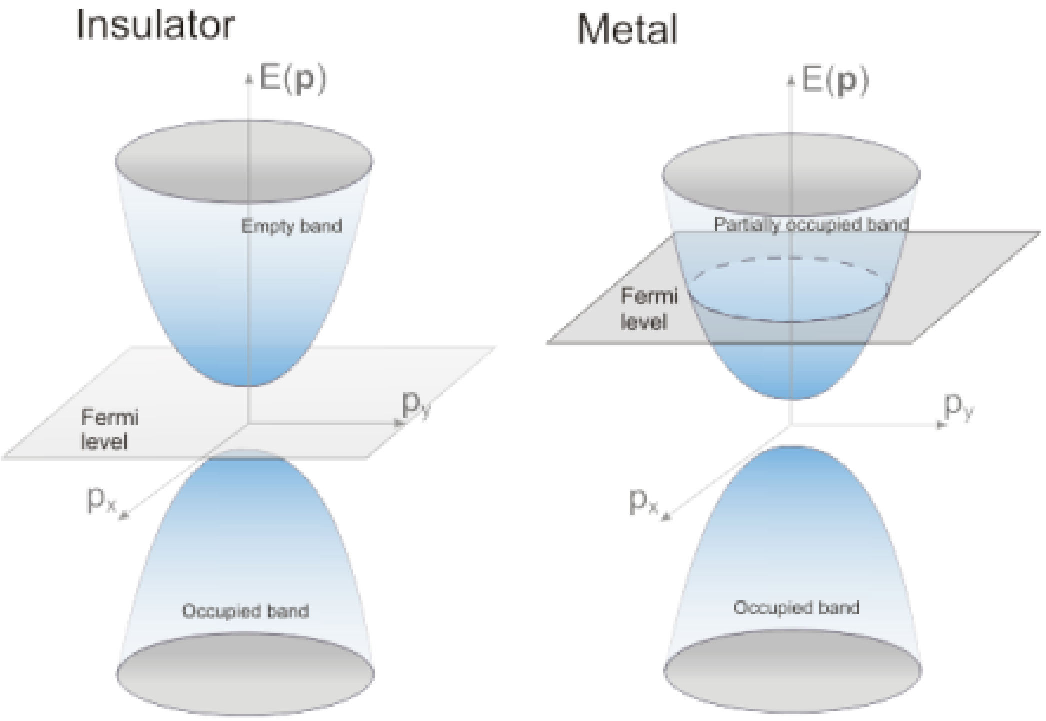


Insulators and metals

- Bloch theorem and band structure:

$$\left[-\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{r})\right]\psi(\mathbf{r})=E\psi(\mathbf{r}), V(\mathbf{r})=V(\mathbf{r}+\mathbf{a})$$

$$\psi_{\mathbf{p}}(\mathbf{r})=u_{\mathbf{p}}(\mathbf{r})\,e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}},\ \text{with}\ u_{\mathbf{p}}(\mathbf{r})=u_{\mathbf{p}}(\mathbf{r}+\mathbf{a})$$



Quantum topological equivalence

- How to define topological invariants for quantum states of matter?
- We need a notion of topological equivalence of quantum states.
- The notion of quantum topological equivalence follows from adiabatic continuity

If we can adiabatically deform $|0\rangle$ into $|0'\rangle$, then $|0\rangle \sim |0'\rangle$

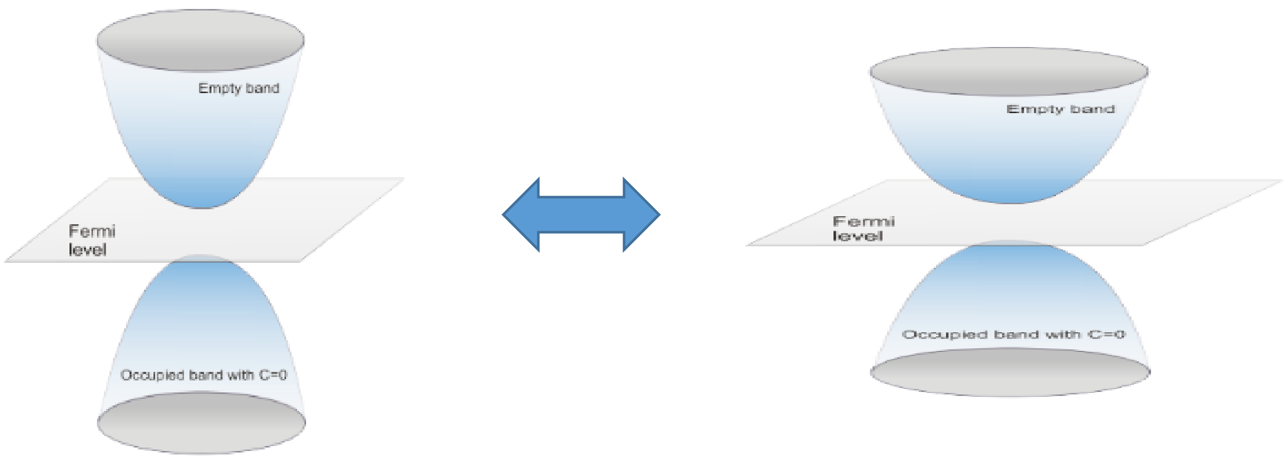
Band topological equivalence

- How to define topological invariants for quantum states of matter?
- We need a notion of topological equivalence of quantum states.

Topological Equivalence : adiabatic continuity

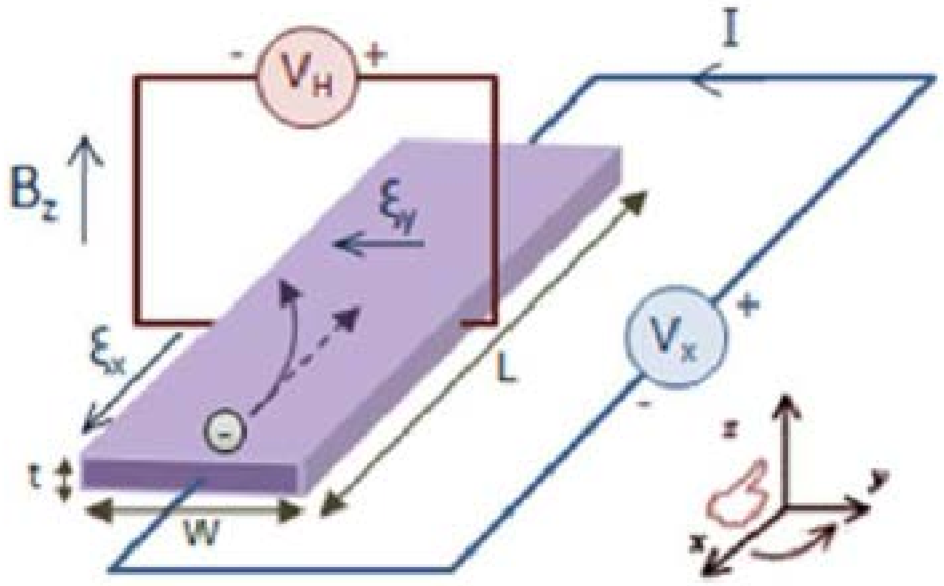
Band structures are equivalent if they can be continuously deformed into one another without closing the energy gap

Insulator I Insulator II



I) The integer Quantum Hall effect

Classical Hall effect (1879)



Classical equation of motion

$$m\left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}\right) = -e(\vec{E} + \vec{v} \wedge \vec{B})$$

Conductivity tensor

$$\sigma_{xx} = \frac{\sigma_0}{1 + (\omega_c \tau)^2}$$

$$\sigma_{yx} = \frac{\sigma_0}{1 + (\omega_c \tau)^2}$$

Drude Conductivity

$$\sigma_0 = \frac{ne^2\tau}{m}$$

$$\omega_c = \frac{eB}{m}$$

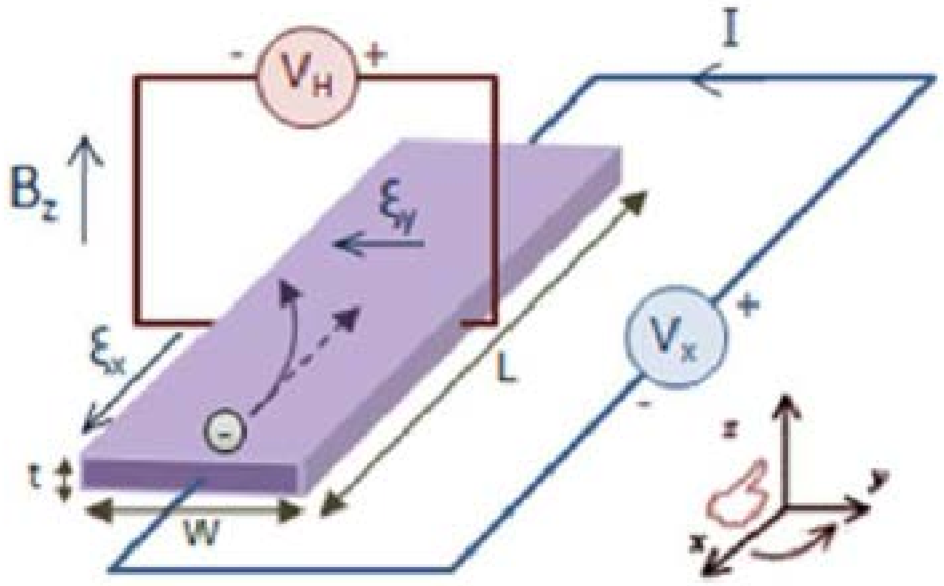
Resistivity tensor

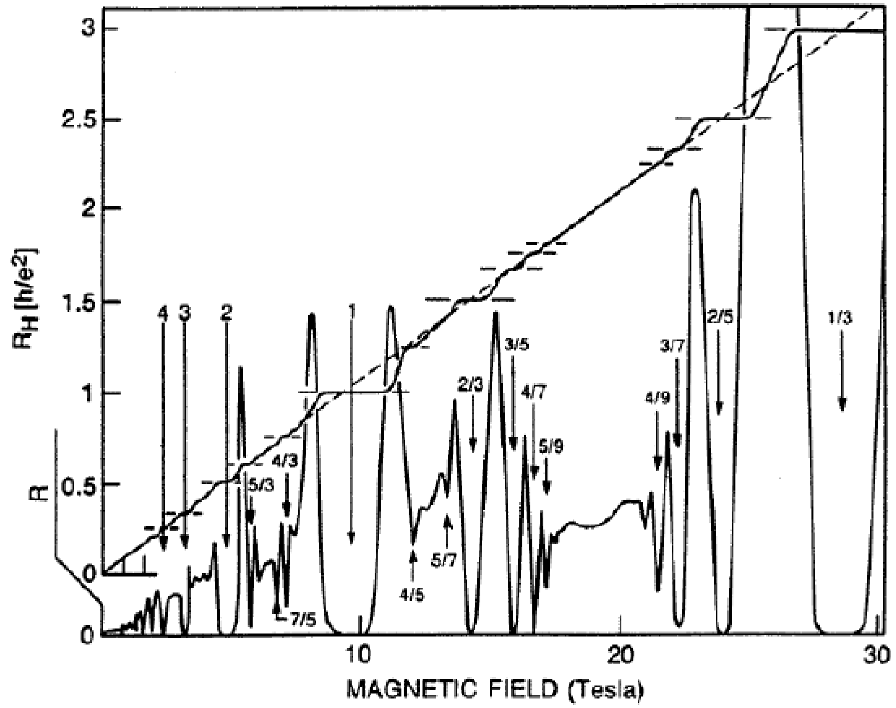
$$\rho_{xx} = \sigma_0^{-1}$$

$$\rho_{xy} = \frac{m\omega_c}{ne^2} = \frac{1}{ne}B.$$

$$\rho_H = \rho_{xy} \propto B$$

Quantum Hall effect (1980)

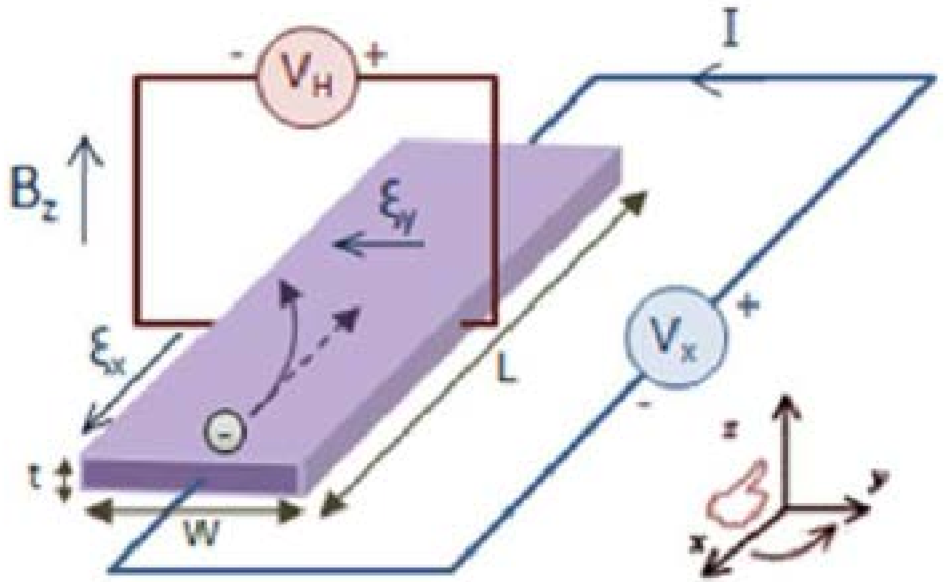


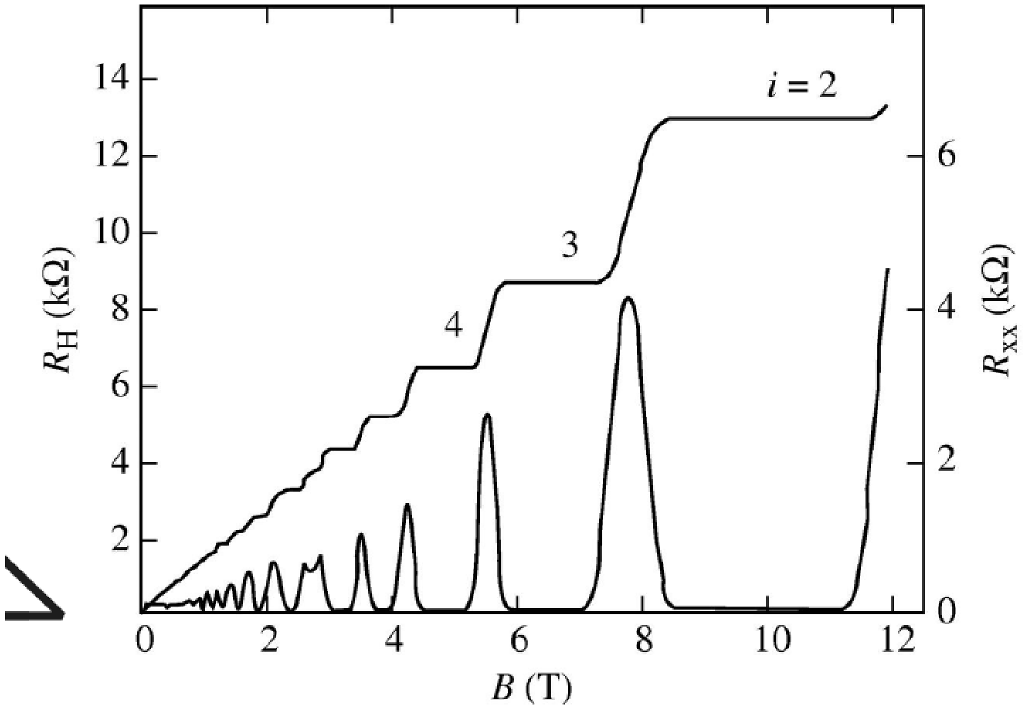


Stormer,
Physica **B177**,
401 (1992)

K. v. Klitzing, G. Dorda, and M. Pepper, PRL 45, 494 (1980)

Quantum Hall effect





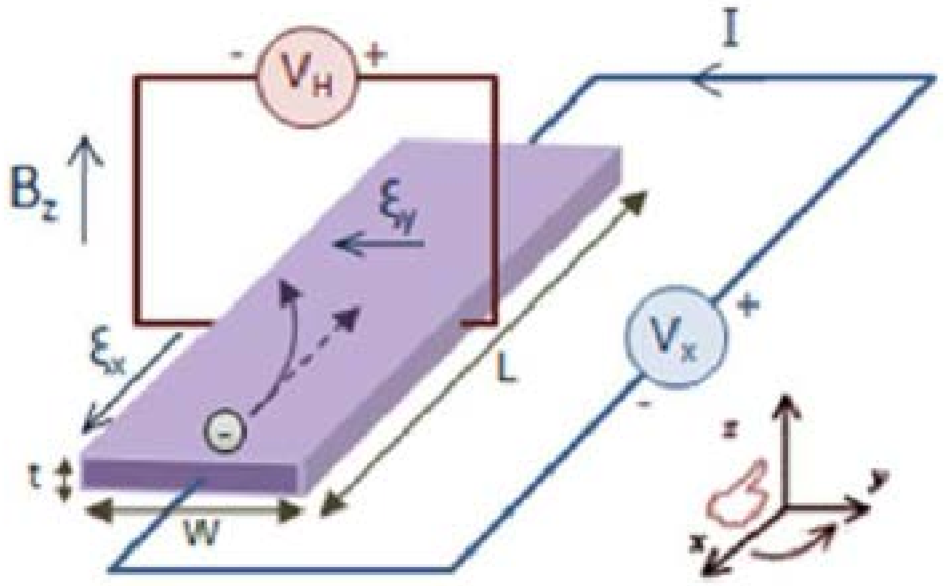
K. v. Klitzing, G. Dorda, and M. Pepper, PRL 45, 494 (1980)

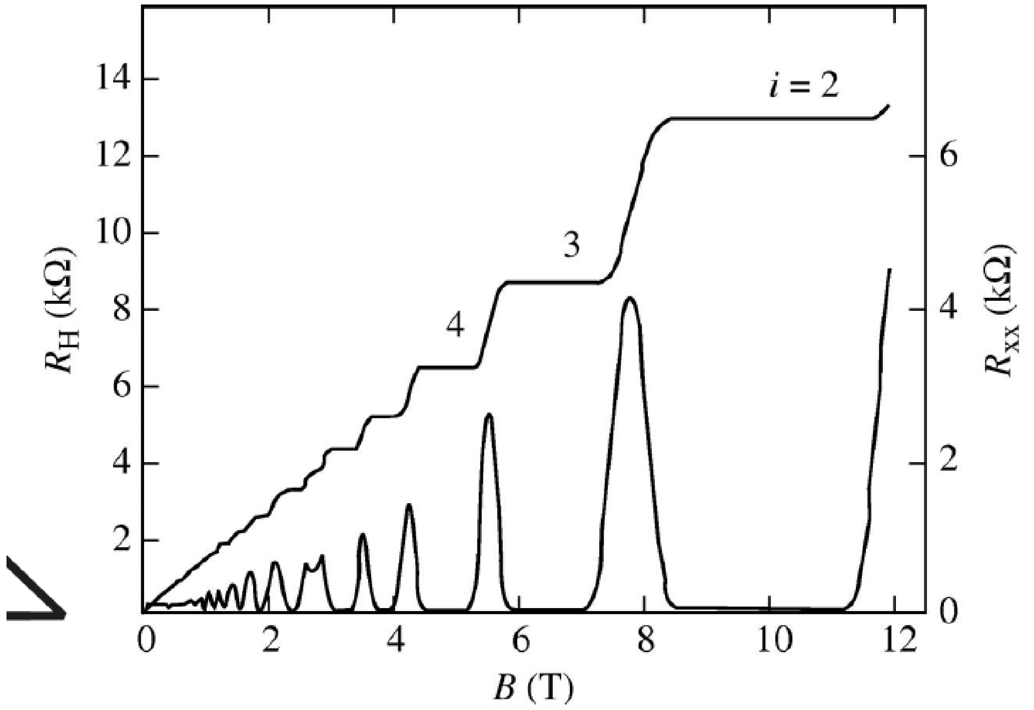
- **Quantization** of the Hall resistance at low temperature : $R_H = \frac{h}{e^2}\frac{1}{n}$ Results independent of geometrical and microscopic details

$R_K = \frac{e^2}{h} \approx 25812.807\,\Omega$ Quantum of resistance; UNIVERSAL constant

Used as a metrological unit : help to redefine the unit of mass !

Quantum Hall effect





K. v. Klitzing, G. Dorda, and M. Pepper, PRL 45, 494 (1980)

→ Quantum Hall conductivity changes by plateaus.

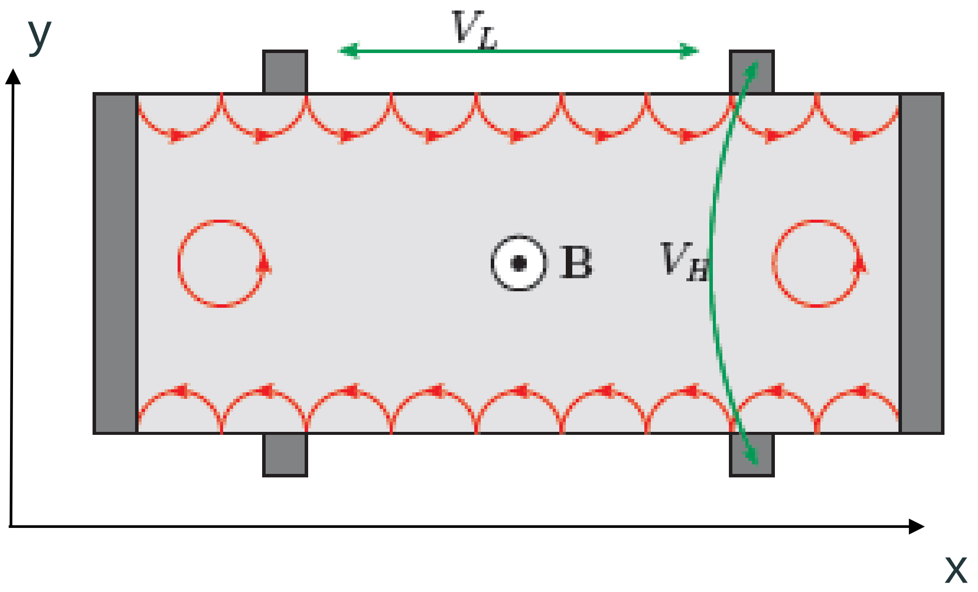
→ Each plateau is perfectly quantized by an integer number in unit of e^2/h

$$J_y = \sigma_{xy} E_x$$

$$\sigma_{xy} = n\frac{e^2}{h}$$

Integer accurate to 10^{-9}

Semi-classical picture

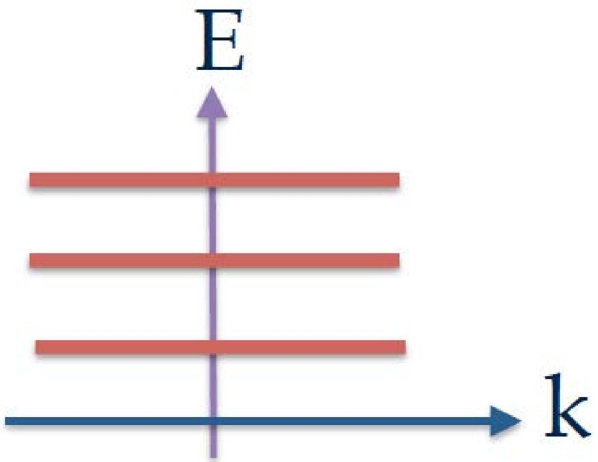


2D Cyclotron Motion,
Landau Levels

Electron in an orbital magnetic field :

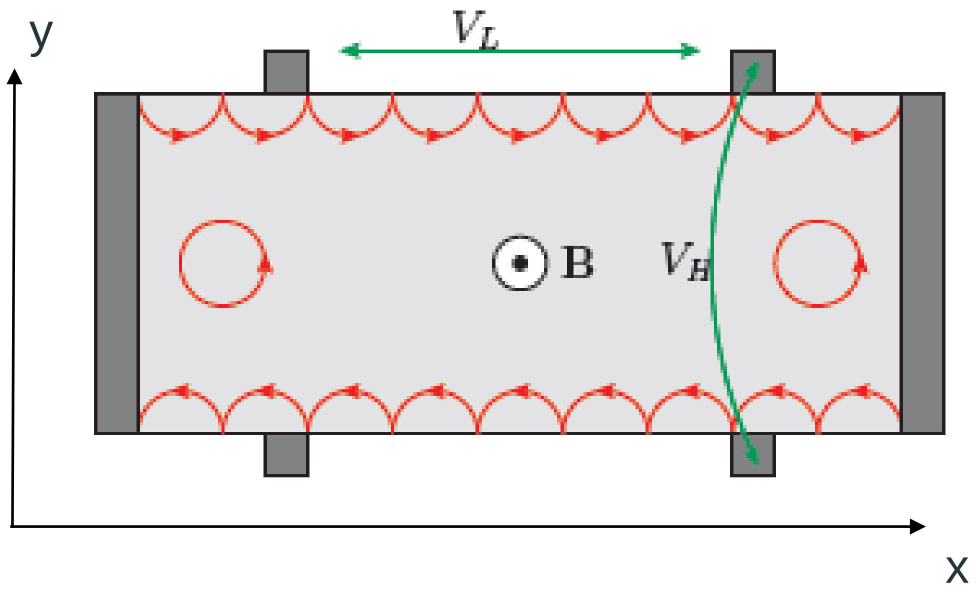
$$H = \frac{1}{2m_e}(\vec{p} + e\vec{A})^2$$

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega_c,$$ Landau levels

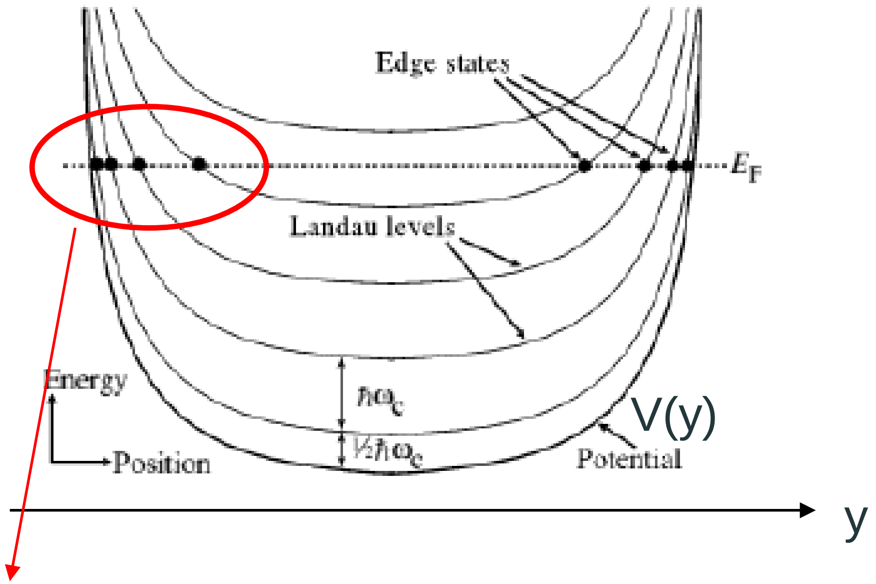


Why such perfect robustness & quantization ?

