Majorana zero modes around skyrmionic textures

Pascal SIMON
University Paris Sud

Dirac equation

Paul Dirac (1928)

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\,\Psi = 0$$

<u>Complex</u> solution Ψ

particle ≠ antiparticle

Ettore Majorana (1906-19??)

Majorana's question (1937): does the Dirac equation necessarily involve complex fields?

La Procura: Ettore Majorana vivo in Venezuela fra il 1955 e il 1959

Majorana disappeared in unknown circumstances during a boat trip from Palermo to Naples on 25 March 1938. Despite several investigations, his body was not found and his fate is still uncertain. He had apparently withdrawn all of his money from his bank account prior to making his trip to Palermo

Le foto: due immagini del giovane Majorana con al centro una foto del 1950 scattata in Germania. Ma la svolta all'inchiesta è stata data da una seconda foto, scattata in Argentina nel 1955: secondo il Ris in questa seconda immagine ci sarebbero «10 coincidenze» tra il volto del fisico italiano e quello del padre

What is a Majorana fermion ?:

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\right)\Psi = 0$$

Majorana's answer: **No**, if Weyl matrices are purely imaginary

$$\gamma^0 = \sigma_y \otimes \sigma_x$$

$$\gamma^1 = i\sigma_x \otimes 1$$

$$\gamma^2 = i\sigma_z \otimes 1$$

$$\gamma^3 = i\sigma_y \otimes \sigma_y$$

$$\Rightarrow \quad \Psi = \Psi^*$$

•Neutral particle equals its own antiparticle.
•Very relevant in neutrino physics.
•Many experimental efforts to search for Majorana neutrinos are underway.

Majorana, Nuovo Cimento **14**, 171 (1937)

SO FAR NO EVIDENCE OF MAJORANA PARTICLES

HOWEVER MANY MAJORANA EXCITATIONS IN CONDENSED MATTER:

ALMOST ALL PROPOSALS ARE BASED ON HYBRID SYSTEMS INVOLVING **SUPERCONDUCTORS** perspective

NATURE PHYSICS | VOL 5 | SEPTEMBER 2009 | www.nature.com/naturephysics

Majorana returns

Frank Wilczek

www.sciencemag.org SCIENCE VOL 332 8 APRIL 2011
Published by AAAS

Search for Majorana Fermions Nearing Success at Last?

Researchers think they are on the verge of discovering weird new particles that borrow a trick from superconductors and could give a big boost to quantum computers

Physics
Physics 3, 24 (2010)

Viewpoint

Race for Majorana fermions

Marcel Franz
Department of Physics and Astronomy, University of British Columbia, Va
Published March 15, 2010

The race for realizing Majorana fermions—elusive particles th
we still await ideal materials to work with.

Physics
Physics 4, 67 (2011)

Viewpoint

Majorana fermions inch closer to reality

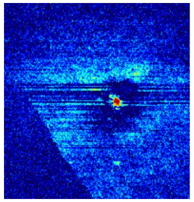
Taylor L. Hughes
University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana, IL 61801, USA
Published August 22, 2011

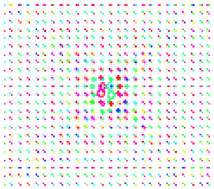
Outline

I) The simplest model exhibiting Majorana fermions: the 1D spinless superconductor (the Kitaev model)

II) A short review on the different experimental realizations

III) Pairs of Majorana zero modes states localized by magnetic defects

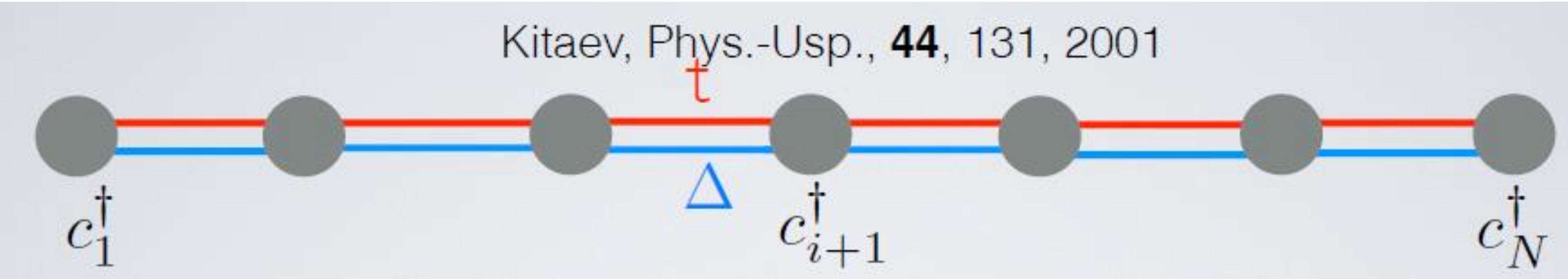




IV) Toward more exotic magnetic textures

I) 1D spinless topological superconductors

Kitaev's model



Majorana decomposition

$$\gamma_i^1 = c_i + c_i^\dagger$$

$$\gamma_i^2 = i(c_i - c_i^\dagger)$$

γ_i^1 γ_i^2 γ_{i+1}^1 γ_{i+1}^2

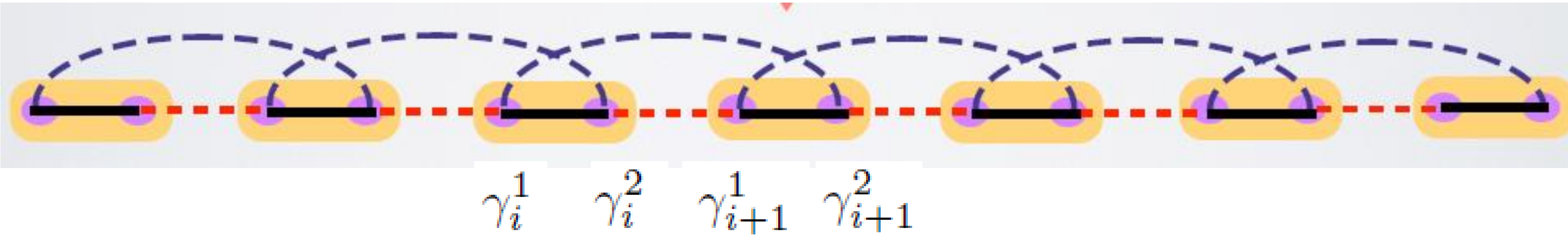
Any fermionic Hamiltonian can be recast in terms of Majorana operators !

Kitaev's model

However, very few can support solutions with isolated localized Majorana fermions

The necessary magic trick for getting a majorana fermion.

Kitaev's model



$$H = -\sum_{i=1}