Semi-classical picture



2D Cyclotron Motion,
Landau Levels

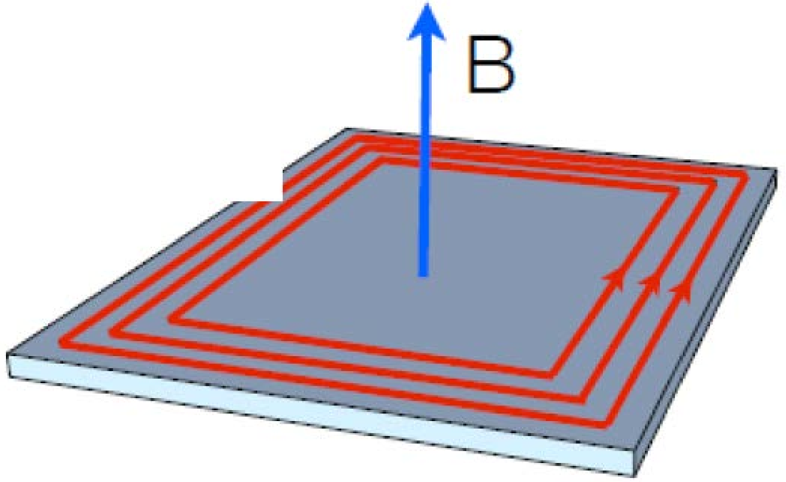


Edge states= skipping orbits

- Landau levels (LLs) bend near sample edge.
- The Fermi level intersects LLs at the edge.
- Nb of edge states at the Fermi level= Nb of occupied bulk LLs

Landau levels with a bulk gap and (protected) edge states

The edges' viewpoint: Robustness of n



- Electrons on same edge move along the same direction.

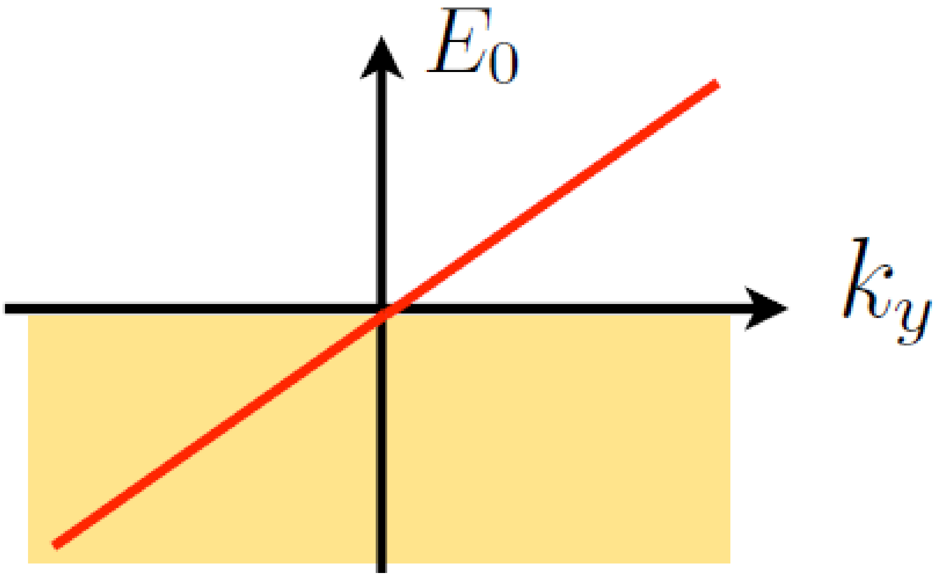
- Electrons on opposite edges move along the opposite directions.

Chirality = Consequence of time reversal symmetry breaking

Robustness against backscattering

- chiral edge state cannot be localized by disorder (no backscattering)

- edge states are therefore **perfect charge conductors**



Only 1 branch (chiral)

The bulk point of view

The quantum Hall effect: a topological property?

Distinction between the integer quantum Hall state and a conventional insulator is a topological property of the band structure

$\mathcal{H}(\mathbf{k})$: Brillouin zone $\longmapsto$ Hamiltonians with energy gap

Classified by Chern number: $n = \frac{i}{2\pi} \sum_{\substack{\text{filled} \\ \text{states}}} \int \mathcal{F} d^2k$ (= topological invariant) $n \in \mathbb{Z}$

Kubo formula : $$\sigma_{xy} = \frac{e^2}{h} \frac{i}{2\pi} \sum_{\substack{\text{filled} \\ \text{states}}} \int \mathcal{F} d^2k = \frac{e^2}{h} n$$

Thouless et al., PRL 49, 405 (1982)

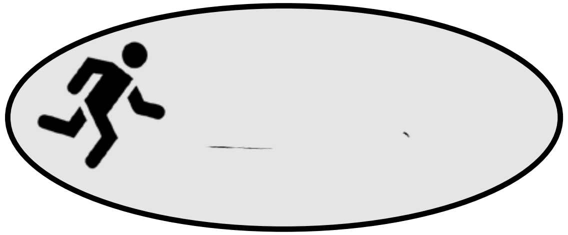
Alternative description: n is a bulk topological invariant

Example of a topological invariant

Can we tell by local measurements whether we are living on the surface of a sphere or a torus?

$g = 0$

$g = 1$

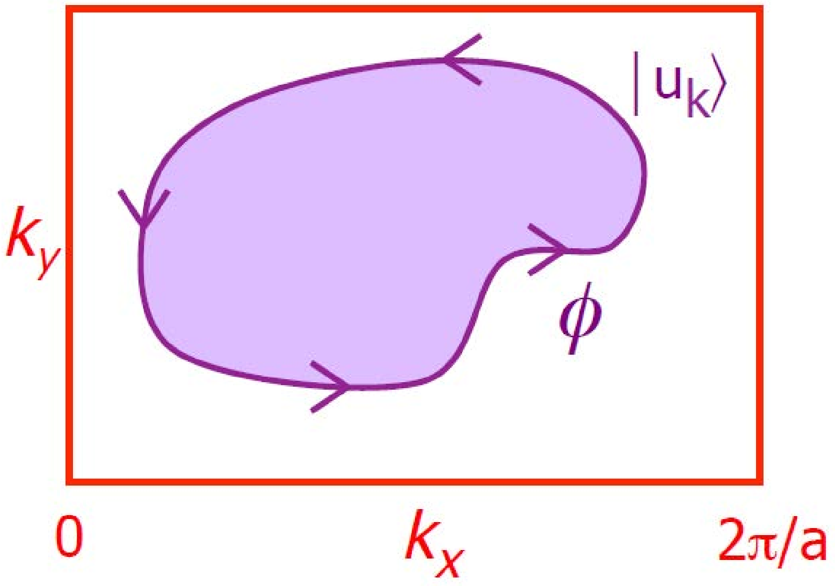


Gaussian curvature, $\boldsymbol{K_g} = \pm \frac{\mathbf{1}}{\boldsymbol{R_1 R_2}}$

Topological invariant = quantity that does not change under continuous deformation

Berry connection & curvature

For a given band, we can introduce :



$$u_{\mathbf{k}}(\mathbf{r}) = e^{-i\mathbf{k}\cdot\mathbf{r}}\psi_{\mathbf{k}}(\mathbf{r})$$

Berry connection:

$$\mathbf{A}(\mathbf{k}) = -\mathrm{Im}\,\langle u_{\mathbf{k}}|\nabla_{\mathbf{k}}|u_{\mathbf{k}}\rangle$$

Berry phase :

$$\phi = \oint \mathbf{A}(\mathbf{k})\cdot d\mathbf{k}$$

Berry curvature

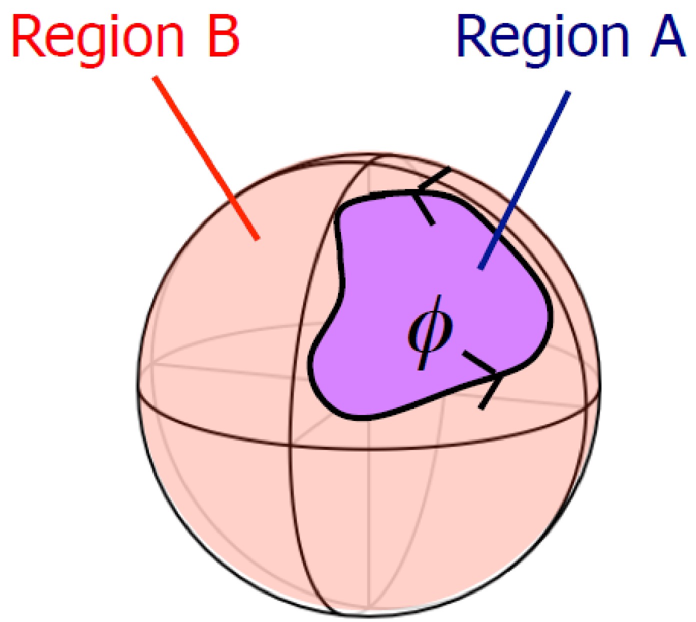
$$\Omega(\mathbf{k}) = \nabla \times \mathbf{A}$$

$$\Rightarrow \quad \Omega_z(\mathbf{k}) = -2\mathrm{Im}\left\langle \frac{du}{dk_x}\middle|\frac{du}{dk_y}\right\rangle$$

Stokes thm :

$$\phi = \int \Omega_z(\mathbf{k})\, d^2k$$

Chern theorem



Berry curvature

($F \equiv \Omega$)

Stokes thm applied to A:

$$\phi = \int_A \mathcal{F}(\lambda)\, dS_\lambda \mod 2\pi$$

Stokes applied to B:

$$\phi = -\int_B \mathcal{F}(\lambda)\, dS_\lambda \mod 2\pi$$

Subtract:

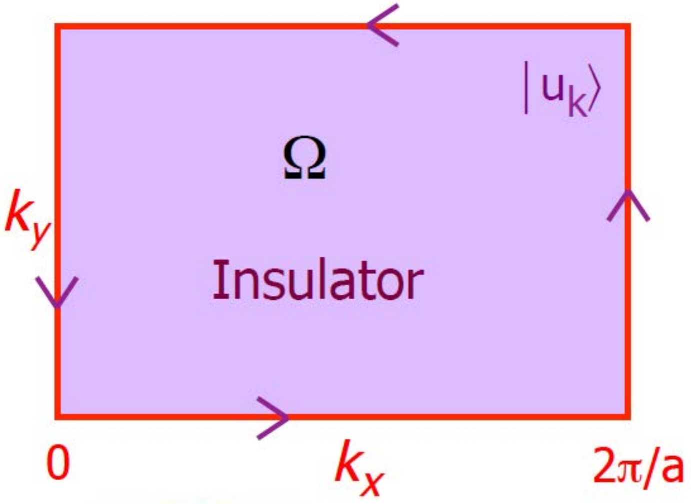
$$0 = \oint \mathcal{F}(\lambda)\, dS_\lambda \mod 2\pi$$

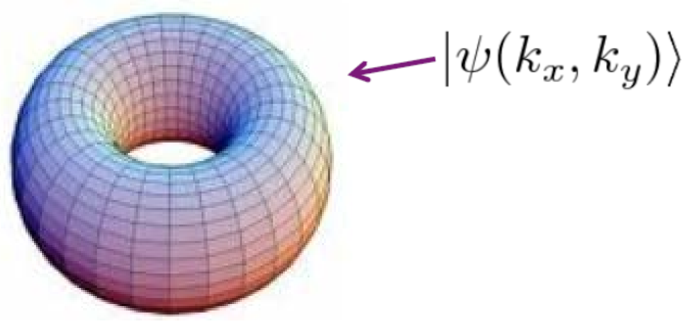
Chern Theorem: $$\oint \mathcal{F}(\lambda)\, dS_\lambda = 2\pi C$$ with C $\in \mathbb{Z}$

C = First Chern number

Application of Chern theorem

Let us apply this result to the Brillouin zone





Application of Chern theorem

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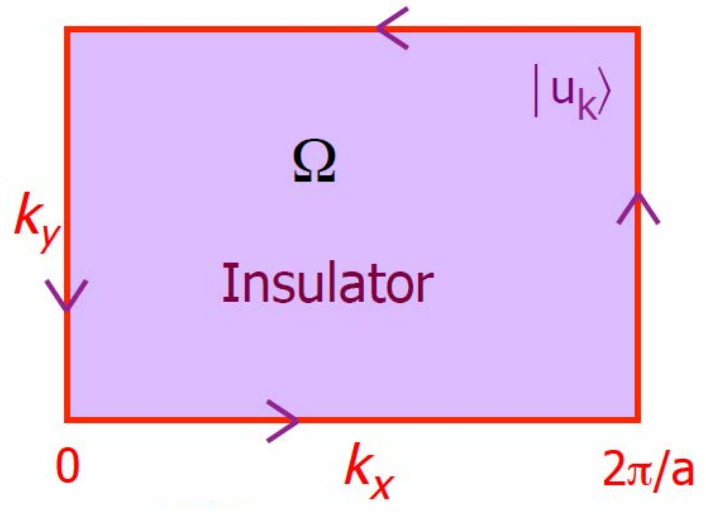
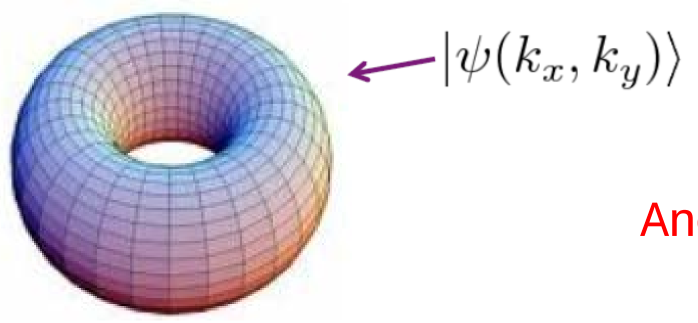


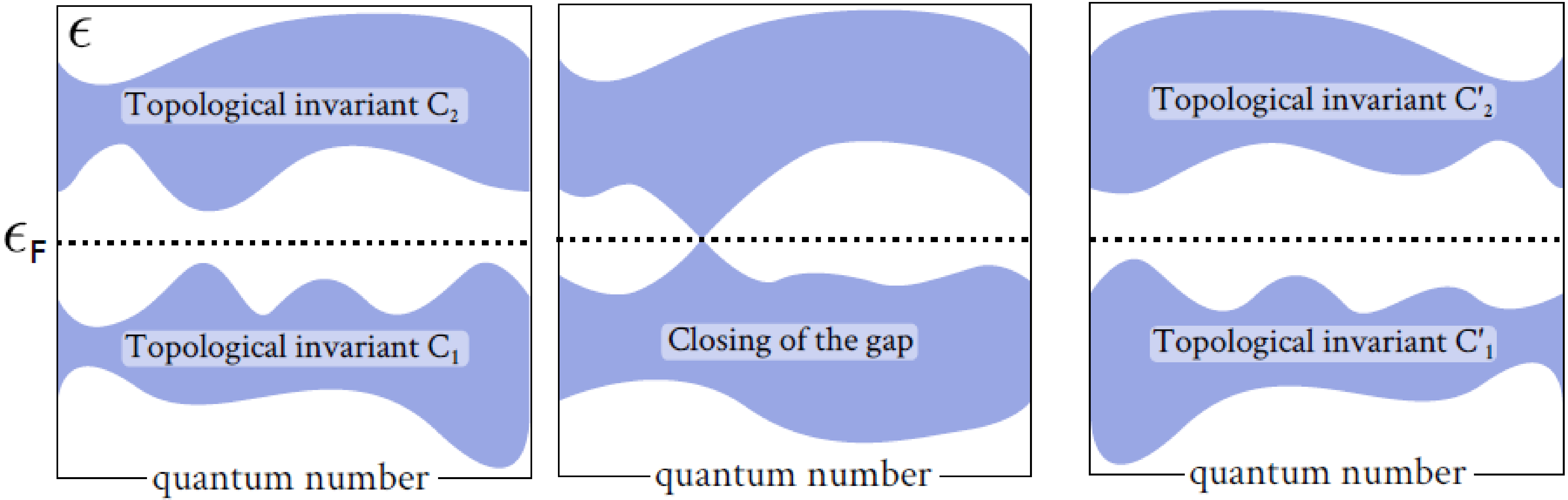


$$\Omega_z(\mathbf{k}) = -2\mathrm{Im}\left\langle \frac{du}{dk_x} \middle| \frac{du}{dk_y} \right\rangle$$

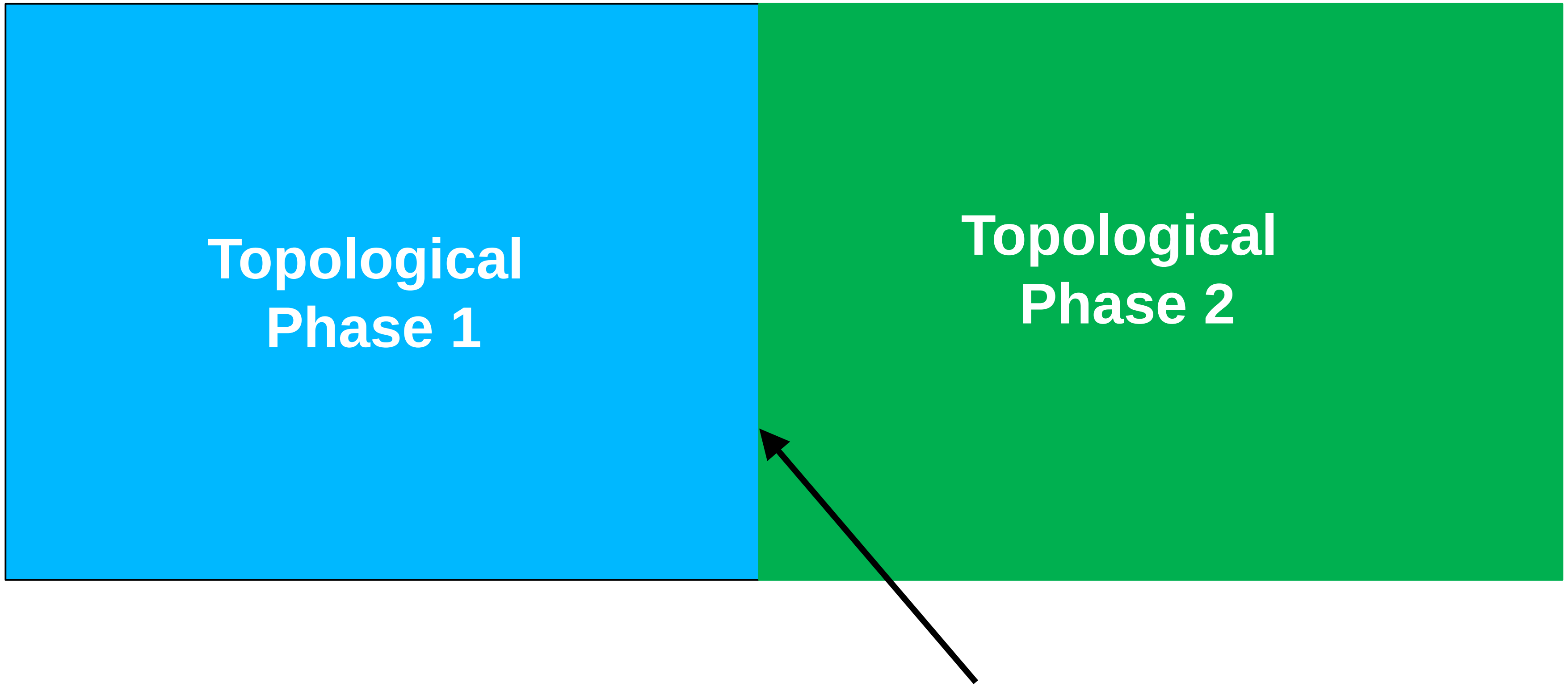
$$\phi = \int_{BZ} \Omega_z(\mathbf{k})\, d^2k = 2\pi C$$

Anomalous Hall conductivity: $\sigma_{xy} = \frac{e^2}{h} C$

Topological phase transition

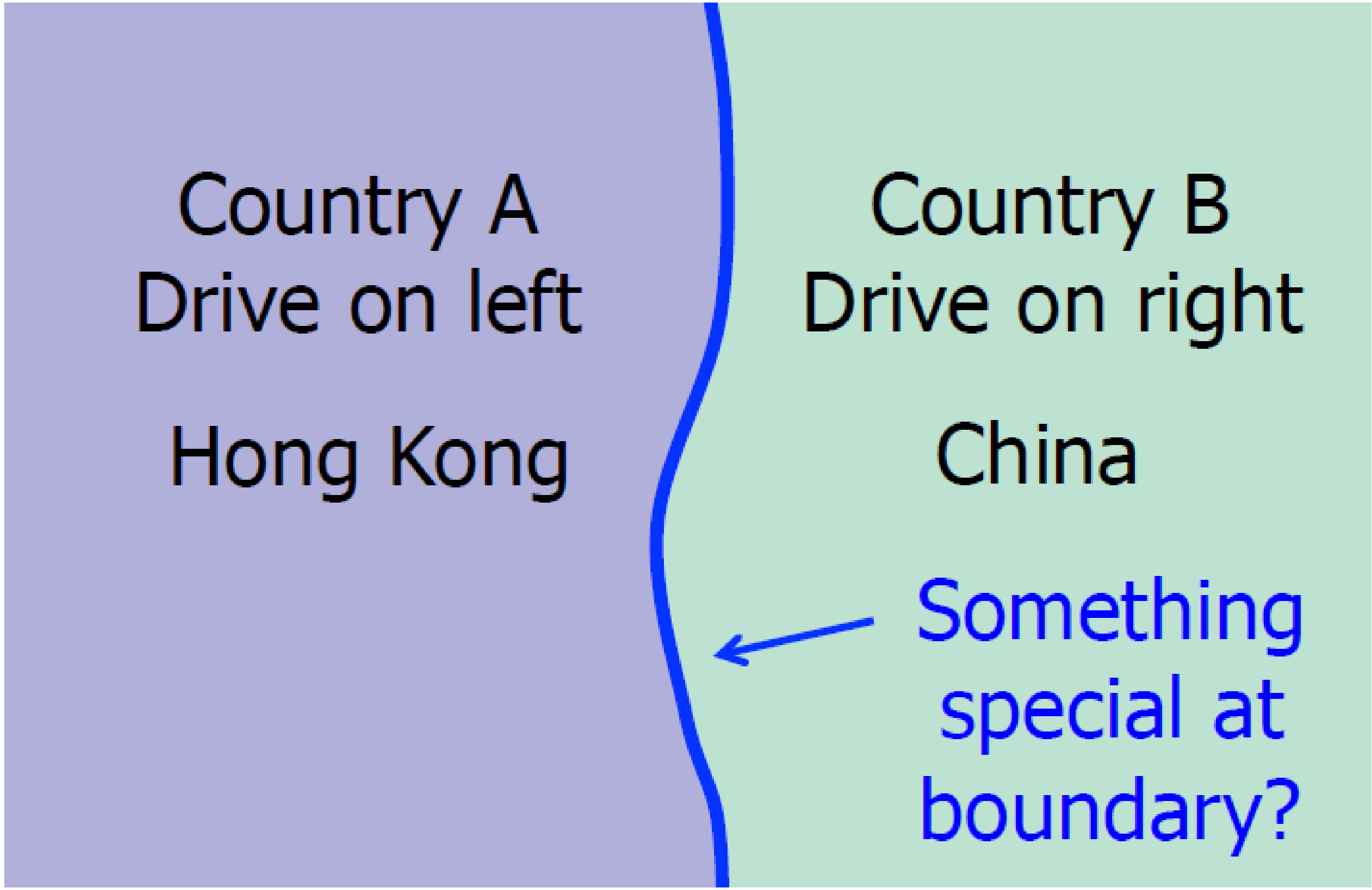


Bulk-edge correspondence



Something special at the boundary

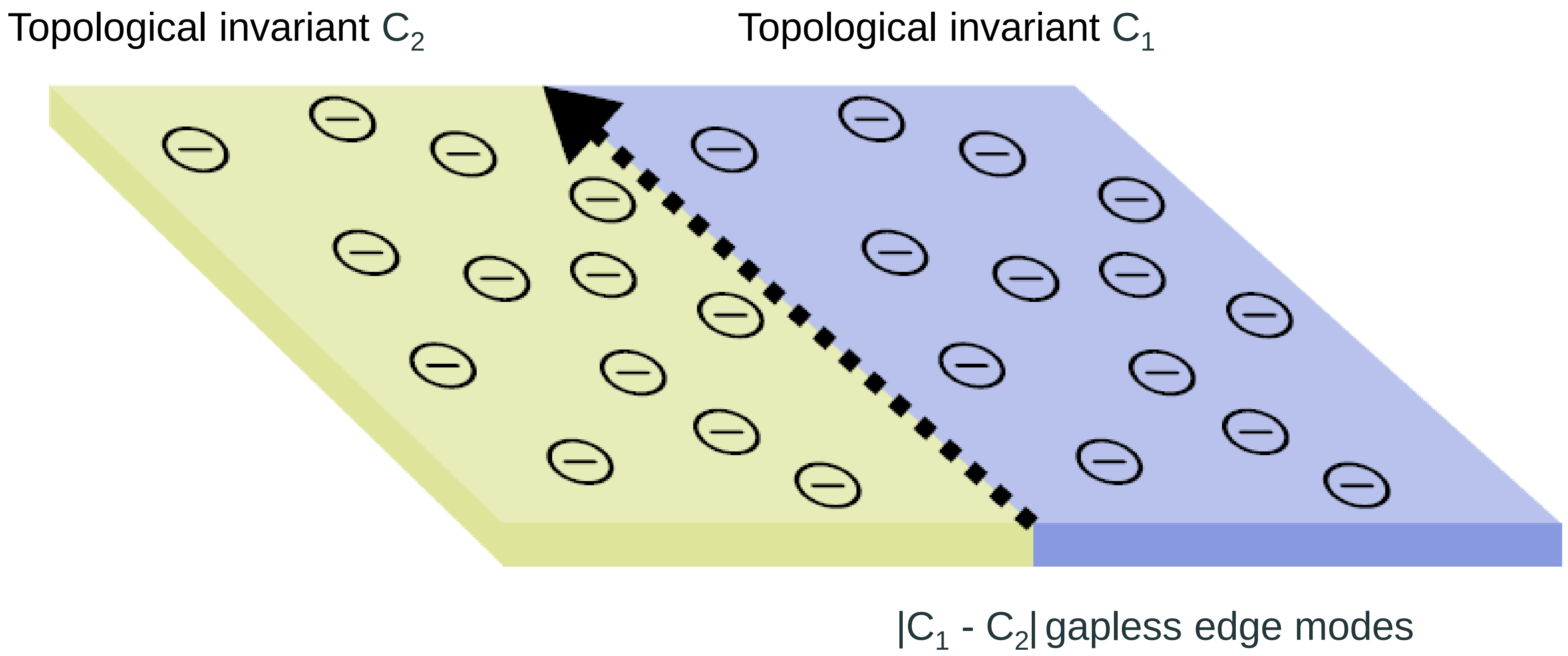
Bulk-edge correspondence



Bulk-edge correspondence

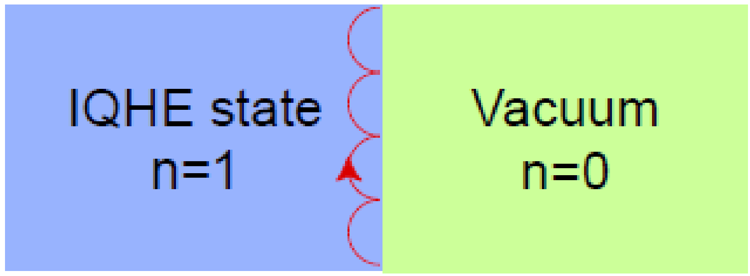
Bulk-edge correspondence

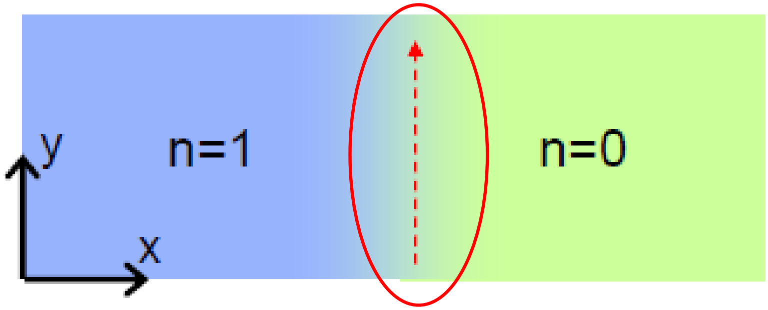
Two materials described by different topological invariants C_1 and C_2 placed in contact ➔ emergence of $|C_1 - C_2|$ gapless edge modes



Chiral edges states in the QHE

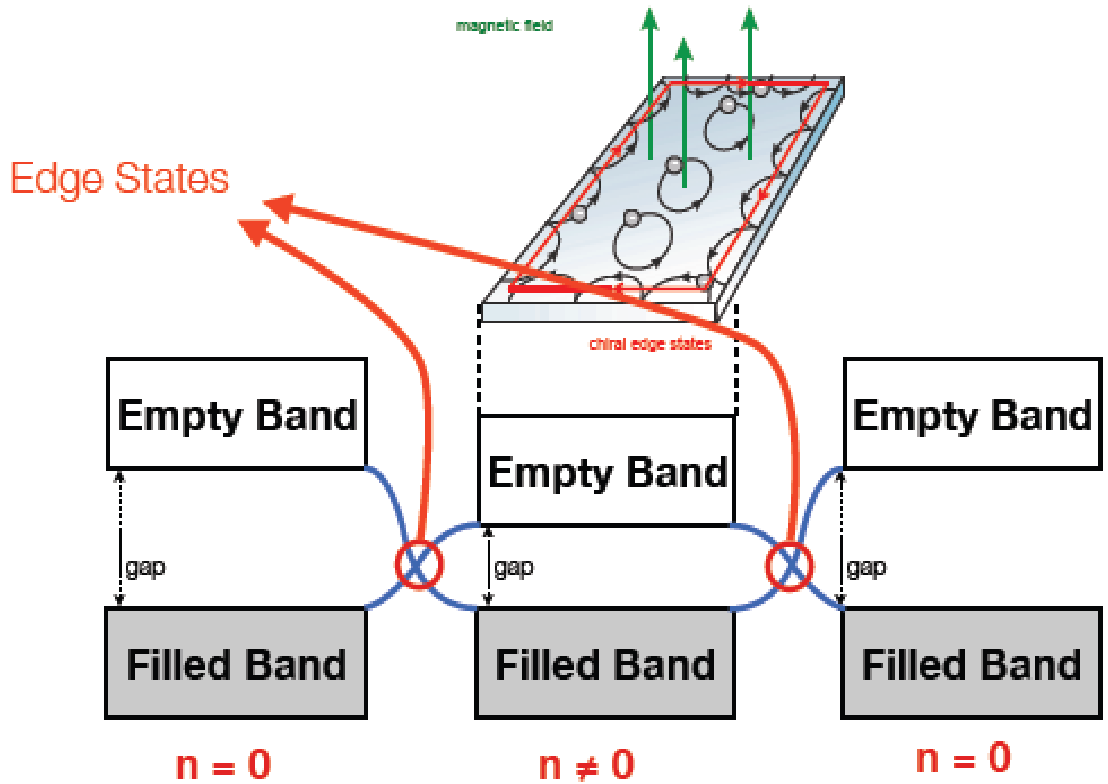
Gapless states must exist at the interface between different topological phases





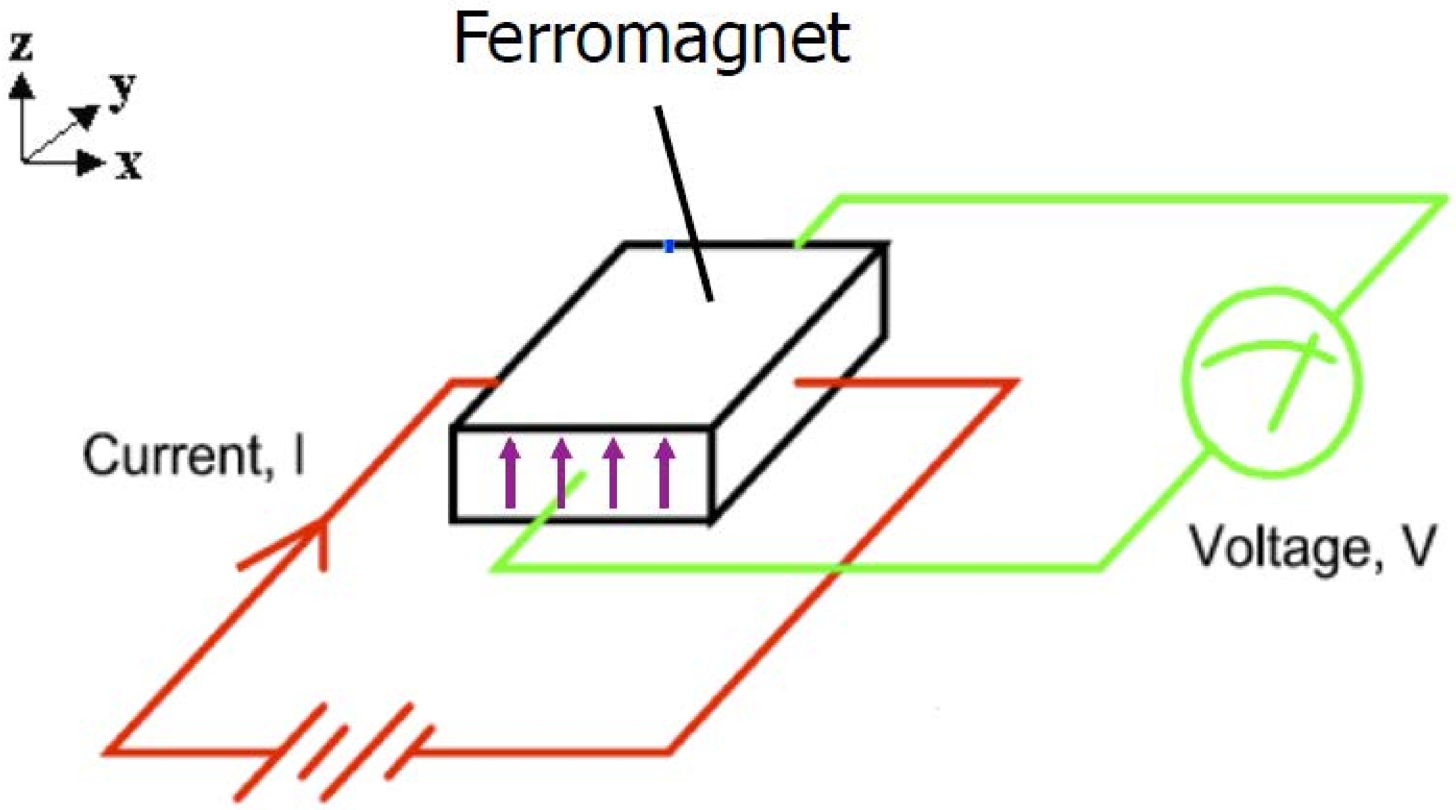
Edge states ~ skipping orbits

Smooth transition : band inversion



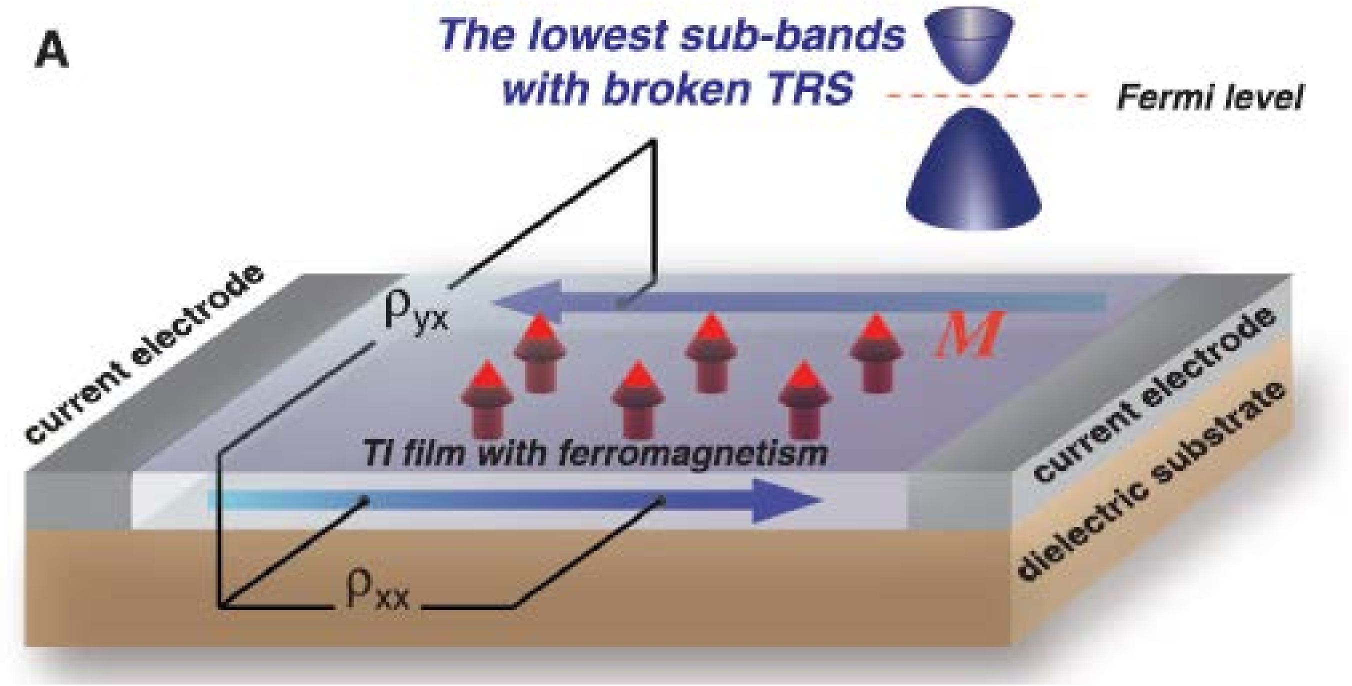
II) The anomalous quantum Hall effect or the 2D Chern insulator

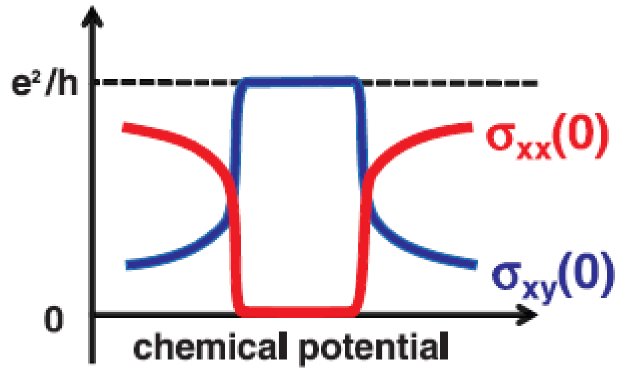
Anomalous Hall effect (1881)

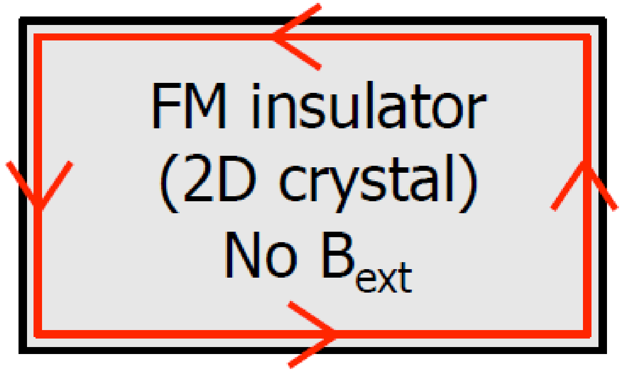


Measure of Hall conductivity in absence of a magnetic field

Quantum anomalous Hall effect (2013 ?)



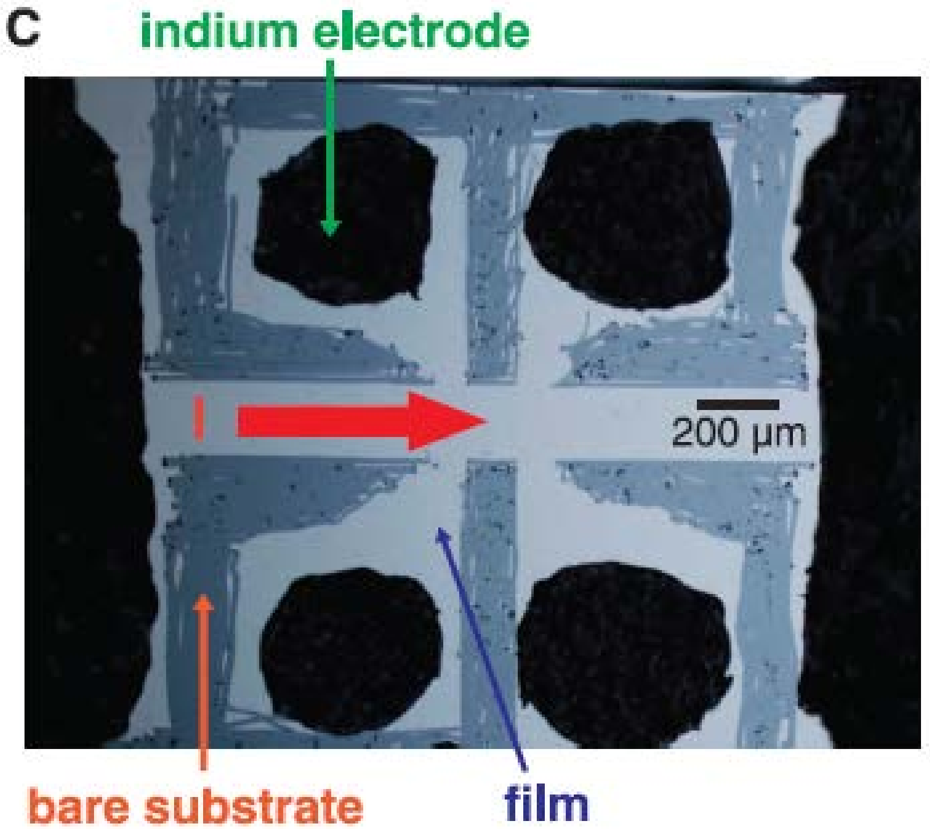


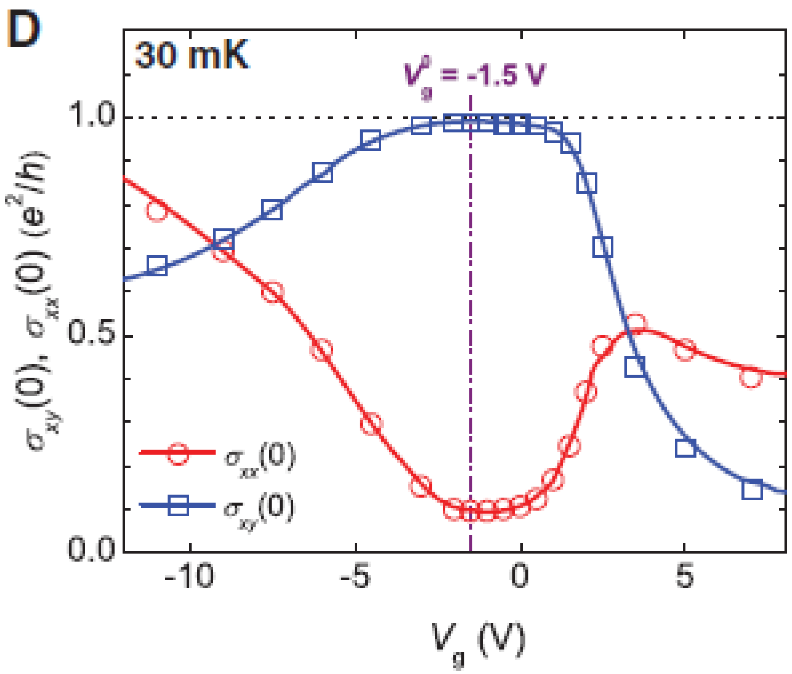


Anomalous Hall conductivity $\sigma_{xy} = \frac{e^2}{h} C$

Like integer quantum Hall effect, but no Bext

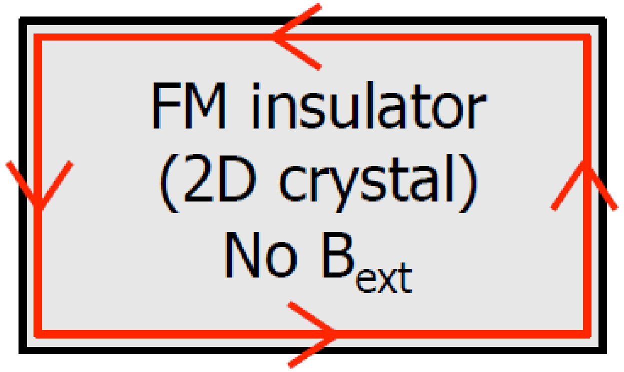
Quantum anomalous Hall effect (2013 ?)

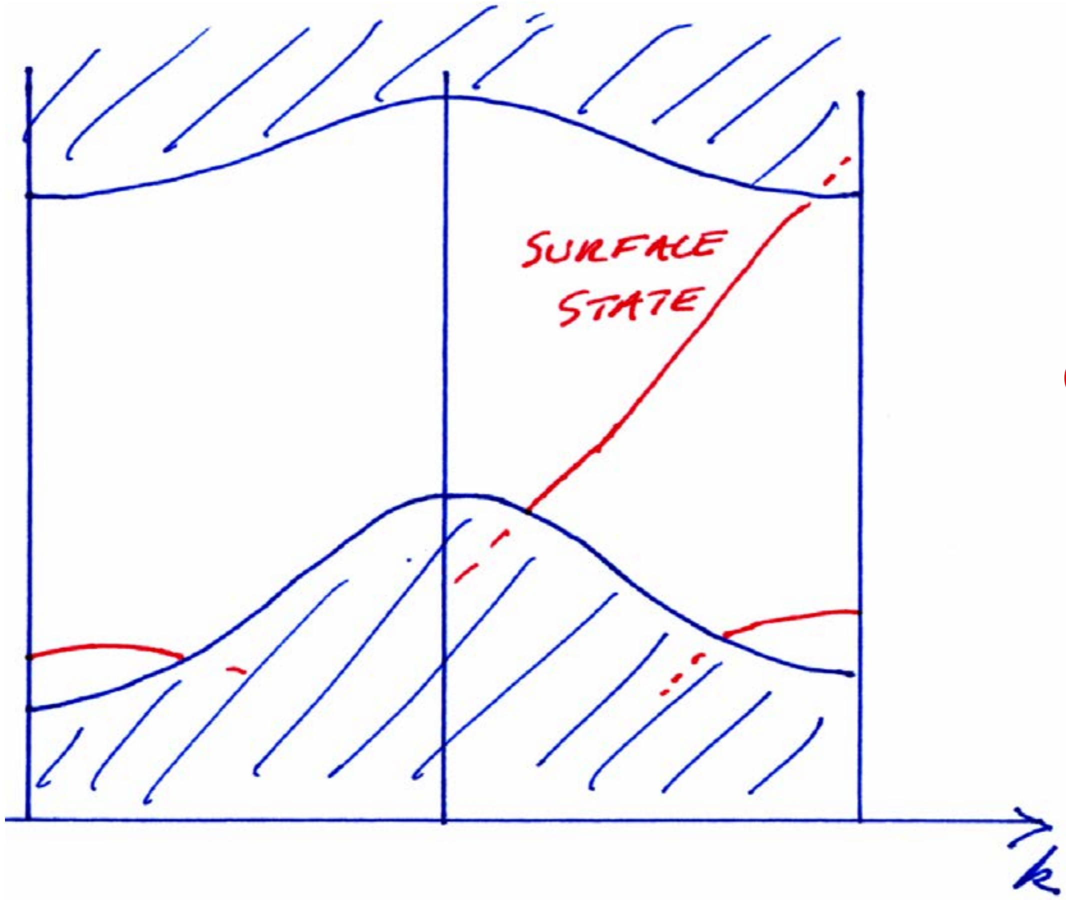




C.-Z. Zhang et al., Science 340, 167 (2013)

Edge states: 2D QAH insulator

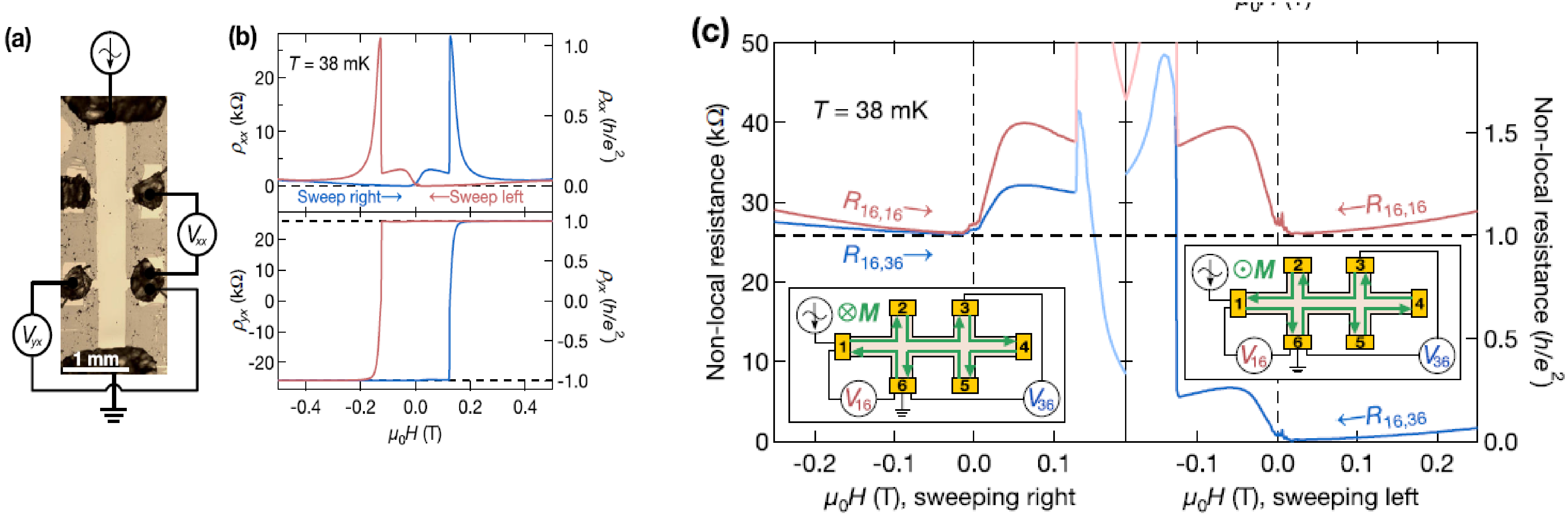




C = +1

Existence of a chiral edge state without magnetic field !

Edge states: 2D QAH insulator



A. J. Bestwick et al., PRL 114, 187201 (2015)

Proof of principle: the Haldane model

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

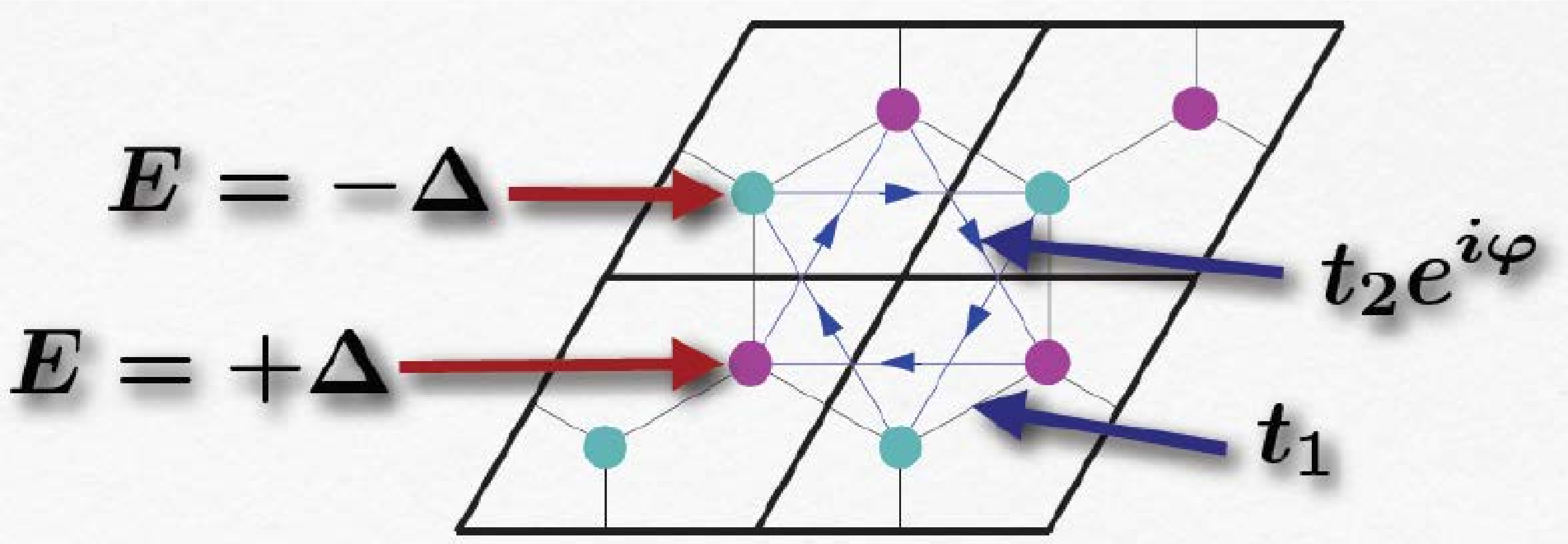
Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

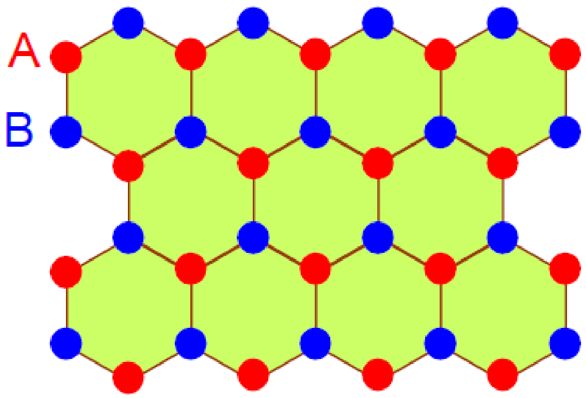
Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.



Graphene



One orbital per site

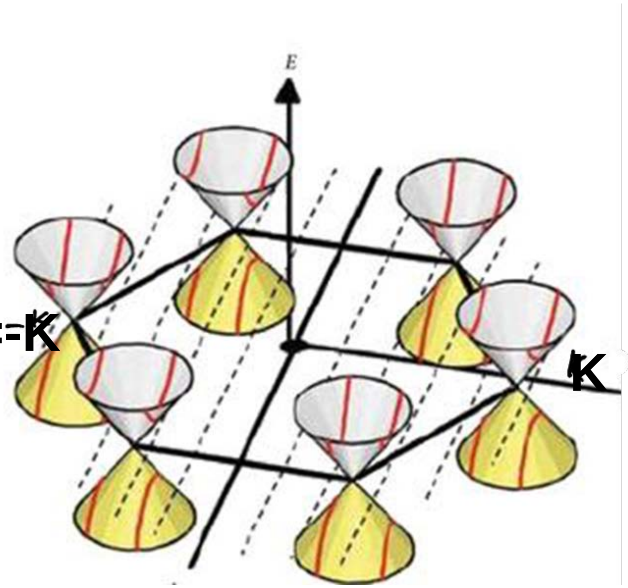
Two atoms per unit cell (A and B)

Spinless

Band structure near Dirac cones

A/B sublattice

$$h(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$$



Emergence of massless Dirac fermions at low energies:

K / K' valley

$$h(\mathbf{q}) \approx v\tau^z\sigma^x q_x + v\sigma^y q_y$$

Momentum measured from Dirac node

Symmetries of graphene

- Inversion symmetry A sublattice ⟷ B sublattice

$$\hat{\mathcal{P}} = \sigma_x \tau_x$$

$$\hat{\mathcal{P}} h(\mathbf{q}) \hat{\mathcal{P}} = h(-\mathbf{q})$$

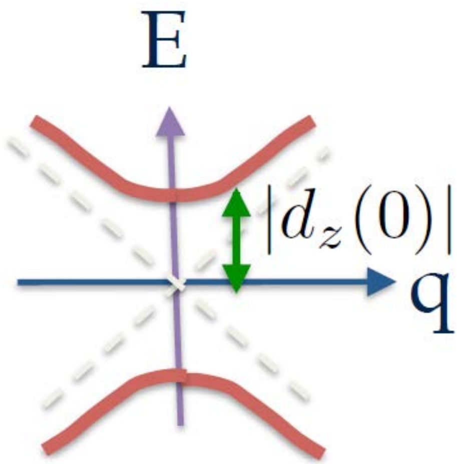
- Time reversal symmetry:

$$\hat{\mathcal{T}} = \tau_x \mathcal{K}$$

$$\hat{\mathcal{T}} h(\mathbf{q}) \hat{\mathcal{T}} = h(-\mathbf{q})$$

Making graphene insulating

$$h(\mathbf{q}) = v\,\tau^z \sigma^x\, q_x + v\,\sigma^y q_y + d_z(\mathbf{q})\sigma^z$$



Need to break either time-reversal symmetry or inversion symmetry

(i) Break inversion symmetry

$$d_z(\mathbf{q}) = m_S$$

Semenoff insulator (1984)

(ii) Break time-reversal symmetry

$$d_z(\mathbf{q}) = m_H \tau^z$$

Haldane insulator (1988)
= Quantum spin Hall insulator
= Chern insulator

Proof of principle: the Haldane model

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Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

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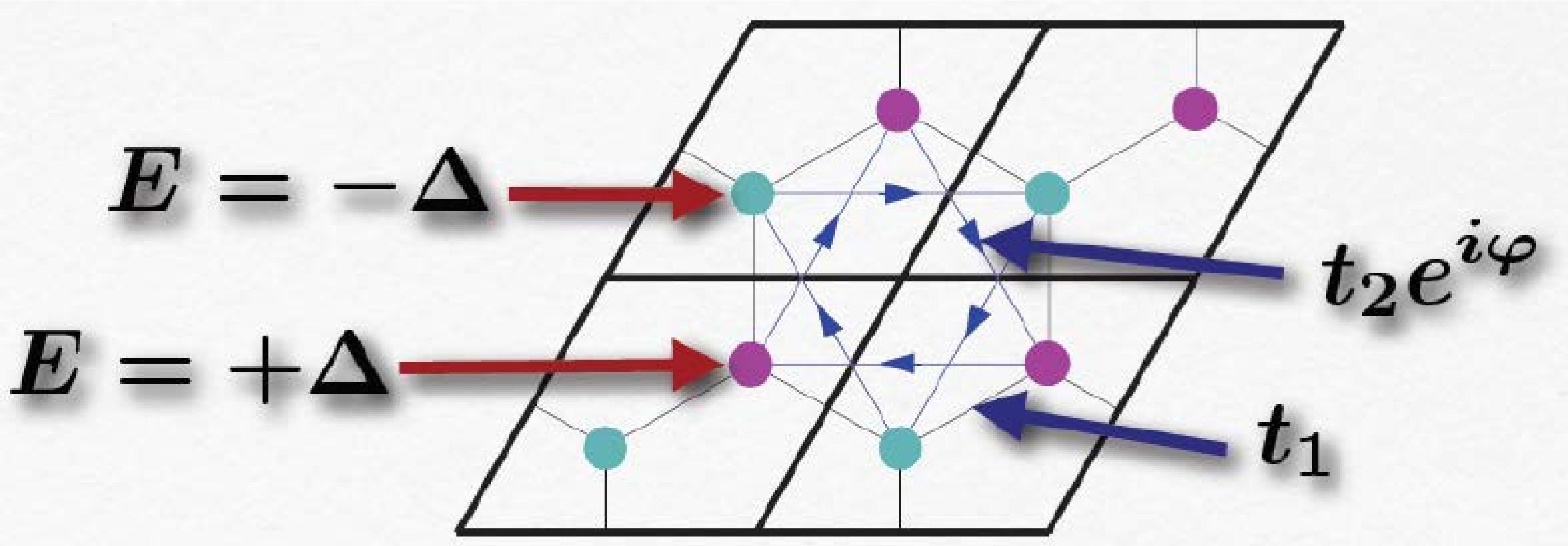
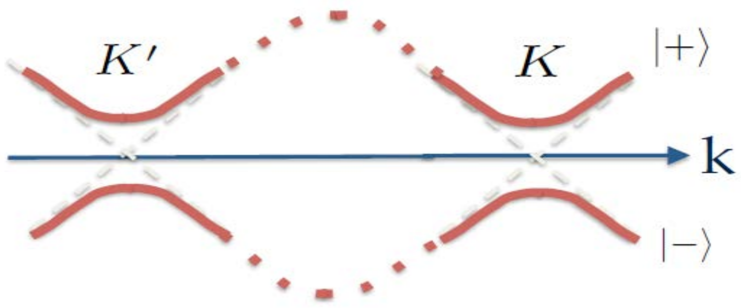
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.



Topological characterization

$$h(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$$



Two strategies:

i) Compute the eigenvectors, Berry connection, Berry phase and Chern number.

ii) Look at **d(k)**

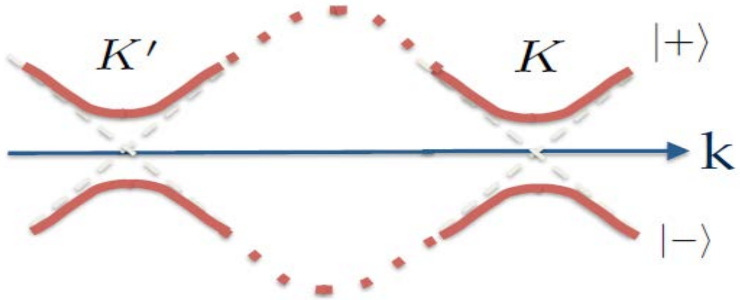
$$E_{\pm} = \pm |\mathbf{d}(\mathbf{k})|$$ Spectrum flattening $$\hat{\mathbf{d}}(\mathbf{k}) = \frac{\mathbf{d}(\mathbf{k})}{|\mathbf{d}(\mathbf{k})|}$$

Mapping: $\hat{\mathbf{d}}(\mathbf{k})$: Brillouin zone $\longmapsto \hat{\mathbf{d}}(\mathbf{k}) \in S^2$

$$\text{“}\pi_2(S^2) = \mathbb{Z}\text{”}$$

Topological characterization

$$h(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$$



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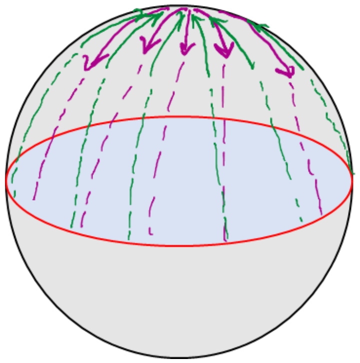
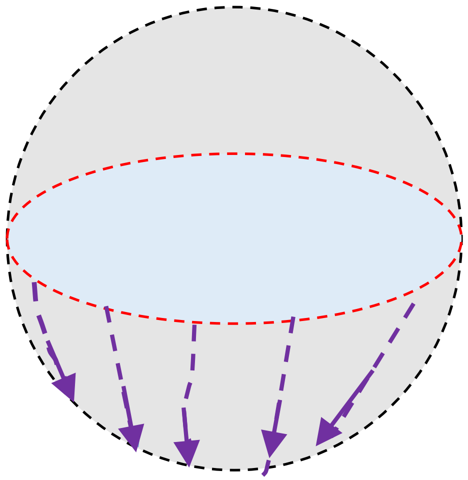
Semenoff insulator



Trivial insulator: $m_K = m_{K'}$

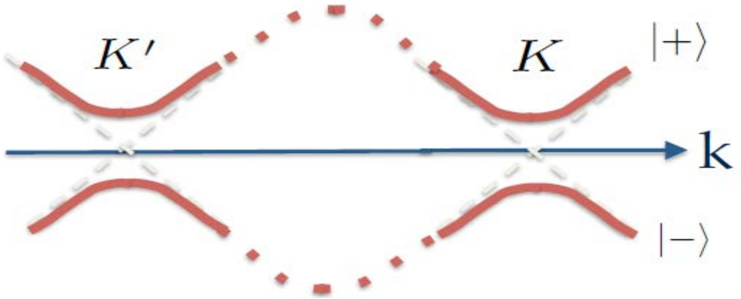
Haldane insulator



$$m_K = -m_{K'}$$

Topological characterization

$$h(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$$



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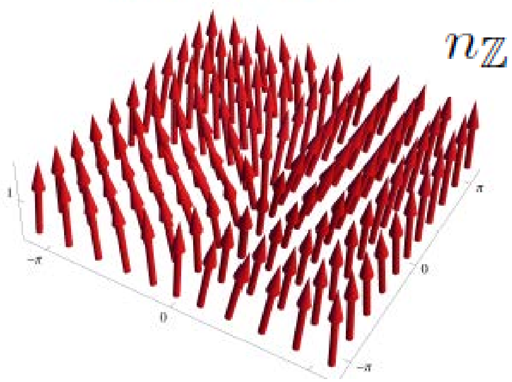
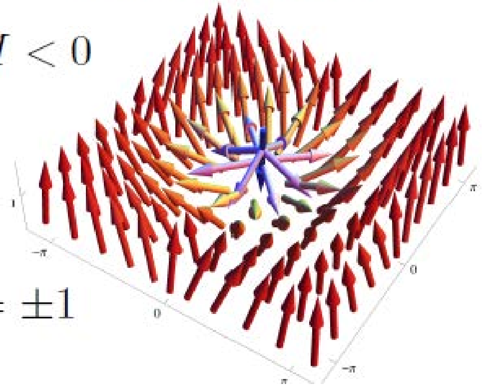
trivial phase



$n_{\mathbb{Z}} = 0$

non-trivial phase

$M < 0$

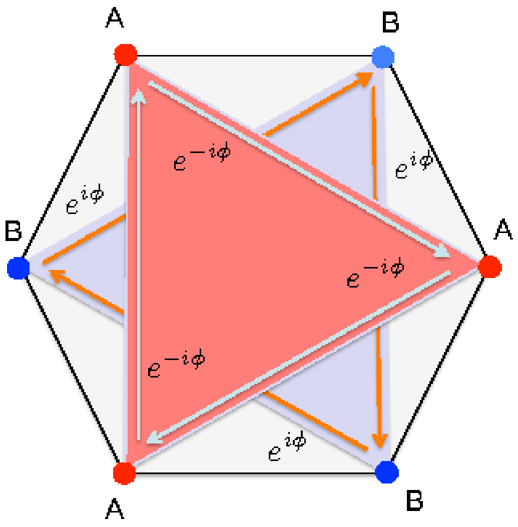


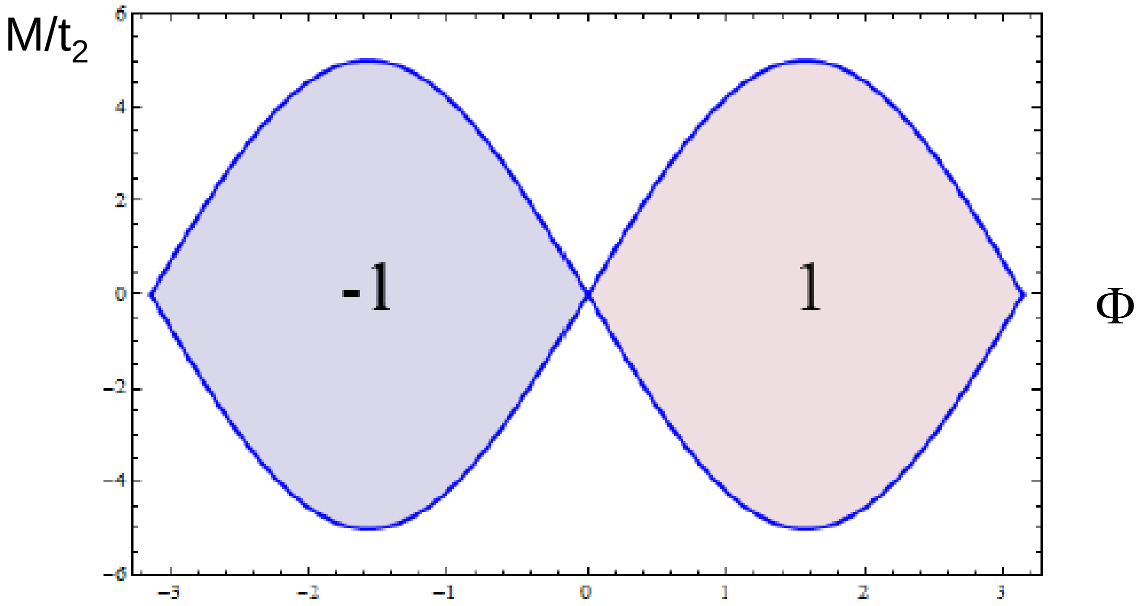
$= \pm 1$

$n_{\mathbb{Z}} = \pm 1 = \mathrm{sgn}(m_H)$

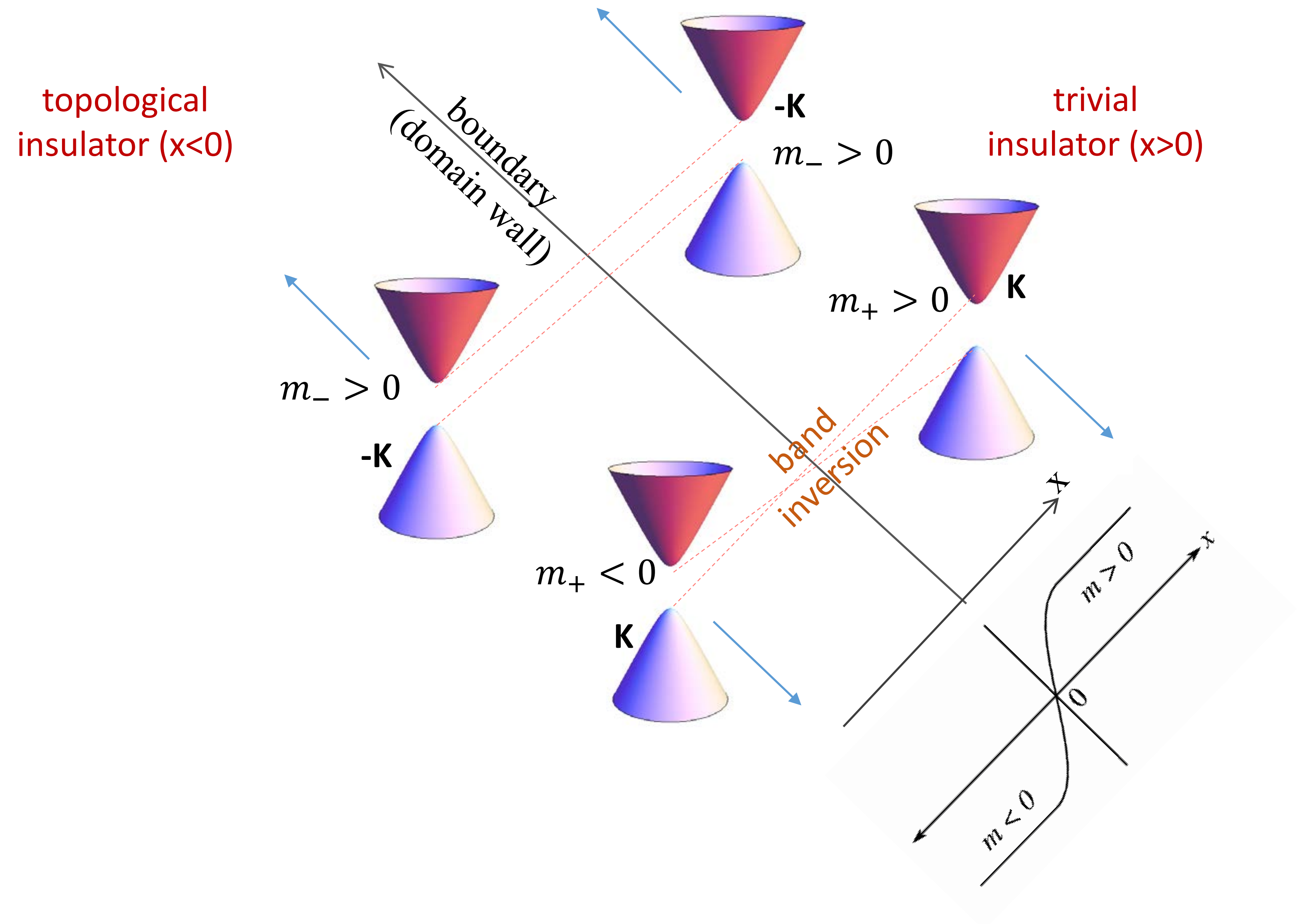
Chern number: $n_{\mathbb{Z}} = \frac{1}{8\pi} \int_{\mathrm{BZ}} d^2\mathbf{k}\, \epsilon^{\mu\nu} \hat{\mathbf{d}} \cdot \left[\partial_{k_\mu} \hat{\mathbf{d}} \times \partial_{k_\nu} \hat{\mathbf{d}}\right]$

Phase diagram of the Haldane model



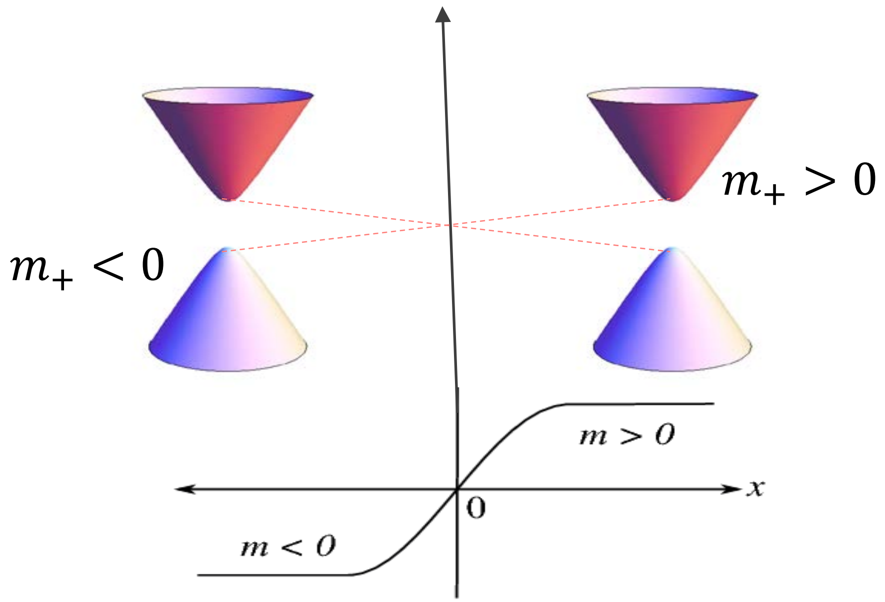


Bulk-boundary correspondence: Application to the Haldane model



Domain wall along the x-axis

Dispersing Jackiw-Rebbi-like edge modes



k_y conserved

$$\mathcal{H}_+ = v_F\left(-i\hat{\sigma}_x\partial_x + i\hat{\sigma}_y k_y\right) + m_+(x)\hat{\sigma}_z$$

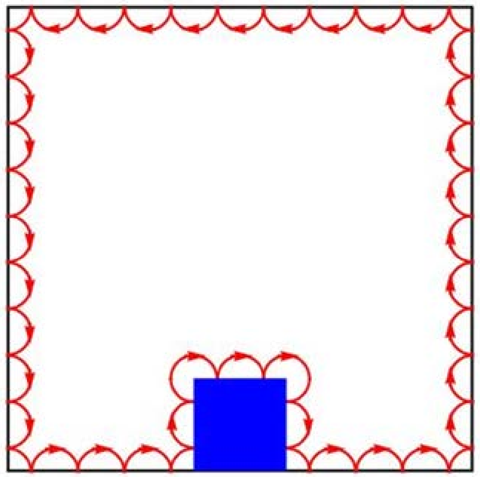
Fixing k_y maps the problem on the 1D Jackiw-Rebbi model, with the edge mode

$$|\psi(k_y)\rangle = e^{ik_y y}\ exp\left[-\frac{1}{v_F}\int_{-\infty}^{x}|m_+(x')|dx'\right]|\chi_+\rangle \quad \text{where} \quad |\chi_+\rangle = \begin{pmatrix}1\\ i\end{pmatrix}$$

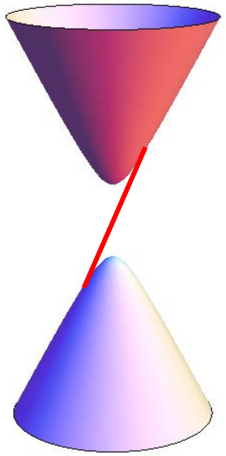
$$\mathcal{H}_+|\psi(k_y)\rangle = v_F k_y|\psi(k_y)\rangle$$ ➡ **CHIRAL STATE**

Properties of the chiral edge mode

$$|\psi(k_y)\rangle = e^{ik_y y}\ exp\left[-\frac{1}{v_F}\int_{-\infty}^{x}|m_+(x')|dx'\right]|\chi_+\rangle$$



Conducting chiral edge



- The chiral mode can not be stopped by any obstacle or edge disorder.
- Normally, any 1D system localizes at low temperature (Anderson insulator). The chiral edge is protected from localization.
- Such a 1D mode can not appear in a pure 1D system, only at a boundary of a higher-dimensional system.
- The chiral edge carries the quantized Hall conductivity (IQHE). $\sigma_{xy}=j_x/E_y= n\, e^2/h$

III) A brief incursion into 2D topological insulators Or The 2D spin quantum Hall insulator

Destroying Dirac points in spinfull graphene

Graphene Hamiltonian with spin & valley indices restored

$$\mathcal{H} = v_F\left(\hat{\mathbb{I}}\otimes\hat{\tau}_z\otimes\hat{\sigma}_x\, q_x + \hat{\mathbb{I}}\otimes\hat{\mathbb{I}}\otimes\hat{\sigma}_y\, q_y\right) + \hat{V}$$

Spin   Valleys (K & K')   Sublattices (A & B)   gap-opening perturbation

1. Inversion (P-) breaking perturbation (trivial insulator, e.g. Boron nitride)

$$\hat{V} = m_p\hat{\mathbb{I}}\otimes\hat{\mathbb{I}}\otimes\hat{\sigma}_z$$

2. T-reversal breaking perturbation (Chern insulator, e.g. Haldane model)

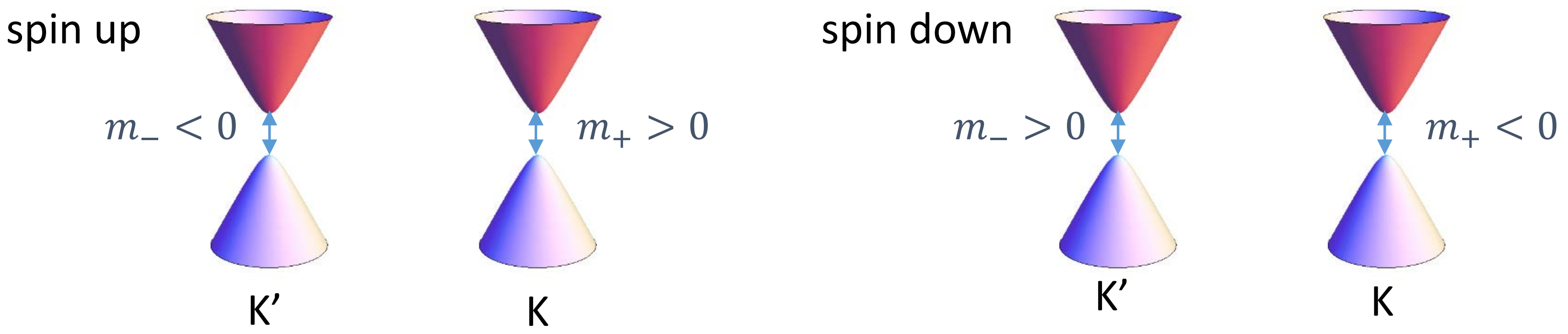
$$\hat{V} = m_T\hat{\mathbb{I}}\otimes\hat{\tau}_z\otimes\hat{\sigma}_z$$

3. Symmetry preserving perturbation (topological insulator, Kane-Mele model)

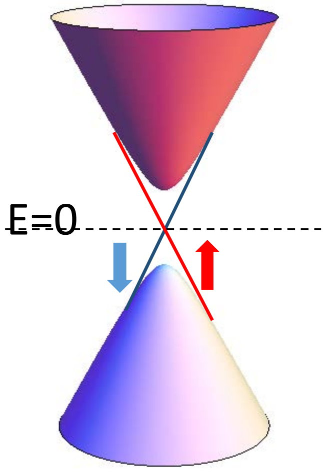
$$\hat{V} = m_{SO}\hat{S}_z\otimes\hat{\tau}_z\otimes\hat{\sigma}_z$$

The Kane-Mele model

Kane-Mele model = Haldane model ^{2}



$$\breve{\mathcal{H}}_{Kane-Mele} = \begin{pmatrix} \hat{\mathcal{H}}_{Haldane} & 0 \\ 0 & \hat{\mathcal{H}}^{*}_{Haldane} \end{pmatrix}$$



Spin-Hall conductivity:

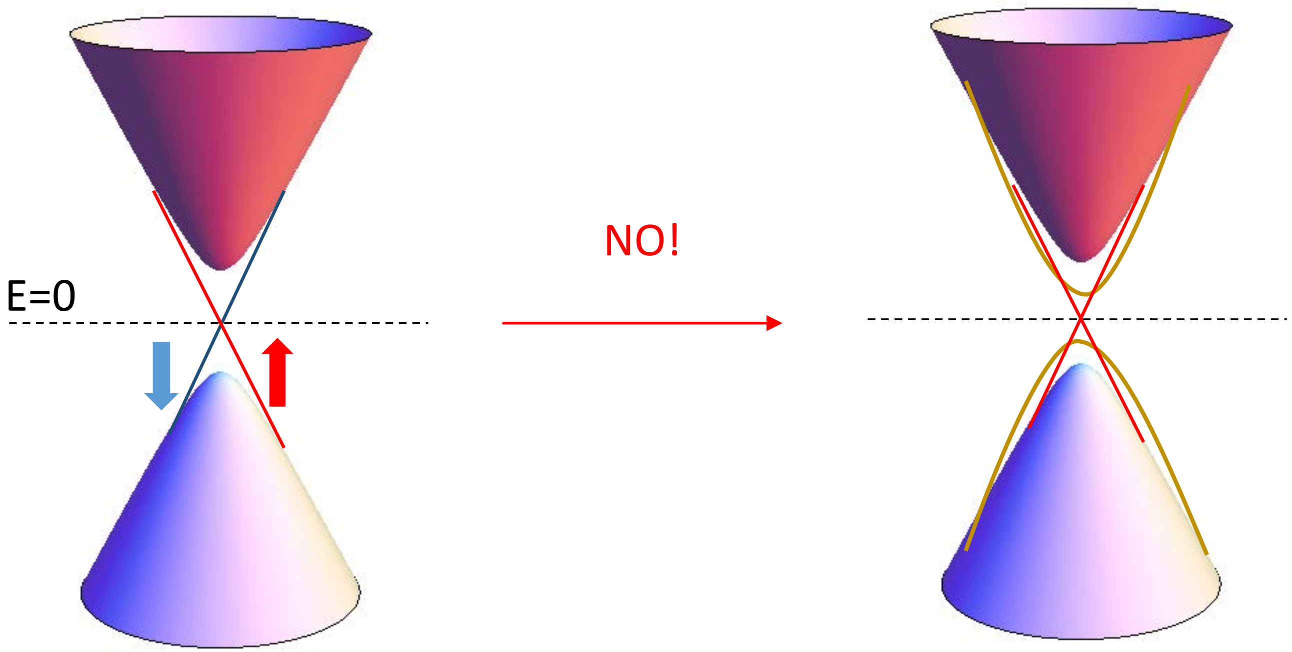
$$\sigma^{s}_{xy} = \sigma^{\uparrow}_{xy} - \sigma^{\downarrow}_{xy} = (n_{\uparrow} - n_{\downarrow})\frac{e^2}{h}$$

Can the degeneracy be lifted?

- Other spin-orbit couplings are possible (e.g., Rashba), which introduce off-diagonal terms and break spin conservation (no notion of spin up or down exists)

$$\begin{pmatrix} \hat{\mathcal{H}}_{Haldane} & 0 \\ 0 & \hat{\mathcal{H}}^*_{Haldane} \end{pmatrix} \longrightarrow \begin{pmatrix} \hat{\mathcal{H}}_{Haldane} & \# \\ \# & \hat{\mathcal{H}}^*_{Haldane} \end{pmatrix}$$

- Can the generic spin-orbit perturbation lift the degeneracy?

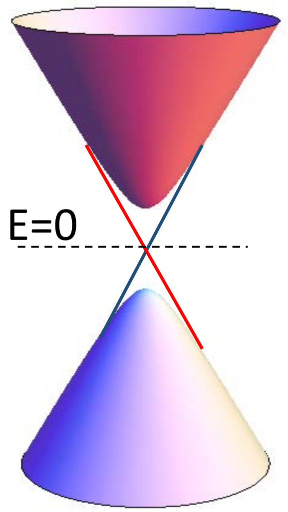


The degeneracy is protected by T-reversal symmetry

- Time-reversal operator = spin-rotation and complex conjugation

$$\mathbb{T}\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix} = e^{i\pi\hat{S}_y}\begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow\end{pmatrix}^* = \begin{pmatrix}\psi_\downarrow^*\\ -\psi_\uparrow^*\end{pmatrix}, \qquad \mathbb{T}^2 = -1$$

- Time-reversal symmetry implies $[\mathcal{H},\mathbb{T}]$=0
- This guarantees double-degeneracy of the spectrum
- Hence, the must be (at least) 2 distinct, degenerate states with energy E connected by $\mathbb{T}$-reversal (Kramers doublet).
- We can't remove degeneracy at E=0, as long as perturbation does not break $\mathbb{T}$-reversal!



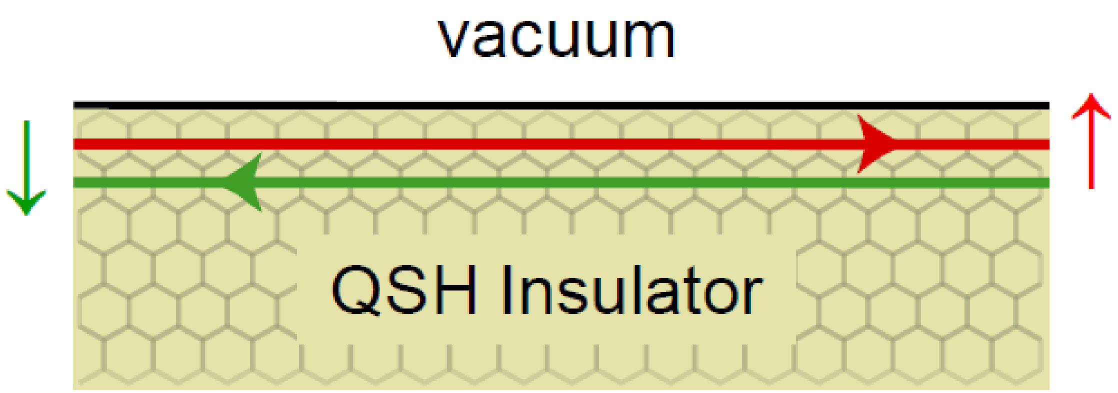
A **(non-Chern) topological invariant** is responsible for this robustness

This is the **Z2 invariant**

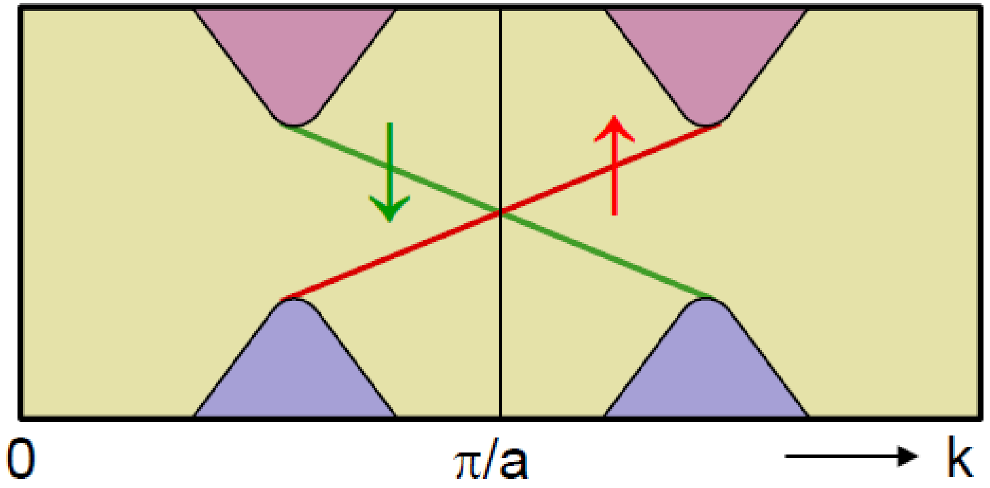
1D Helical edges states

Bulk energy gap, but gapless edge states

"Spin Filtered" or "helical" edge states



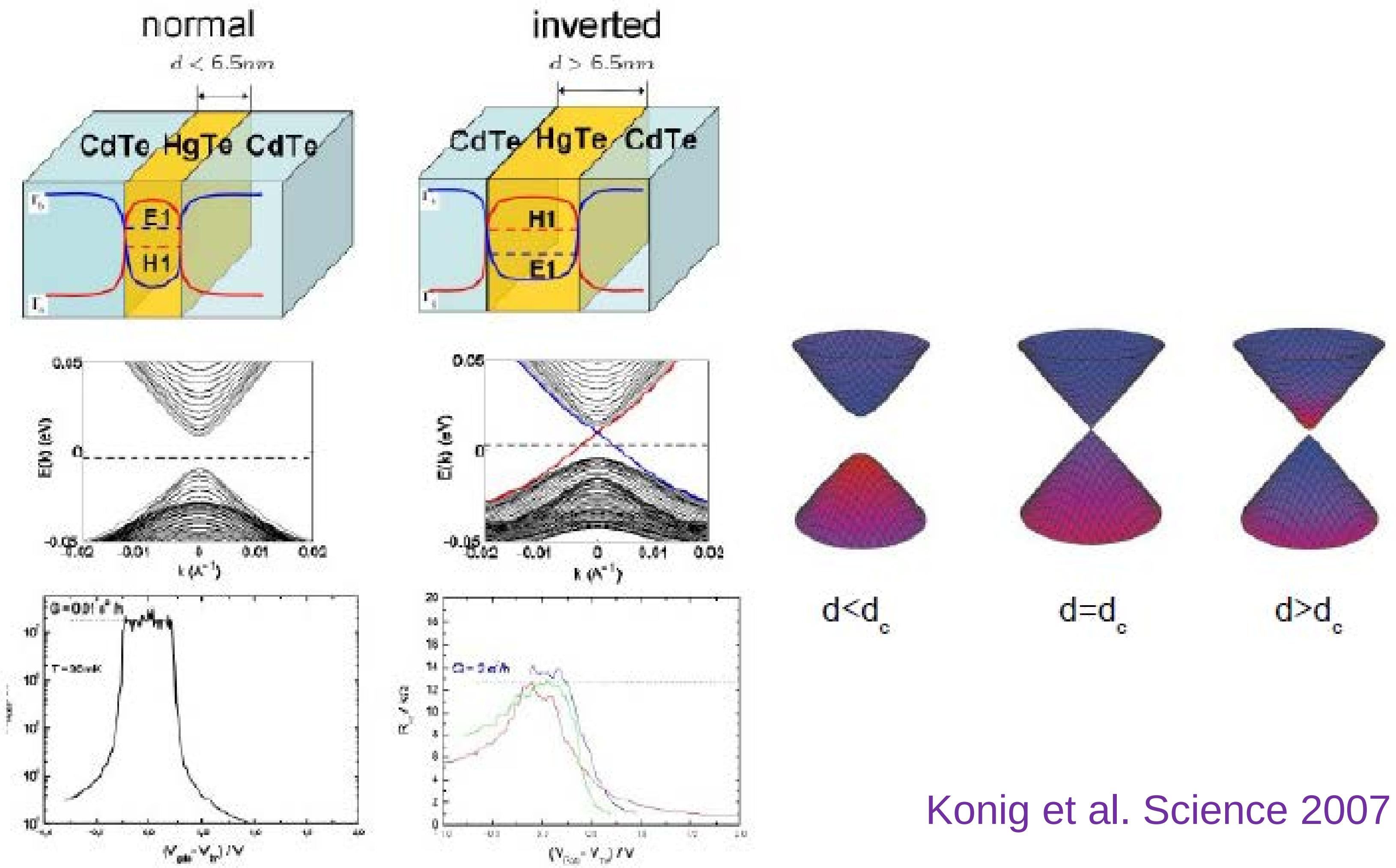
Edge band structure



Edge states form a unique 1D electronic conductor

- HALF an ordinary 1D electron gas
- Protected by Time Reversal Symmetry

Conductance in HgTe/ CdTe heterojunctions



Konig et al. Science 2007

See also Multiterminal conductance probes (Roth et al., Science 325, 294 (2009))

Spin polarization of the quantum spin Hall edge states (Brune et al.,Nature Physics 8, 486 (2012))

See also quantum spin Hall effect in WTe_2, S. Wu et al., Science 359, 76 (2018)

IV) 2D chiral topological superconductors

s-wave superconductors

Symmetry of pairing: Pauli exclusion principle imposes that the pairing function must be antisymmetric.

$$\Delta_{\alpha,\beta}(k) \propto \langle c_\alpha(k) c_\beta(-k) \rangle = -\Delta_{\beta,\alpha}(-k)$$

$$\Delta_{\alpha,\beta}(k) = f_{\alpha,\beta}\Delta(k)$$

Singlet pairing: spin part odd, orbital part even

$$\Delta(k) = \Delta(-k)$$

s-wave superconductor

p-wave superconductors

Symmetry of pairing: Pauli exclusion principle imposes that the pairing function must be antisymmetric.

$$\Delta_{\alpha,\beta}(k) \propto \langle c_\alpha(k) c_\beta(-k) \rangle = -\Delta_{\beta,\alpha}(-k)$$

$$\Delta_{\alpha,\beta}(k) = f_{\alpha,\beta}\Delta(k)$$

Triplet pairing: spin part even, orbital part odd

$$\Delta(k) = -\Delta(-k)$$

p-wave superconductor

Bogoliubov-de Gennes formalism

Bogoliubov - de Gennes (BdG) formalism of superconductivity: essentially BCS theory adapted to describe **quasiparticle excitations** in superconductors.

$$H = \sum_{\mathbf{p},\sigma} c^\dagger_{\mathbf{p}\sigma} \left(\frac{p^2}{2m} - \mu \right) c_{\mathbf{p}\sigma} \equiv \sum_{\mathbf{p},\sigma} c^\dagger_{\mathbf{p}\sigma} \epsilon(p) c_{\mathbf{p}\sigma}$$

Ground state: $$|\Omega\rangle = \prod_{\mathbf{p}:\, \epsilon(p)<0} \prod_{\sigma} c^\dagger_{\mathbf{p}\sigma} |0\rangle$$

Use fermionic anti-commutation relations:

$$H = \frac{1}{2} \sum_{\mathbf{p}\sigma} \left[c^\dagger_{\mathbf{p}\sigma} \epsilon(p) c_{\mathbf{p}\sigma} - c_{\mathbf{p}\sigma} \epsilon(p) c^\dagger_{\mathbf{p}\sigma} \right] + \frac{1}{2} \sum_{\mathbf{p}} \epsilon(p)$$

$$= \frac{1}{2} \sum_{\mathbf{p}\sigma} \left[c^\dagger_{\mathbf{p}\sigma} \epsilon(p) c_{\mathbf{p}\sigma} - c_{-\mathbf{p}\sigma} \epsilon(-p) c^\dagger_{-\mathbf{p}\sigma} \right] + \frac{1}{2} \sum_{\mathbf{p}} \epsilon(p).$$

Bogoliubov-de Gennes formalism

Bogoliubov - de Gennes (BdG) formalism of superconductivity: essentially BCS theory adapted to describe quasiparticle excitations in superconductors.

TRICK: redundant description to treat electrons and holes at the same footing

Nambu basis $\Psi_{\mathbf{p}} \equiv (c_{\mathbf{p}\uparrow}\ c_{\mathbf{p}\downarrow}\ c^{\dagger}_{-\mathbf{p}\uparrow}\ c^{\dagger}_{-\mathbf{p}\downarrow})^T$

$$H = \sum_{\mathbf{p}} \Psi^{\dagger}_{\mathbf{p}} H_{\mathrm{BdG}}(\mathbf{p}) \Psi_{\mathbf{p}} + \text{constant},$$

with

$$H_{\mathrm{BdG}}(\mathbf{p}) = \frac{1}{2}\begin{pmatrix} \epsilon(p) & 0 & 0 & 0 \\ 0 & \epsilon(p) & 0 & 0 \\ 0 & 0 & -\epsilon(-p) & 0 \\ 0 & 0 & 0 & -\epsilon(-p) \end{pmatrix}.$$

Be careful: Different choice of Nambu basis are used by different authors and sometimes may evolve along a paper and/or simply not be specified ….

Particle-hole symmetry

Intrinsic built-in particle-hole redundancy :

$$\Xi H_{BdG}(\mathbf{k})\Xi^{-1} = -H_{BdG}(-\mathbf{k})$$

The p/h operator Ξ anticommutes with the Hamiltonian

$$\Xi = \tau^{x} \otimes \mathbb{1}_2 \, \mathcal{K} \quad \Rightarrow \quad \Xi^2 = +1$$

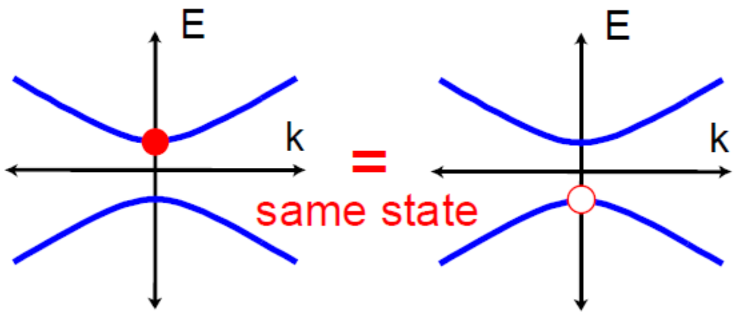
Pauli matrix acts in p/h space

Complex conjugation (antiunitary operator)

If φ_E is an eigenstate with energy +E then

$\varphi_{-E} = \Xi \varphi_E$ is an eigenstate with energy - E

$$\Rightarrow \quad \gamma_E^{\dagger} = \gamma_{-E}$$



Why such artificial redundency ?

Easier to handle mean field superconductivity

s-wave superconductor

$$H_\Delta = \Delta c^\dagger_{\mathbf{p}\uparrow} c^\dagger_{-\mathbf{p}\downarrow} + \Delta^* c_{-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow}$$

$$= \frac{1}{2}\left[\Delta\left(c^\dagger_{\mathbf{p}\uparrow} c^\dagger_{-\mathbf{p}\downarrow} - c^\dagger_{-\mathbf{p}\downarrow} c^\dagger_{\mathbf{p}\uparrow}\right) + \Delta^*\left(c_{-\mathbf{p}\downarrow} c_{\mathbf{p}\uparrow} - c_{\mathbf{p}\uparrow} c_{-\mathbf{p}\downarrow}\right)\right]$$

Coupling between particle and hole sectors

$$H + H_\Delta = \sum_{\mathbf{p}} \Psi^\dagger_{\mathbf{p}} H_{\mathrm{BdG}}(\mathbf{p}, \Delta) \Psi_{\mathbf{p}}$$

$$H_{\mathrm{BdG}}(\mathbf{p}, \Delta) = \frac{1}{2}\begin{pmatrix} \epsilon(p) & 0 & 0 & \Delta \\ 0 & \epsilon(p) & -\Delta & 0 \\ 0 & -\Delta^* & -\epsilon(-p) & 0 \\ \Delta^* & 0 & 0 & -\epsilon(-p) \end{pmatrix}$$

Or more compactly

$$H_{\mathrm{BdG}}(\mathbf{p}, \Delta) = \epsilon(p)\tau^z \otimes I_{2\times 2} - (\mathrm{Re}\Delta)\tau^y \otimes \sigma^y - (\mathrm{Im}\Delta)\tau^x \otimes \sigma^y$$

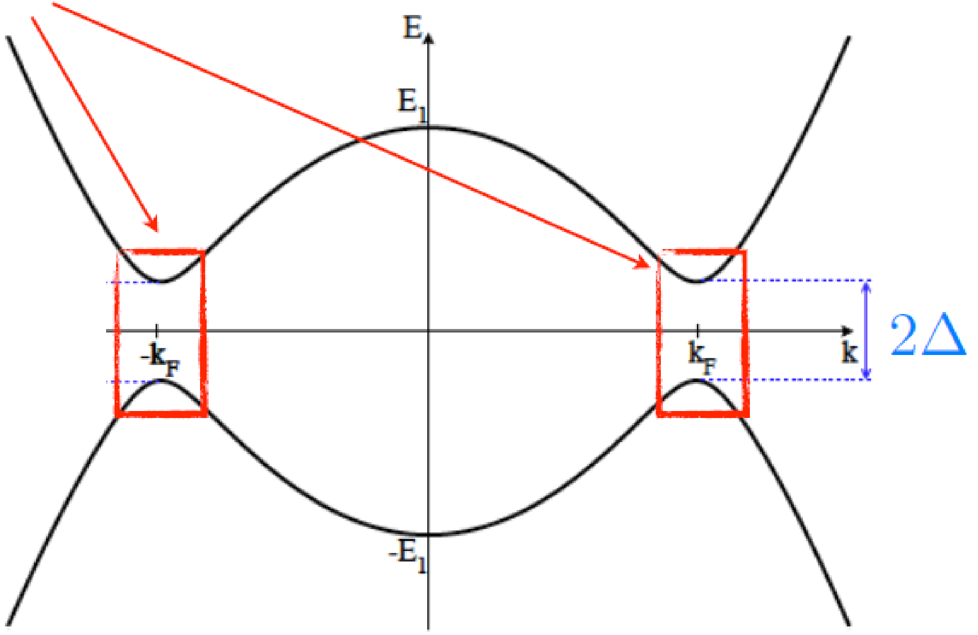
particle-hole Pauli matrix

spin Pauli matrix

BdG Spectrum

$$H_{\mathrm{BdG}}(\mathbf{p},\,\Delta) = \frac{1}{2}\begin{pmatrix} \epsilon(p) & 0 & 0 & \Delta \\ 0 & \epsilon(p) & -\Delta & 0 \\ 0 & -\Delta^* & -\epsilon(-p) & 0 \\ \Delta^* & 0 & 0 & -\epsilon(-p) \end{pmatrix} \Rightarrow E_\pm = \pm\sqrt{\epsilon(\mathbf{p})^2 + |\Delta|^2}$$

Coupling between particle and hole sectors



Behaves like an single-particle insulator ⇒ all topological concepts apply

BdG Spectrum

Spectrum is similar to a band insulator with particle-hole symmetry. A key difference, however, is that excitations in the superconductor are superpositions of electrons and holes

$$\gamma^\dagger_{+,\mathbf{p}\uparrow} = e^{i\theta/2} \sin \alpha_\mathbf{p}\, c^\dagger_{\mathbf{p}\uparrow} + e^{-i\theta/2} \cos \alpha_\mathbf{p}\, c_{-\mathbf{p}\downarrow},$$

$$\gamma^\dagger_{+,\mathbf{p}\downarrow} = -e^{i\theta/2} \sin \alpha_\mathbf{p}\, c^\dagger_{\mathbf{p}\downarrow} + e^{-i\theta/2} \cos \alpha_\mathbf{p}\, c_{-\mathbf{p}\uparrow}$$

$$\gamma^\dagger_{-,\mathbf{p}\uparrow} = e^{i\theta/2} \sin \beta_\mathbf{p}\, c^\dagger_{\mathbf{p}\uparrow} + e^{-i\theta/2} \cos \beta_\mathbf{p}\, c_{-\mathbf{p}\downarrow},$$

$$\gamma^\dagger_{-,\mathbf{p}\downarrow} = -e^{i\theta/2} \sin \beta_\mathbf{p}\, c^\dagger_{\mathbf{p}\downarrow} + e^{-i\theta/2} \cos \beta_\mathbf{p}\, c_{-\mathbf{p}\uparrow}$$

Only two independent excitations (owing to BdG redundancy)

$$\gamma^\dagger_{+,\mathbf{p}\uparrow} = \gamma_{-,-\mathbf{p}\downarrow}$$

$$\gamma^\dagger_{+,\mathbf{p}\downarrow} = \gamma_{-,-\mathbf{p}\uparrow}$$

Coherence factors give the difference
Between particle and hole weights

$$\tan \alpha_\mathbf{p} = \frac{\epsilon(p) + \sqrt{\epsilon(p)^2 + |\Delta|^2}}{|\Delta|}$$

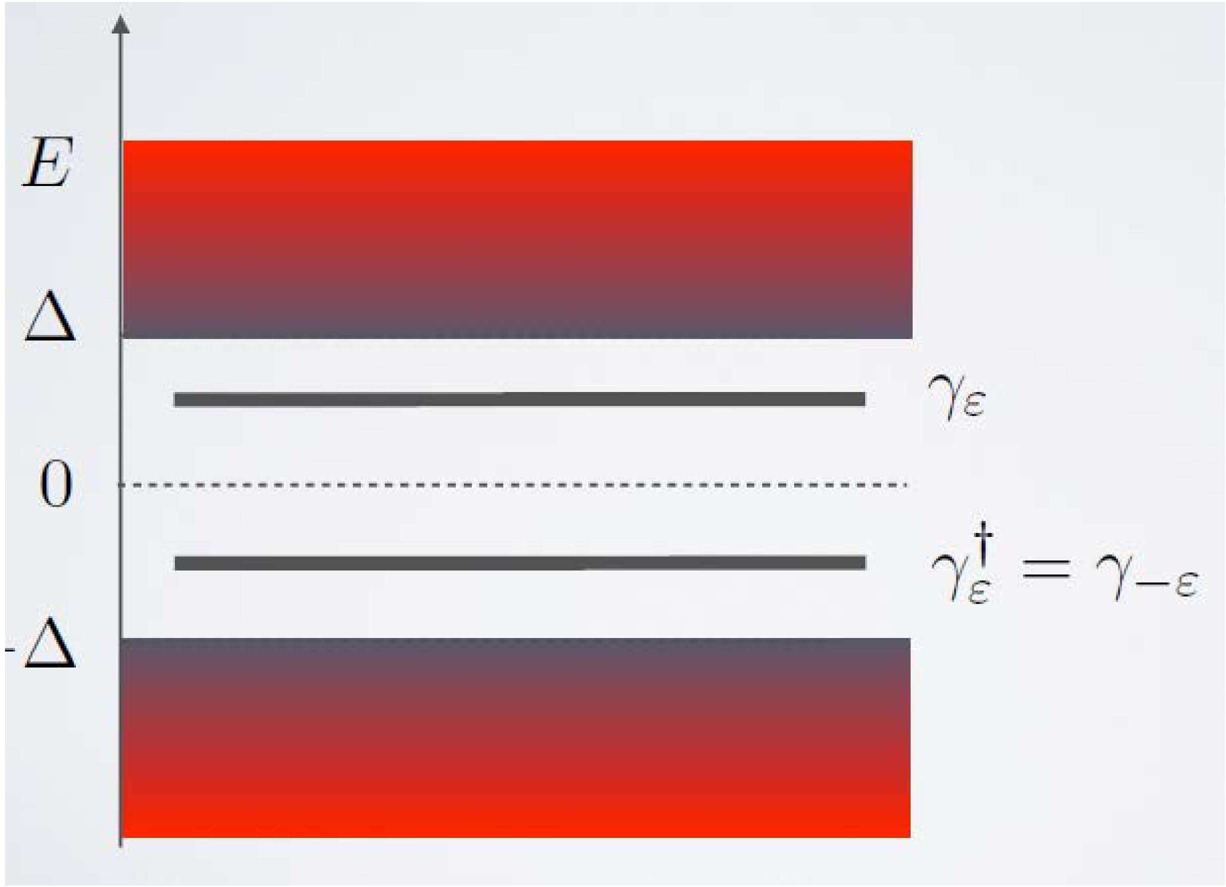
$$\tan \beta_\mathbf{p} = \frac{\epsilon(p) - \sqrt{\epsilon(p)^2 + |\Delta|^2}}{|\Delta|}$$

NOTE

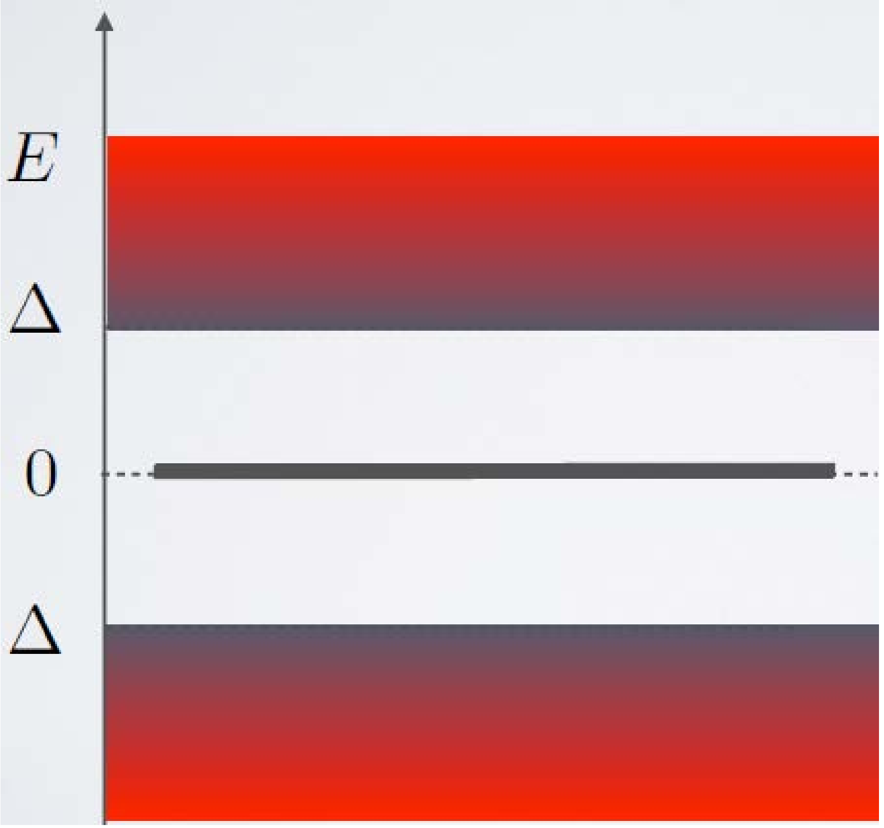
At zero energy (mid-gap) **equal weight** superpositions of electrons and holes

BdG Spectrum

Eigenvalues of the Bogoliubov-De Gennes equation
come in pairs due to the built-in particle-hole symmetry

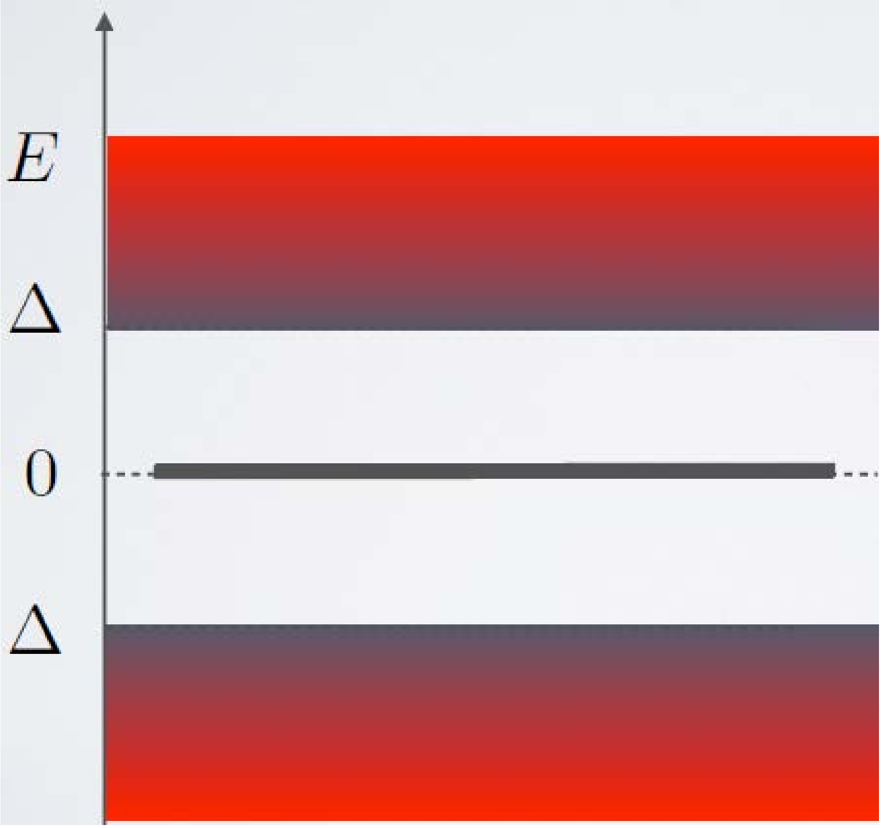


Non-degenerate zero modes correspond to charge neutral superpositions of electrons and holes = Majorana fermions



$$\gamma_0^\dagger = \gamma_0$$

Spin degeneracy is a problem: we need spinless superconductors such as p-wave (early proposals). Intrinsic p-wave superconductors very scarce in nature: Sr_2RuO_4, (Matsumoto and Sigrist 1999), 5/2 fractional QHE (Read & Green, 2000), Kitaev 2001, etc

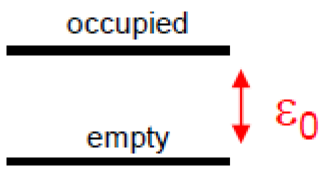


$$\gamma_0^\dagger = \gamma_0$$

About Majorana fermions'properties

Two Majorana fermions define a single two level system

$$\begin{cases} \gamma_1 = \Psi + \Psi^\dagger \\ \gamma_2 = -i(\Psi - \Psi^\dagger) \end{cases} \Rightarrow H = 2i\varepsilon_0\gamma_1\gamma_2 = \varepsilon_0\Psi^\dagger\Psi$$



- 2 degenerate states (full/empty) = 1 qubit

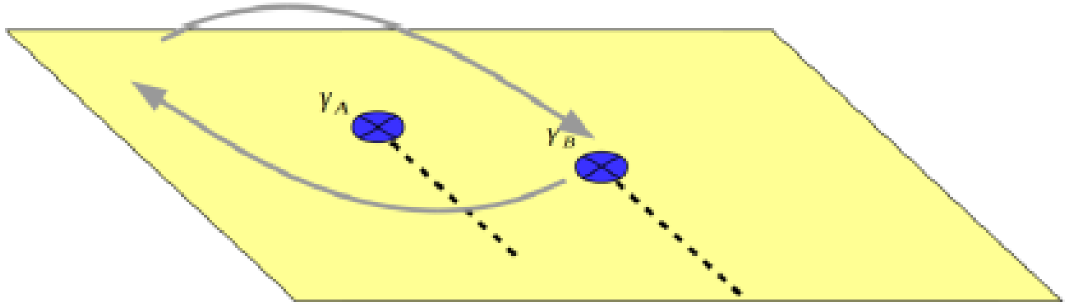
- 2N separated Majoranas = N qubits
- Quantum Information is stored non locally : Immune from local decoherence

Braiding performs unitary operations:
Non-Abelian statistics :

Interchange rule

$$\gamma_i \rightarrow \gamma_j$$

$$\gamma_j \rightarrow -\gamma_i$$



- exchange (=π rotation): $\gamma_b \rightarrow \gamma_a$ $\quad$ $\gamma_a \rightarrow -\gamma_b$
- braid around (=2π rotation): $\gamma_a \rightarrow -\gamma_a$ $\quad$ $\gamma_b \rightarrow -\gamma_b$

Where to find Majorana excitations ?

In spinless topological superconductors

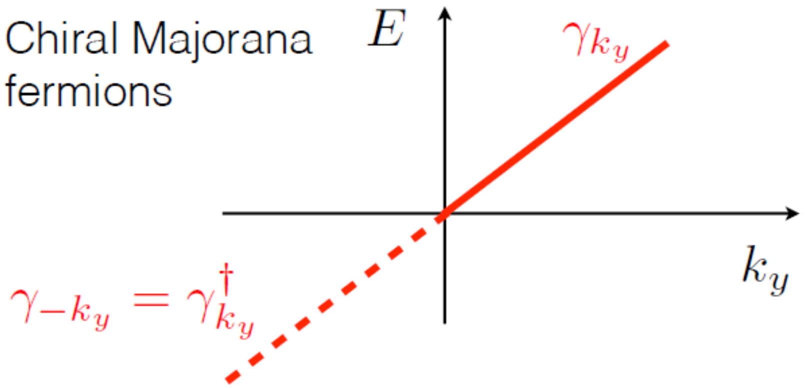
Two-dimensional chiral p+ip superconductors

- The simplest nontrivial time-reversal breaking superconductor in 2D is the spinless p+ip superconductor

$$H_{\mathrm{BdG}} = \frac{1}{2}\sum_{\mathbf{p}} \Psi^{\dagger}_{\mathbf{p}} \begin{pmatrix} \frac{p^2}{2m}-\mu & 2i\Delta(p_x+ip_y) \\ -2i\Delta^*(p_x-ip_y) & -\frac{p^2}{2m}+\mu \end{pmatrix} \Psi_{\mathbf{p}}$$

Weak (topological) pairing phase μ > 0

Diagonalize the Hamiltonian on the half plane x>0



$$H_{\mathrm{edge}} = \sum_{k_y \geq 0} v_F k_y \gamma_{-k_y}\gamma_{k_y}$$

Half of a chiral fermion !

$$\gamma(x) \sim \int dk_y e^{ik_y x}\gamma_{k_y}$$

$$\gamma^{\dagger}(x) = \gamma(x)$$

Two-dimensional chiral p+ip superconductors

Lattice BdG model:

$$\mathcal{H}_{\mathrm{BdG}} = \left(2t\left[\cos k_x + \cos k_y\right] - \mu\right)\tau_z + \Delta_0\left(\tau_x \sin k_x + \tau_y \sin k_y\right) = \boldsymbol{m}(\boldsymbol{k}) \cdot \boldsymbol{\tau}$$

$$E = \pm\left|\boldsymbol{m}(\boldsymbol{k})\right|$$

Spectrum flattening $\hat{\boldsymbol{m}}(\boldsymbol{k}) = \dfrac{\boldsymbol{m}(\boldsymbol{k})}{|\boldsymbol{m}(\boldsymbol{k})|}$

classified by Chern number: (winding number)

$$n = \frac{1}{8\pi}\int_{\mathrm{BZ}} d^2\mathbf{k}\, \epsilon^{\mu\nu}\, \hat{\mathbf{m}} \cdot \left[\partial_{k_\mu}\hat{\mathbf{m}} \times \partial_{k_\nu}\hat{\mathbf{m}}\right]$$

Mapping: $\hat{\boldsymbol{m}}(\boldsymbol{k})$: Brillouin zone $\longmapsto$ $\hat{\boldsymbol{m}}(\boldsymbol{k}) \in S^2$ "$\pi_2(S^2) = \mathbb{Z}$"

Non-trivial Chern number for $\mu \in [-4t, 4t]$

Two-dimensional chiral p+ip superconductors

- Intrinsic realizations of 2D p+ip superconductivity are scarce although there are a few important cases. They include:

1. The 5/2 fractional quantum Hall effect state that can be mapped onto a 2D p+ip superconductor (Read and Green, *"Paired states of fermions in two dimensions with breaking of parity and time reversal symmetries and the fractional quantum Hall effect"*, Phys. Rev. B, **61**, 10267 (2000).

2. The intrinsic p+ip superconductor Sr2RuO4, see Mackenzie and Maeno, *"The superconductivity of Sr2RuO4 and the physics of spin triplet pairing"*, Rev. Mod. Phys. 75, 657, (2003); Das Sarma et al, *"Proposal to stabilize and detect halfquantum vortices in strontium ruthenate thin films: Non-Abelian braiding statistics of vortices in a px+ipy superconductor",* Phys. Rev. B,**73**, 220502 (2006); etc

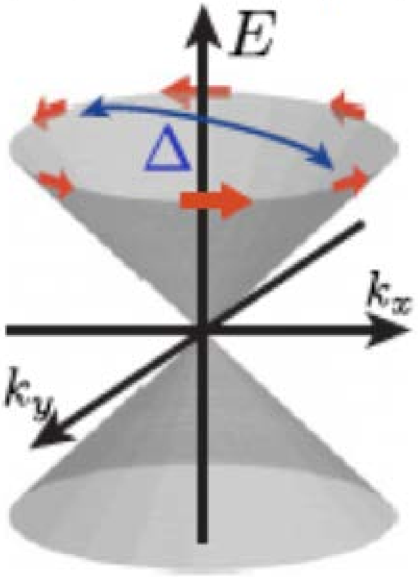
Engineering 2D p+ip topological superconductors

One can also engineer systems that realize a topological phase supporting Majorana fermions in two dimensions by inducing an effective p+ip superconducting pairing in a spinless 2D electron gas.

Simplest case surface state of 3D TIs: The spin is bound to the momentum thanks to spin orbit

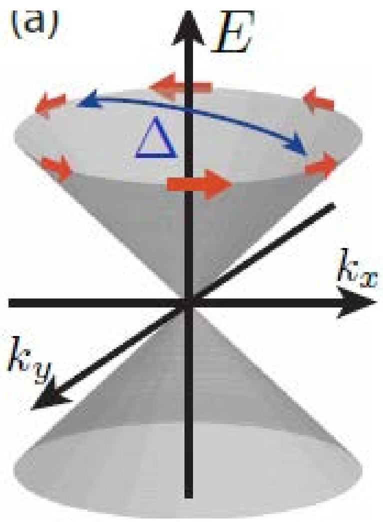
$$H_{3DTI} = \int d^2 r \psi^\dagger [-iv(\partial_x \sigma^y - \partial_y \sigma^x) - \mu] \psi$$

$$\epsilon_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - \mu$$



- For any chemical potential residing within the bulk gap there is only one single Fermi surface (Dirac cones non-degenerate).
- Electrons along the Fermi surface are not spin-polarized (momentum-spin locking) so p+ip pairing can be effectively induced by s-wave proximity effect.

Engineering 2D p+ip topological superconductors



$$\epsilon_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - \mu$$

$$H_{3DTI} = \int d^2\mathbf{r}\psi^\dagger[-iv(\partial_x\sigma^y - \partial_y\sigma^x) - \mu]\psi$$

$$H_\Delta = \int d^2\mathbf{r}\Delta(\psi_\uparrow\psi_\downarrow + H.c.)$$

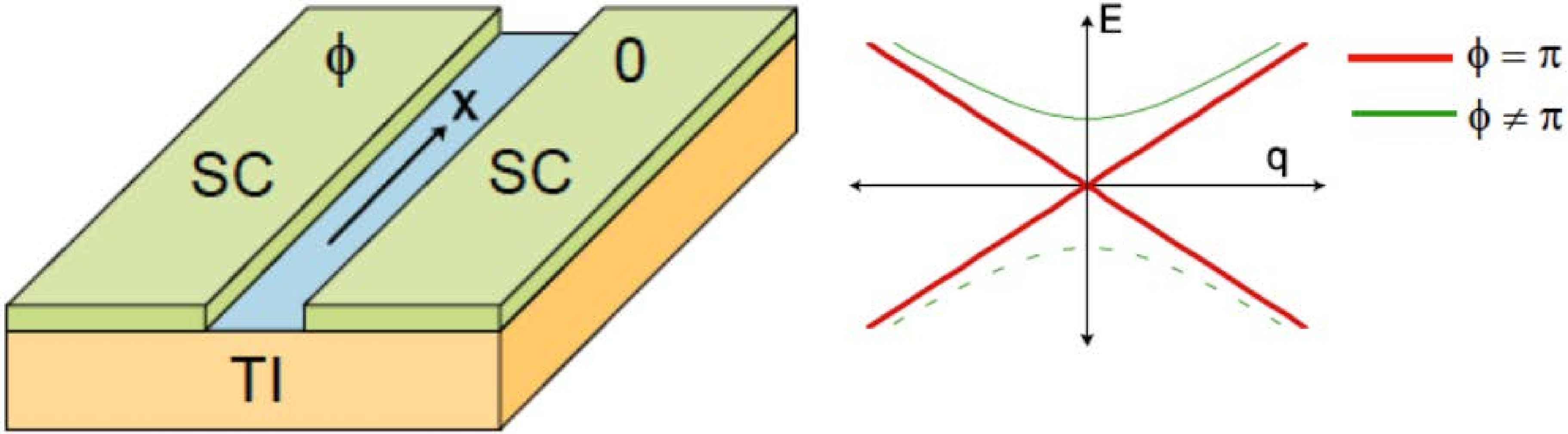
Write it in the basis that diagonalize the kinetic part

$$H = \sum_{s=\pm}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\Big\{\epsilon_s(\mathbf{k})\psi_s^\dagger(\mathbf{k})\psi_s(\mathbf{k}) + \left[\frac{\Delta}{2}\left(\frac{k_x + ik_y}{|\mathbf{k}|}\right)\psi_s(\mathbf{k})\psi_s(-\mathbf{k}) + H.c.\right]\Big\}$$

Realize a 2D time-reversal invariant topological superconductor

Alicea, Reports on progress in physics 2012

Engineering 2D helical p+ip Topological Superconductors



$$H_{edge} = -i\hbar v_F(\gamma^L \partial_x \gamma^L - \gamma^R \partial_x \gamma^R) + i\Delta cos(\frac{\phi}{2})\gamma_L \gamma_R$$

SNS junctions host gapless **non-chiral Majorana edge states** at $\phi = \pi$

Fu and Kane, PRL **100**, 096407, 2008

Engineering 2D spinless p+ip topological superconductors

$$H = \sum_{s=\pm} \int \frac{d^2\mathbf{k}}{2\pi} \{\epsilon_s(\mathbf{k})\psi_s^\dagger(\mathbf{k})\psi_s(\mathbf{k}) + [\frac{\Delta}{2}(\frac{k_x + ik_y}{|\mathbf{k}|})\psi_s(\mathbf{k})\psi_s(-\mathbf{k}) + h.c]\}$$

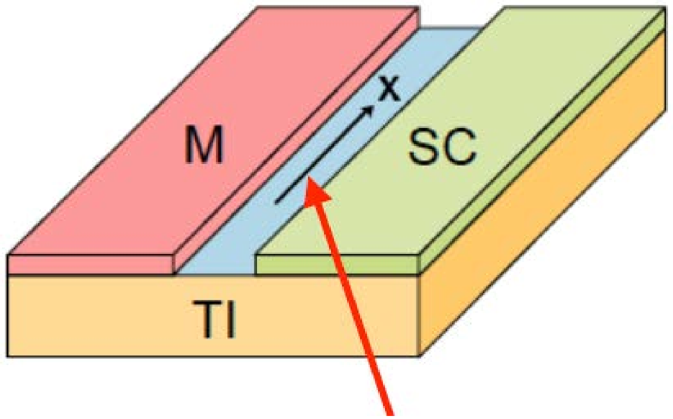
$$H_Z = -h \int d^2r \psi^\dagger \sigma^z \psi$$

$$\epsilon_\pm(\mathbf{k}) = \pm v|\mathbf{k}| - \mu$$

$$H_{edge} = -i\hbar v_F \gamma \partial_x \gamma$$

Remains topological while:

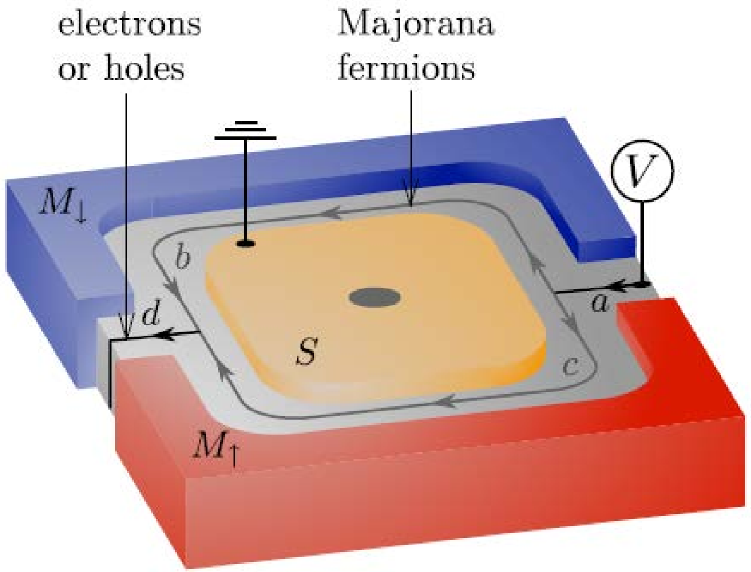
$$h < \sqrt{\Delta^2 + \mu^2}$$



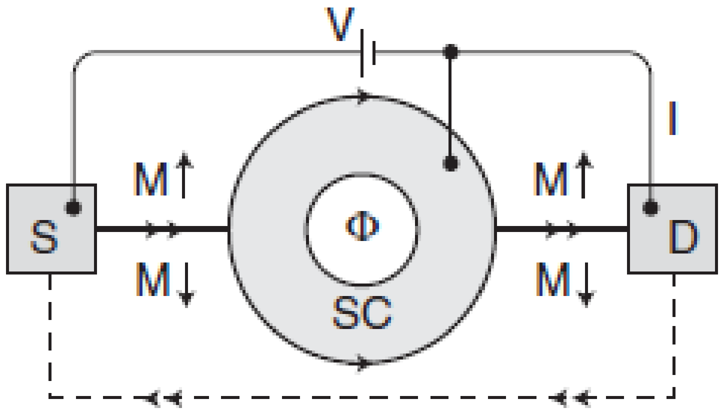
Time-reversal breaking of any form will generate **chiral Majorana edge states** at the boundary between topologically superconducting and magnetically gapped regions in the surface of a 3D TI.

Fu and Kane, PRL 100, 096407, 2008

Majorana interferometer



Akhmerov, Nilsson, and Beenakker, PRL 102, 216404 (2009)

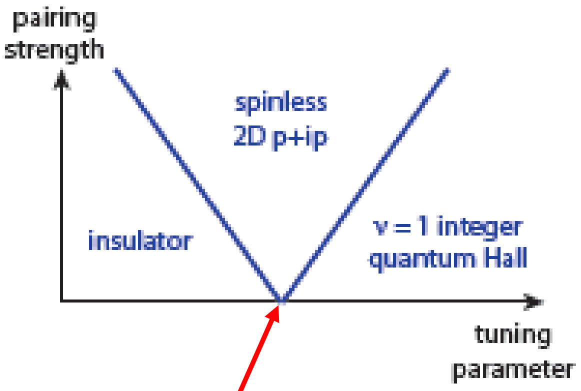


Fu and Kane, PRL 102, 216403 (2009)

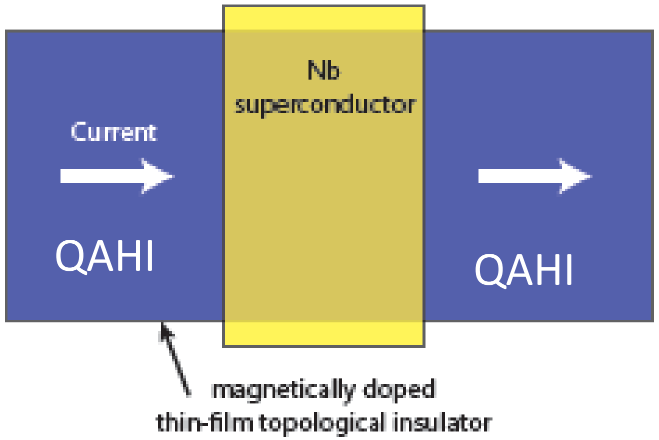
$$I = (-1)^n \frac{e}{h} \frac{\pi k_B T \sin(eV \delta L / v_M)}{\sinh(\pi k_B T \delta L / v_M)}$$

$$\Phi = nh/2e$$

Engineering 2D spinless p+ip TSC



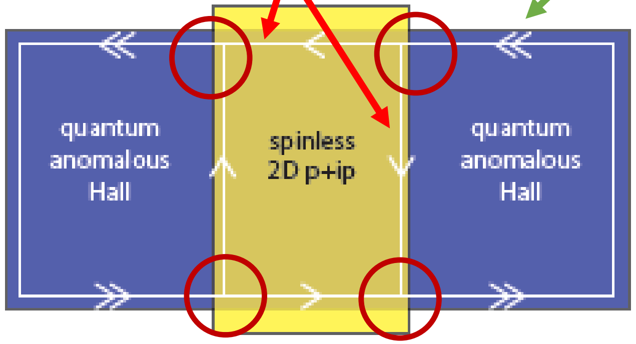
QCP between an insulator and
The integer Quantum Hall regime



Qi, Hughes, Zhang, PRB B 82, 184516 (2010)

Chiral Majorana edge states

Polarized electron edge states

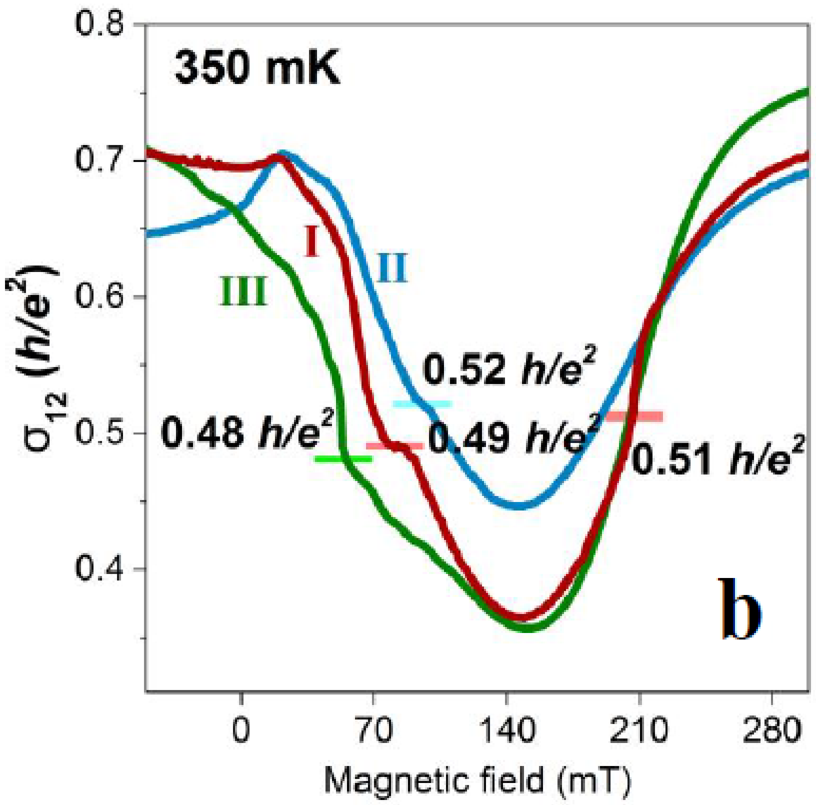


Two terminal conductance:

$$G = \frac{1}{2}\frac{e^2}{h}$$

Chung, Qi, Maciejko, Zhang,PRB 83, 100512 (2011)

Engineering 2D spinless p+ip TSC



Plateaus at half conductance quantum
As a signature of Majorana edge states

He et al., Science (2017)

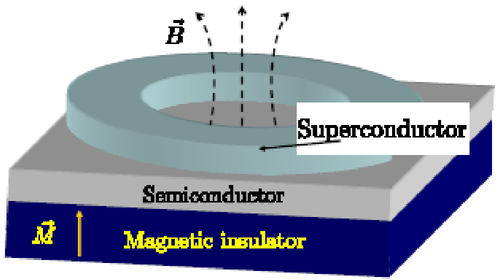
Engineering 2D spinless p+ip TSC

PRL 104, 040502 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

A generic new platform for topological quantum computation using semiconductor heterostructures

Jay D. Sau[1], Roman M. Lutchyn[1], Sumanta Tewari[1,2], and S. Das Sarma[1]
[1]*Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*
[2]*Department of Physics and Astronomy, Clemson University, Clemson, SC 29634*

We show that a film of a semiconductor in which s-wave superconductivity and a Zeeman splitting are induced by proximity effect, supports zero-energy Majorana fermion modes in the ordinary vortex excitations. Since time reversal symmetry is explicitly broken, the edge of the film constitutes a chiral Majorana wire. The heterostructure we propose – a semiconducting thin film sandwiched between an s-wave superconductor and a magnetic insulator – is a generic system which can be used as the platform for topological quantum computation by virtue of the existence of non-Abelian Majorana fermions.



$$H_0 = \frac{p^2}{2m^*} - \mu + V_z\sigma_z + \alpha(\vec{\sigma}\times\vec{p})\cdot\hat{z}.$$

$$\hat{H}_p = \int d\mathbf{r}\,\{\Delta_0(\mathbf{r})\hat{c}^\dagger_\uparrow(\mathbf{r})\hat{c}^\dagger_\downarrow(\mathbf{r}) + \mathrm{H.c}\},$$

Majorana fermions in a tunable semiconductor device

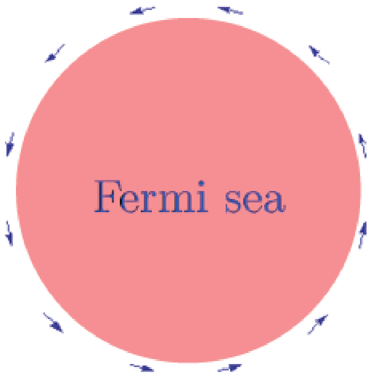
Jason Alicea[1]
[1]*Department of Physics, California Institute of Technology, Pasadena, California 91125*

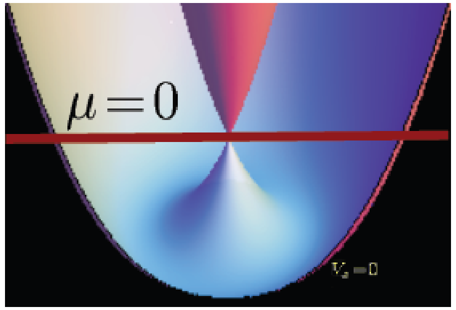
(Dated: December 10, 2009)

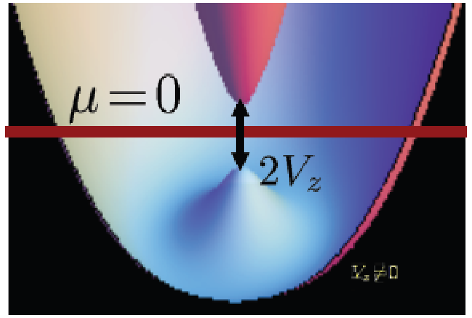
Majorana bound states in SM/SC heterostructures

Semiconductor with Rashba interaction

$$H_0 = \begin{pmatrix} \frac{p^2}{2m} - \mu & \alpha i(p_x - ip_y) \\ -\alpha i(p_x + ip_y) & \frac{p^2}{2m} - \mu \end{pmatrix}$$







spin orientation changes around Fermi surface

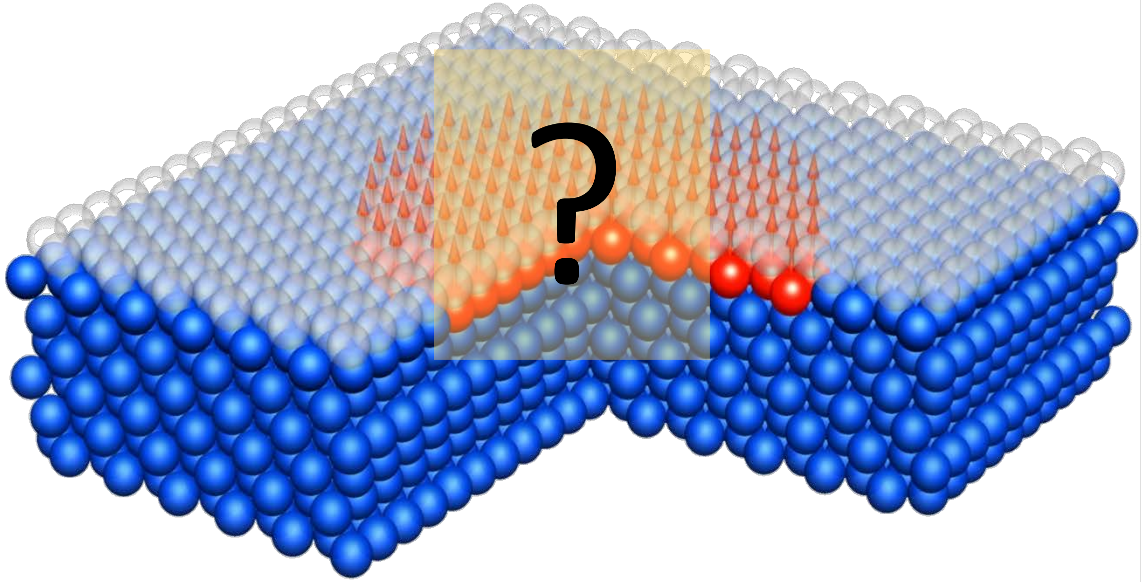
Sau, Lutchyn, Tewari, Das Sarma, PRL'10;

Single Fermi surface !

$$\epsilon_{\pm}(k) = \frac{k^2}{2m} - \mu \pm \sqrt{V_z^2 + \alpha^2 k^2}.$$

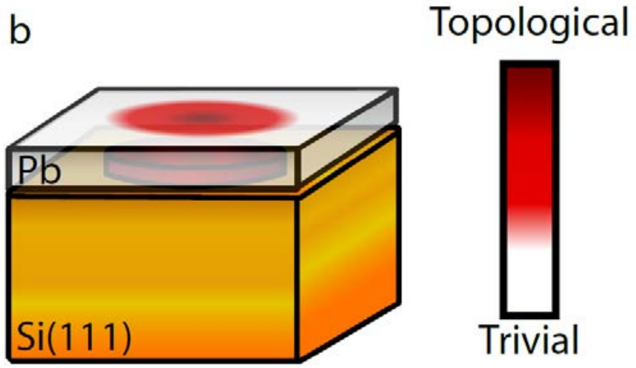
Rashba SO+ Zeeman term break TR and inversion Symmetry

Interplay between a magnetic cluster and 2D superconductivity

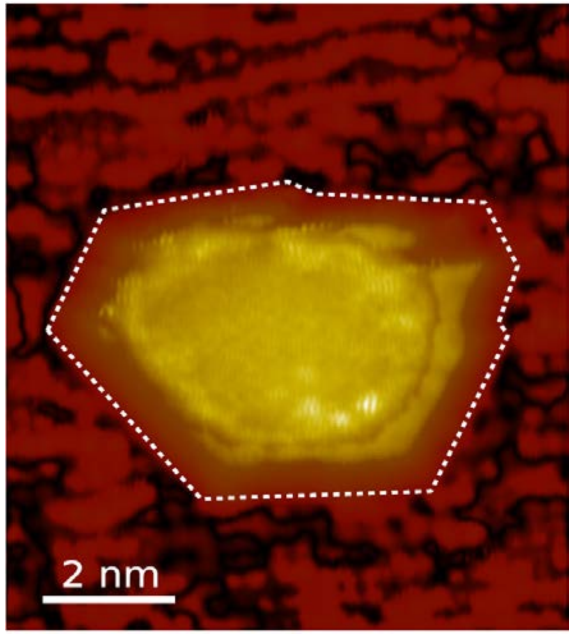


What happens above a magnetic cluster sitting below a Pb monolayer ?

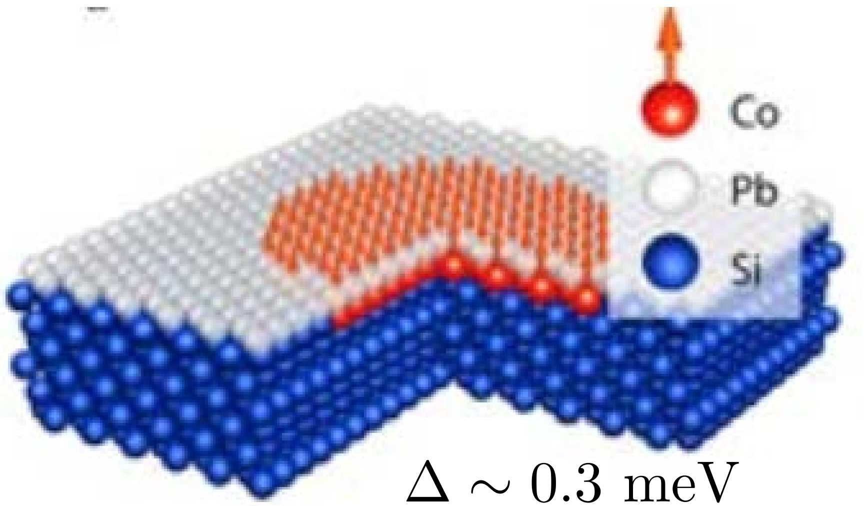
System studied: Magnetic nano-cluster embedded in Pb/Si(111)



Magnetic clusters under the Pb layer to create topological superconductivity over the cluster

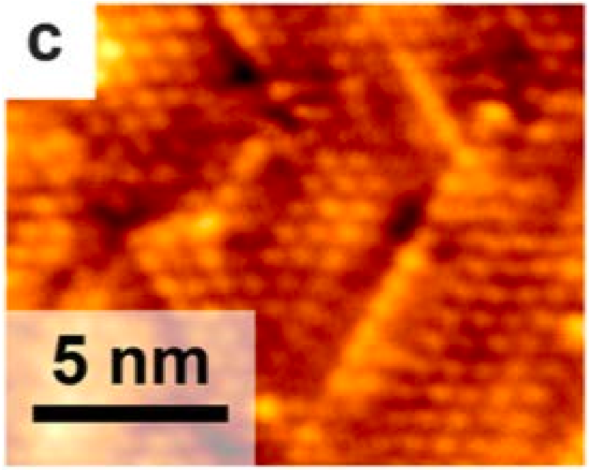


Pb/Co/Si(111) system



$\Delta \sim 0.3$ meV
$\alpha \sim 300$ meV

Topograph 16nm×13nm

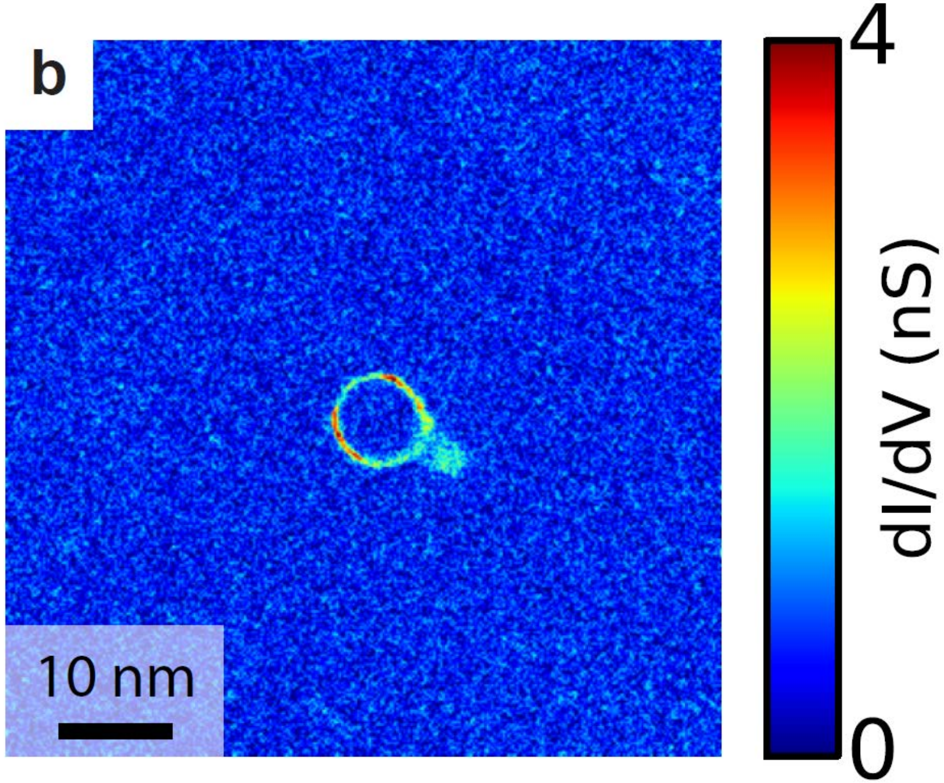


- Island radius: $R \sim 10nm$
- Coherence length: $\xi \sim 40nm$

➡ **Lengthscale separation** $L \gg \xi > R \gg l_F \sim l_{so}$

Magnetic nano-cluster: Majorana dispersive state ?

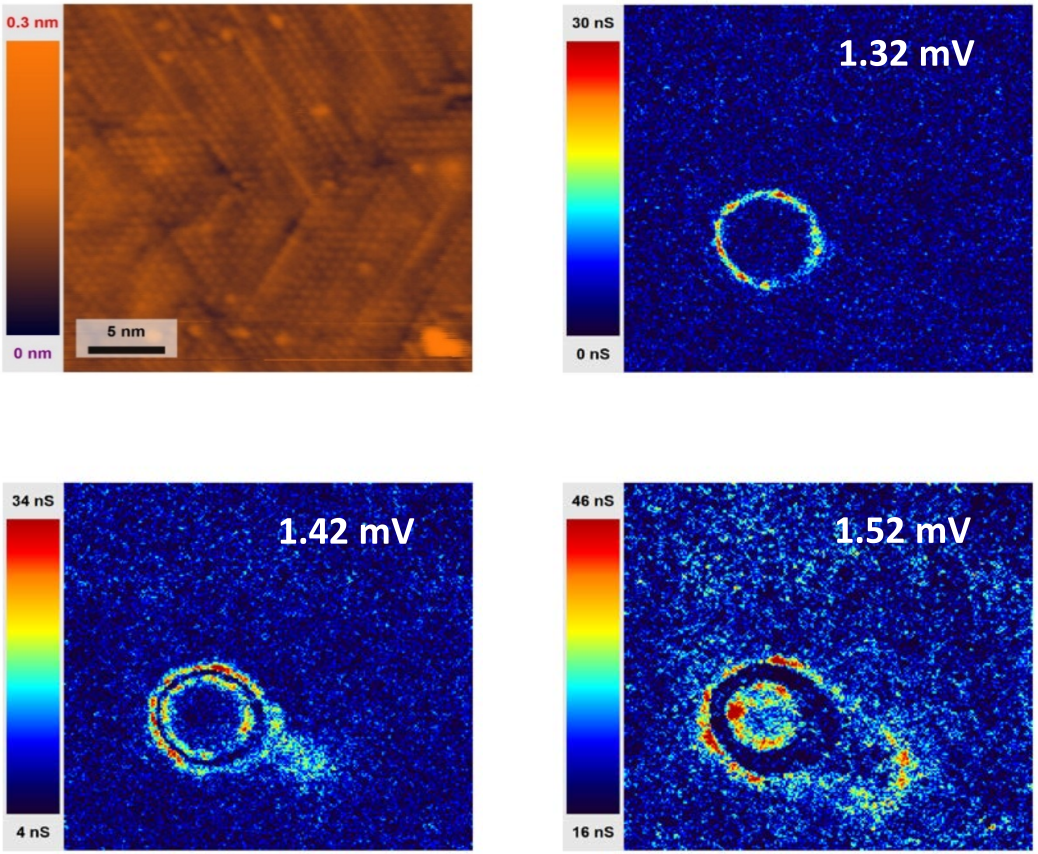
Observation of perfectly circular structure at the Fermi energy



300 mK conductance map at **V=0 meV** using a superconducting tip

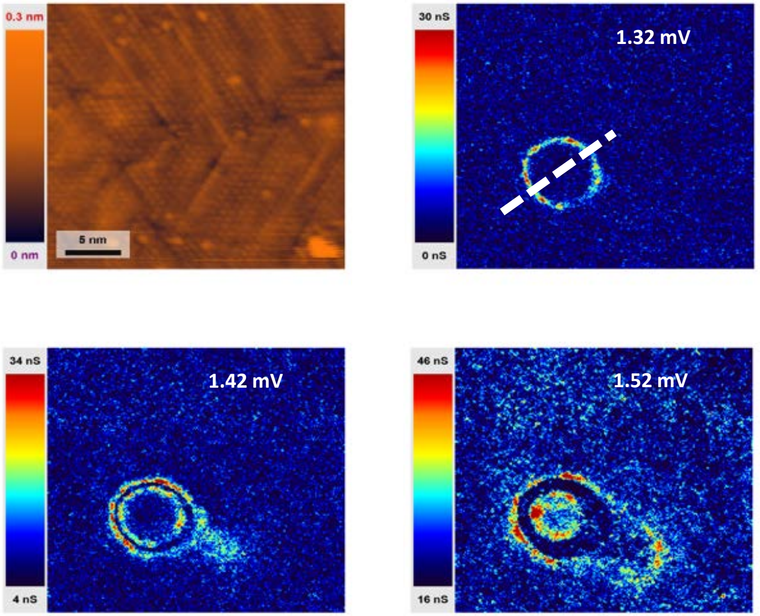
G. Ménard et al., Nat Comm. (2017)

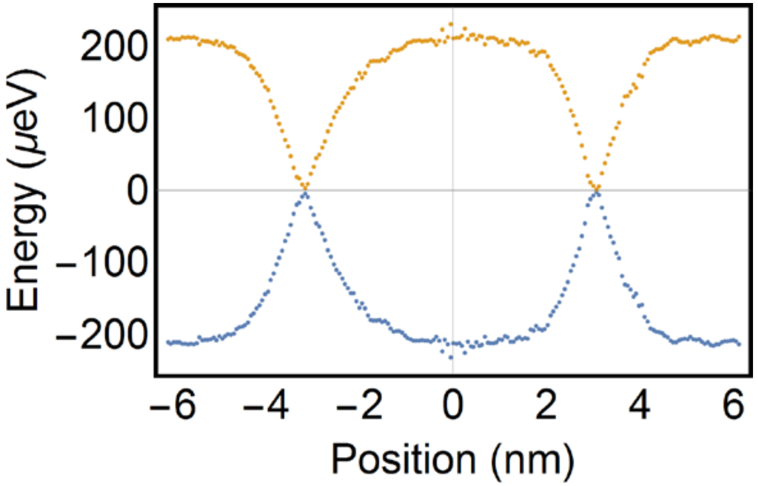
Splitting of rings at finite energy

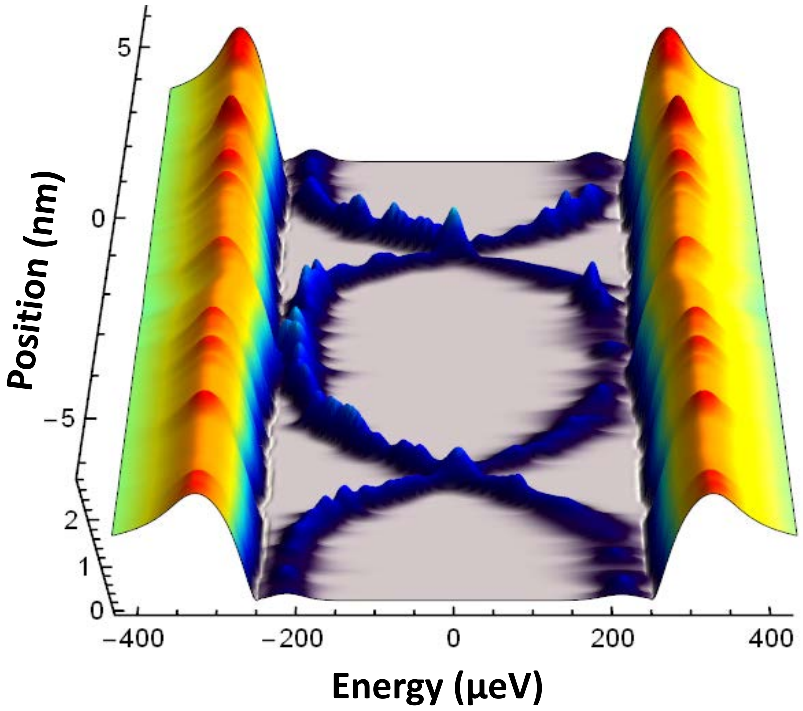


G. Ménard et al., Nat Comm. (2017)

Edges dispersions in Pb/Co/Si(111

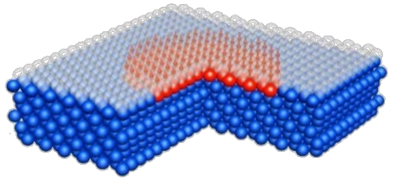






G. Ménard et al., Nat Comm. (2017)

Modeling of the cluster area



Let us first consider an **homogeneous** situation :

$$H = \xi_k \tau_z + \Delta_S \tau_x + V_z \sigma_z + \left(\alpha \tau_z + \frac{\Delta_T}{k_F} \tau_x \right) (\sigma_x k_y - \sigma_y k_x)$$

Zeeman ($V_z \sigma_z$)

Rashba SO ($\alpha \tau_z$)

Triplet superconducting order parameter ($\frac{\Delta_T}{k_F} \tau_x$)

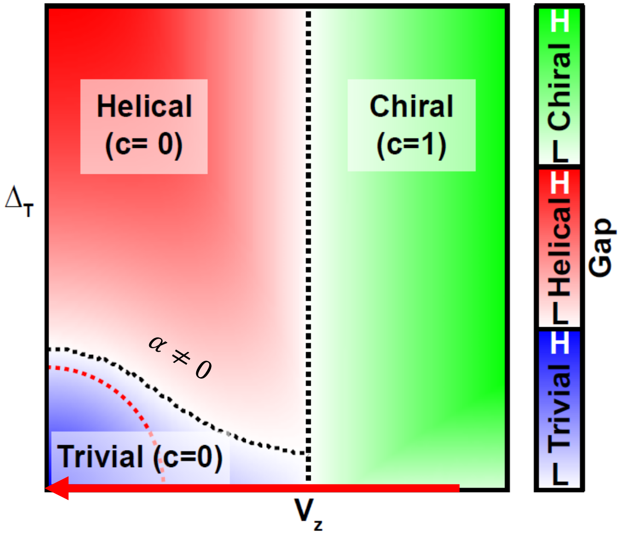
Spectrum:

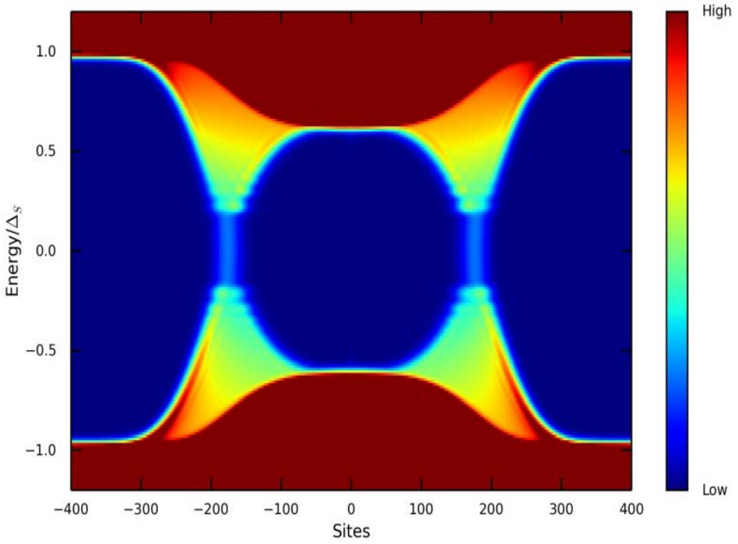
$$E_\pm^2(k) = V_z^2 + (\alpha k)^2 + \Delta_S^2 + \Delta_T^2 \frac{k^2}{k_F^2} + \xi_k^2 \pm 2\sqrt{V_z^2(\Delta_S^2 + \xi_k^2) + \frac{k^2}{k_F^2} \left(\Delta_S \Delta_T + \alpha k_F \xi_k \right)^2}$$

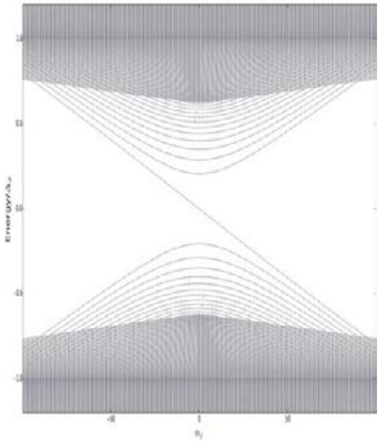
Ghosh, Sau, Tewari, Das Sarma, PRB (2010)
Tanaka, Sato, Nagaosa, JPSJ (2012)

Simplest model: purely chiral case (Singlet pairing, Rashba and Zeeman)

$$H = \xi_k \tau_z + V_Z \sigma_z + \alpha \tau_z + \Delta_S \tau_x$$



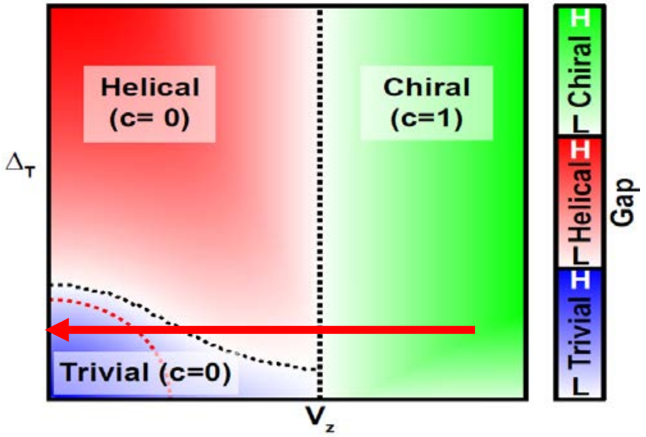


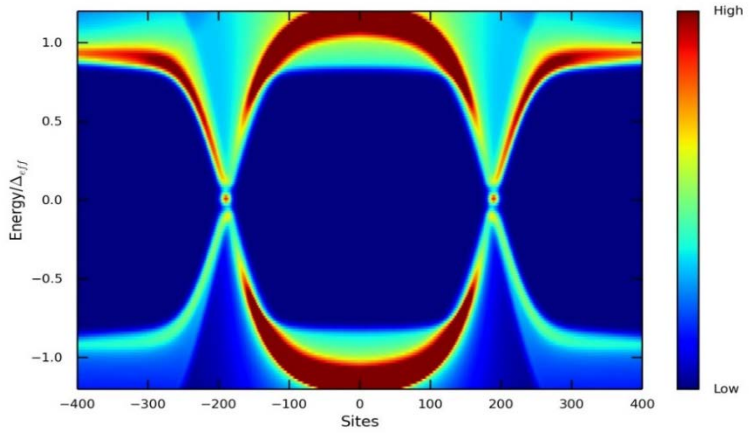


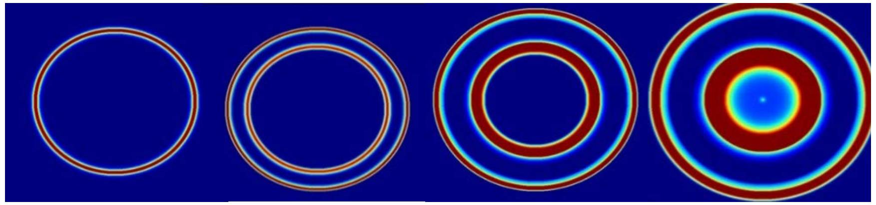
G. Ménard et al., Nat Comm. (2017)

Time reversal broken helical state
(Singlet & triplet pairing and Zeeman)

$$H = \xi_k \tau_z + V_Z \sigma_z + \alpha \tau_z + \Delta_S \tau_x + \frac{\Delta_T}{k_F} \tau_x (\sigma_x k_y - \sigma_y k_x)$$







G. Ménard et al., Nat Comm. (2017)

2D time reversal invariant topological SC

(also known as "helical superconductor")

BdG Hamiltonian in the presence of time-reversal symmetry:

Simplest model:
(spinless chiral p-wave SC)[2]

$$\mathcal{H}_{\rm BdG}(\mathbf{k}) = \begin{pmatrix} \varepsilon(\mathbf{k})\sigma_0 & \Delta_t[\mathbf{d}(\mathbf{k})\cdot\vec{\sigma}](i\sigma_y) \\ \Delta_t(-i\sigma_y)[\mathbf{d}(\mathbf{k})\cdot\vec{\sigma}] & -\varepsilon(\mathbf{k})\sigma_0 \end{pmatrix}$$

→ Two copies of chiral p-wave SC (like in TI)

Z2 topological invariant

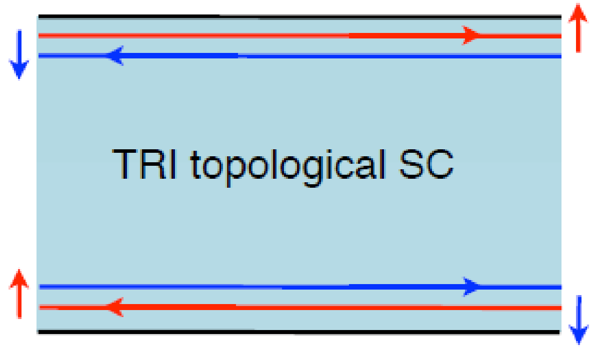
Bulk-boundary correspondence:

By analogy to chiral p-wave SC: (for $|\mu| < 4t$)
two counter-propagating Majorana edge modes

-- protected by TRS and PHS

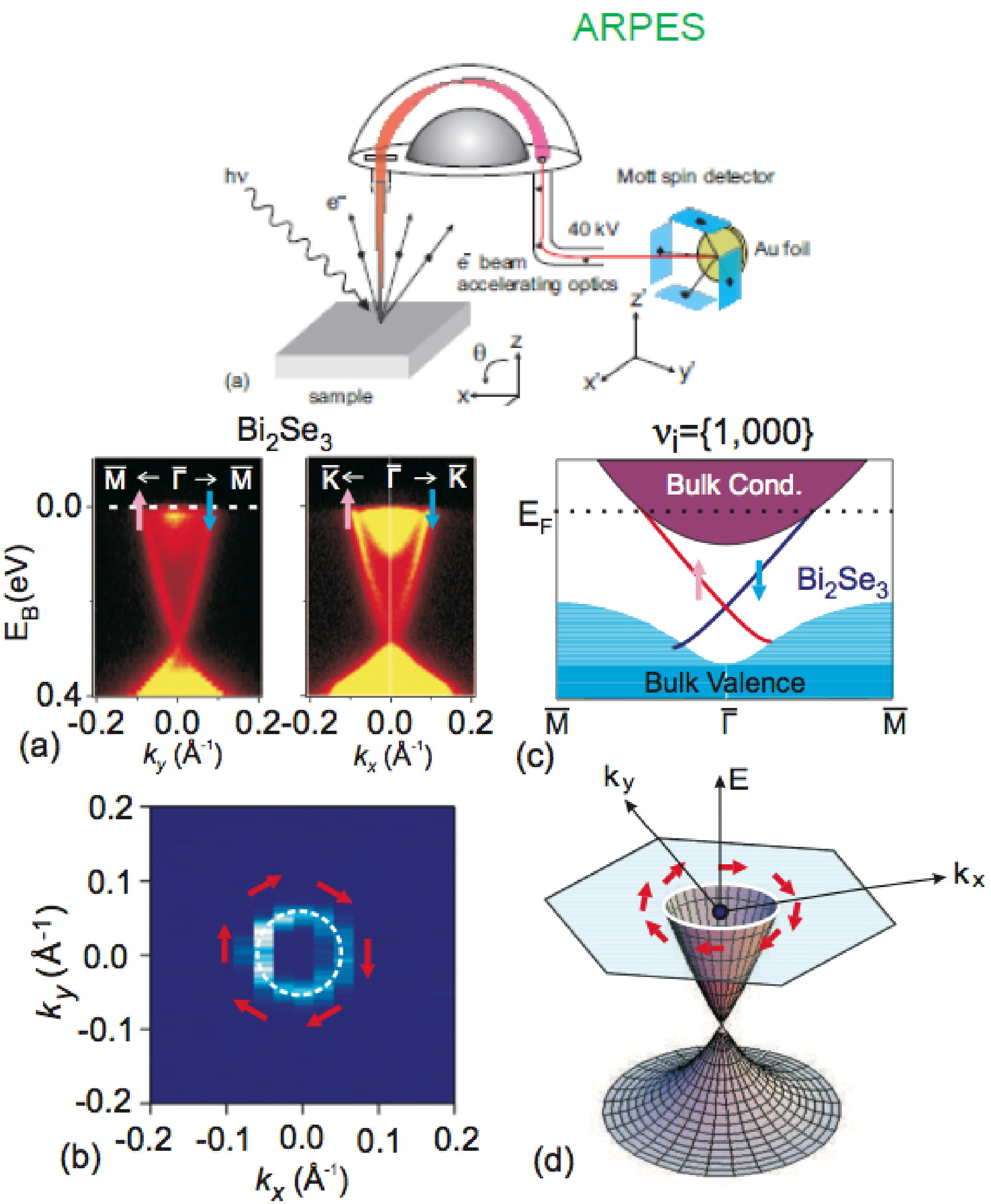
-- two-dimensional analog of B phase of ^{3}He

helical Majorana edge states:



V) How about 3D ?

Back to 3D topological insulators



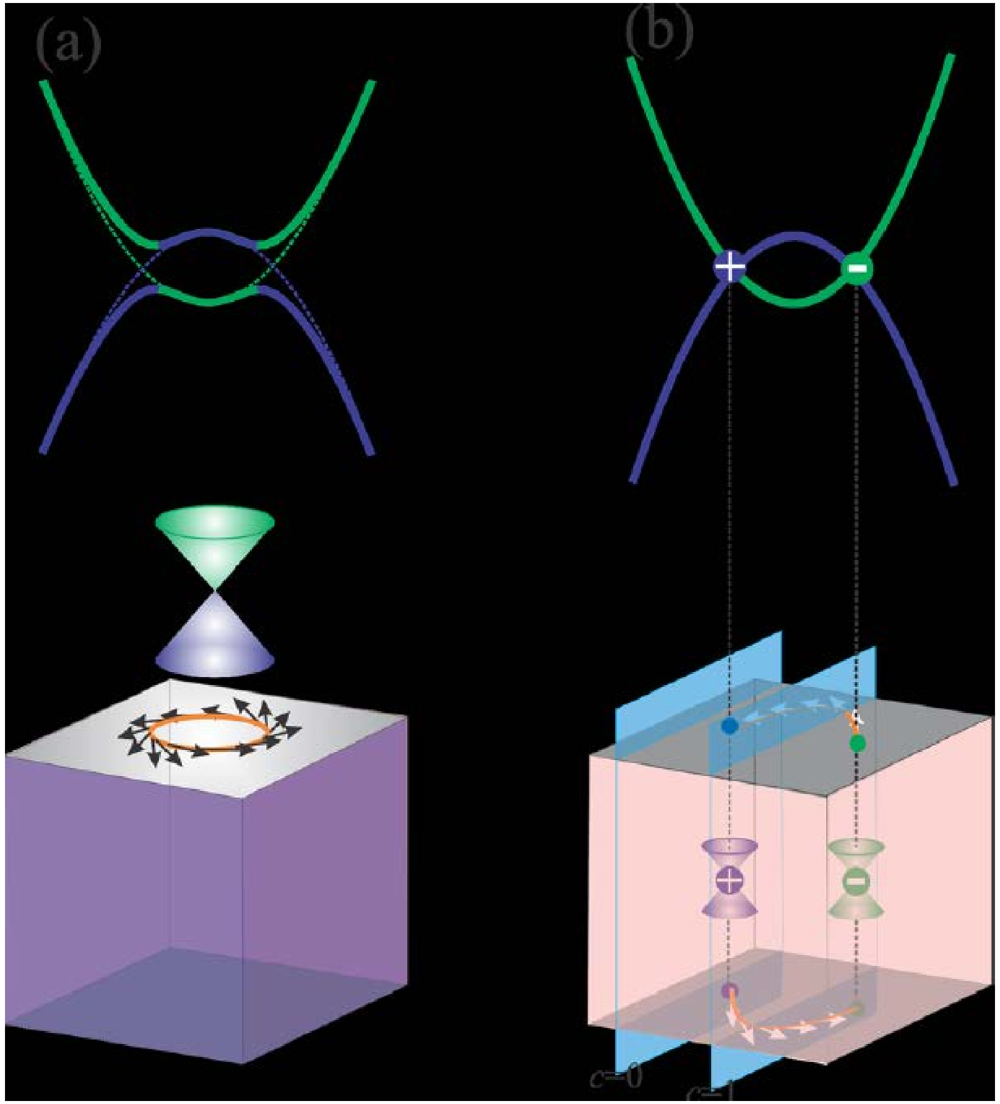
- A single Dirac cone with momentum-spin locking:

Helical Dirac fermions

$$H_{\text{surf}}(k_x, k_y) = C + A_2\left(\sigma^x k_y - \sigma^y k_x\right).$$

Hsieh *et al.*, Nature **452**, 970 (2008),
Xia *et al.*, Nature Phys. **5**, 398 (2009),
Zhang *et al.*, Nature Phys. **5**, 438 (2009),
Chen *et al.*, Science **325**, 178 (2009),
many more…

From Topological Insulator to Weyl Semimetal



Wan, Turner, Vishwanath, Savrasov, PRB2011

Weyl Semi-Metal and its Berry Flux

In the vicinity of Weyl Point: $q=k-k_D$

$$H(q)=\sum_{i=xyz} v_i \bullet q\sigma_i \quad \Rightarrow \quad E_\pm(q)=\pm\sqrt{\sum_{i=xyz}(v_i \bullet q)^2}$$

The Berry curvature is evaluated to be $$\Omega(k)=i\sum_{n=1}^{N_{occ}}\langle\nabla_k u_{kn}|\times|\nabla_k u_{kn}\rangle$$

Integrating over a small sphere surrounding one Weyl point produces flux that is given by the chiral charge *c*

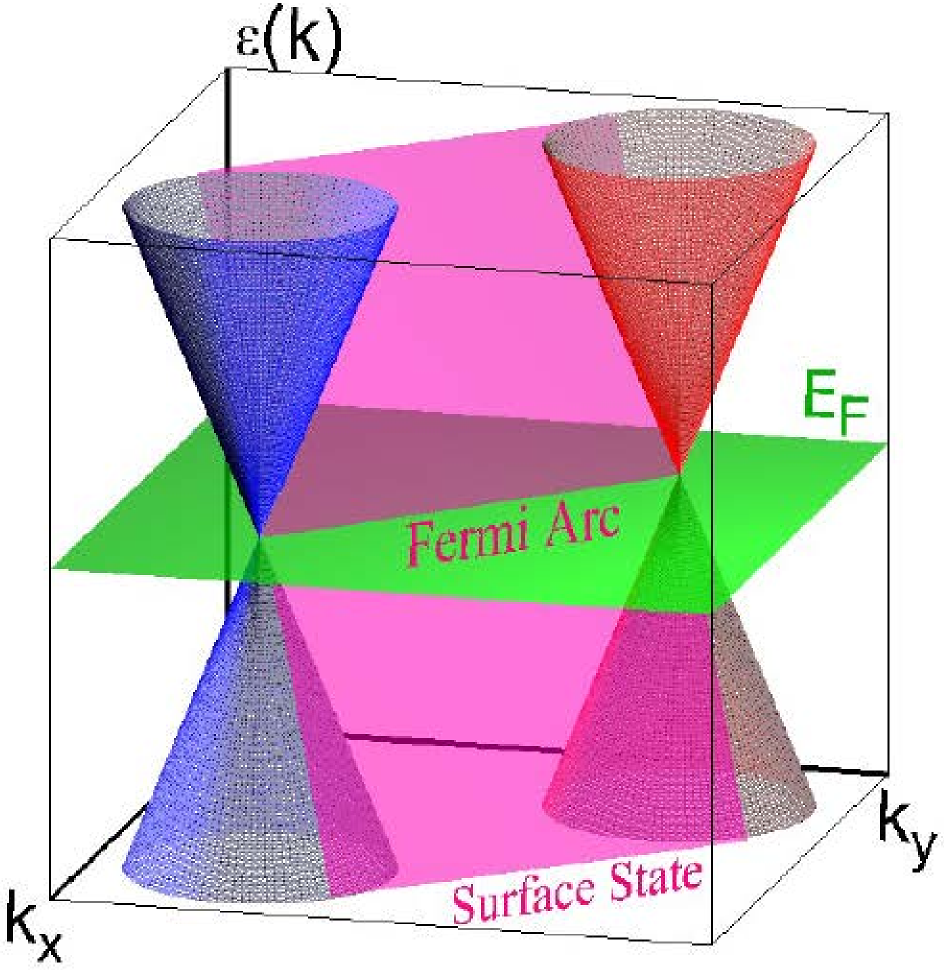
$$c=\frac{1}{2\pi}\oint_S dS\Omega(k)=\text{sign}(v_1 \bullet v_2 \times v_3)$$

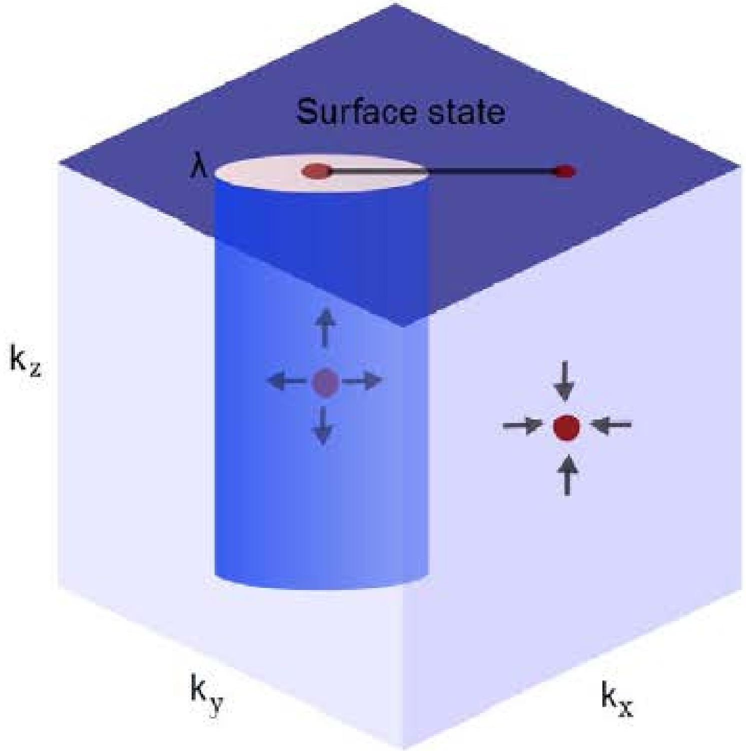
A Weyl point acts as a **magnetic monopole** at the origin:

$$\Omega(q)=c\frac{1}{2}\frac{q}{q^3}$$

whose charge *c* = chirality

Fermi arcs in Weyl materials





Fermi Arc connects Weyl points of opposite chirality

THE ADLER–BELL–JACKIW ANOMALY AND WEYL FERMIONS IN A CRYSTAL

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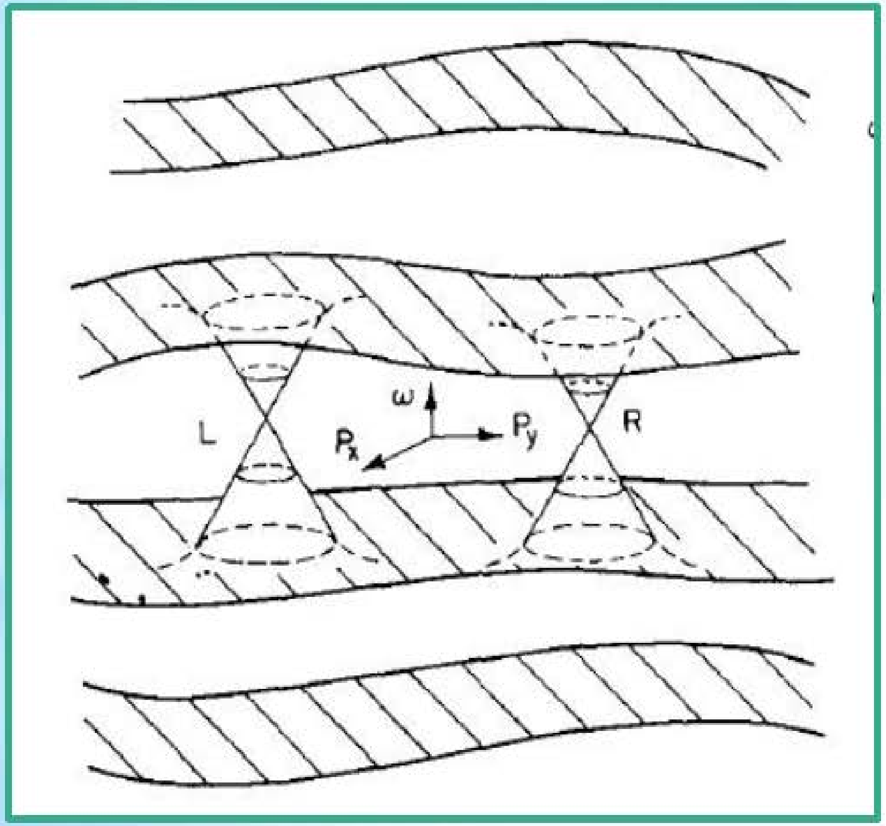
Department of Physics, Brown University, Providence, RI 02912, USA

Received 14 June 1983

Nielsen, Ninomiya and Dirac/Weyl semimetals

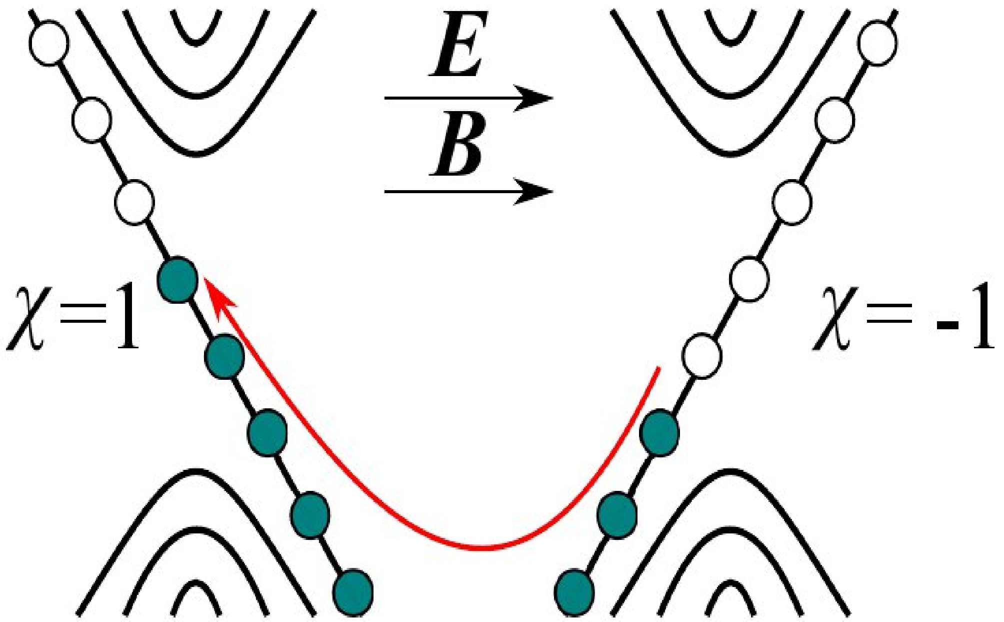


Weyl points separated in momentum space

In compact BZ, equal number of right/left handed Weyl points

Axial anomaly = flow of charges from/to left/right Weyl point

Charge pumping in Weyl materials



If electric field ***E*** is applied along ***B***, all states move along the field according to:

$$d\boldsymbol{k}/dt = -e\boldsymbol{E}$$

Zeroth Landau level is chiral for each Weyl point

Therefore, motion of the states along ***E*** corresponds to electrons disappearing from right-moving band and reappearing in the left-moving one.

ARTICLE

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Signatures of the Adler–Bell–Jackiw chiral anomaly in a Weyl fermion semimetal

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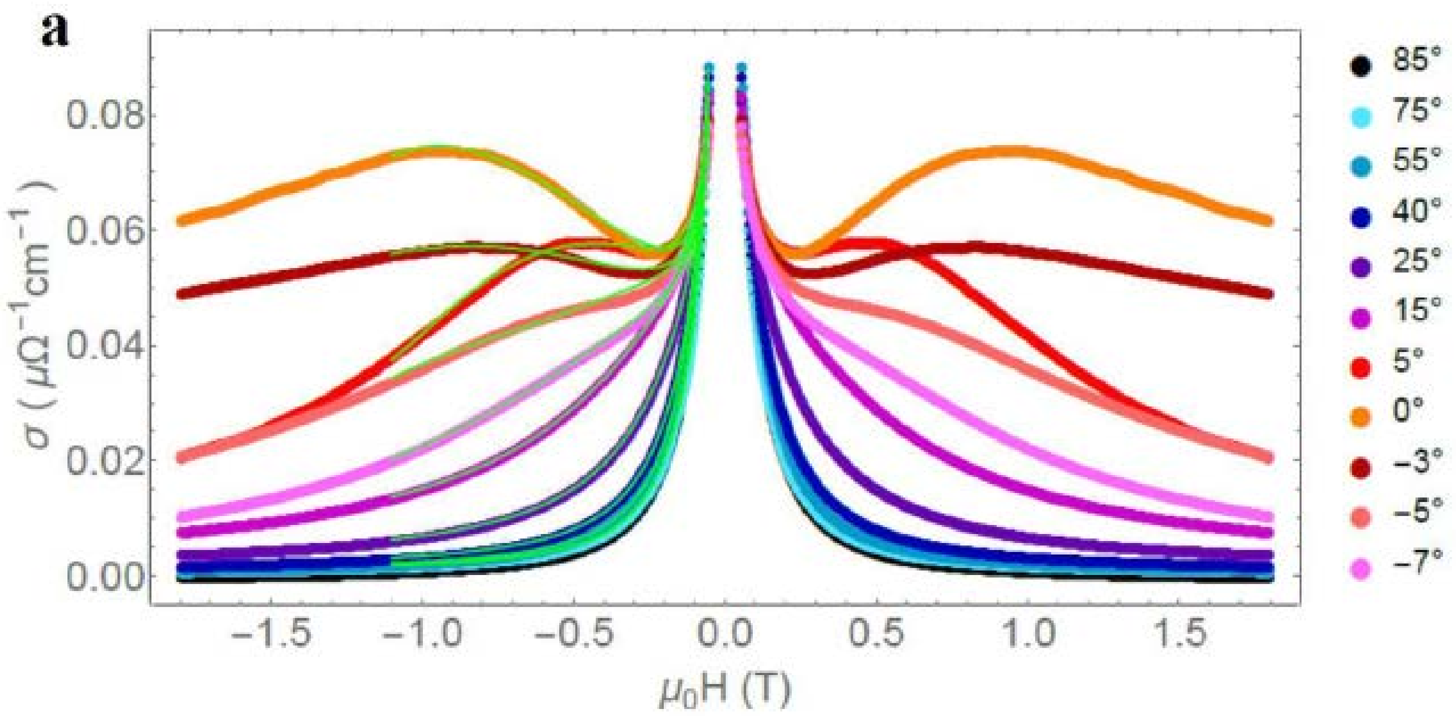
Weyl semimetals provide the realization of Weyl fermions in solid-state physics. Among all the physical phenomena that are enabled by Weyl semimetals, the chiral anomaly is the most unusual one. Here, we report signatures of the chiral anomaly in the magneto-transport measurements on the first Weyl semimetal TaAs. We show negative magnetoresistance under parallel electric and magnetic fields, that is, unlike most metals whose resistivity increases under an external magnetic field, we observe that our high mobility TaAs samples become more conductive as a magnetic field is applied along the direction of the current for certain ranges of the field strength. We present systematically detailed data and careful analyses, which allow us to exclude other possible origins of the observed negative magnetoresistance. Our transport data, corroborated by photoemission measurements, first-principles calculations and theoretical analyses, collectively demonstrate signatures of the Weyl fermion chiral anomaly in the magneto-transport of TaAs.

Magnetoconductance of TaAs (arxiv:1503.02630)



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Observation of the Chiral-Anomaly-Induced Negative Magnetoresistance in 3D Weyl Semimetal TaAs

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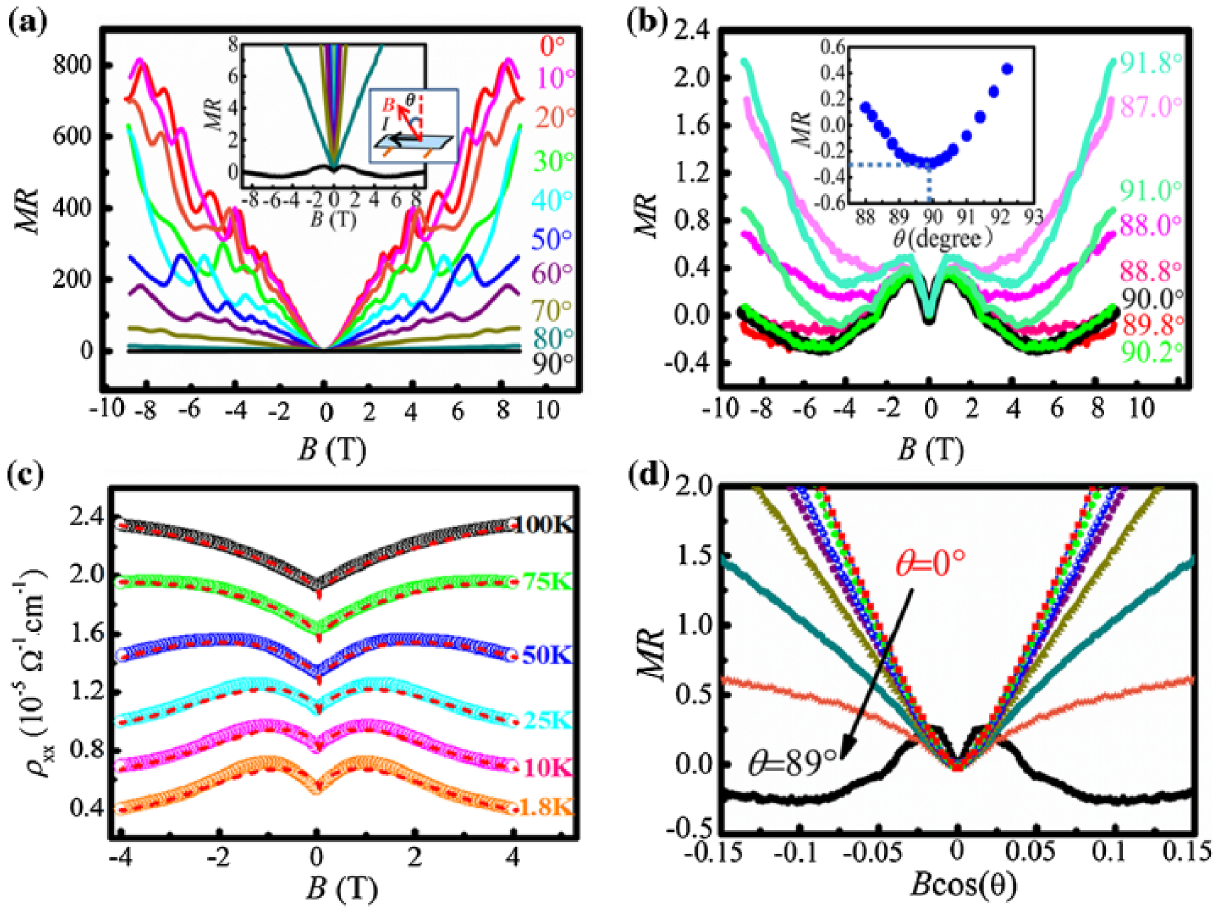

Conclusions

- Topological band theory of non-interacting materials is now well-established
    - Topological invariants and bulk-boundary correspondence
    - Robustness wrt disorder and weak interactions
    - Non-trivial electromagnetic (and gravitational) response

- Huge progress in ``materials''
    - Even more materials have been predicted to be topological (c 25%)
    - More complicated heterostructures
    - Toward applications ?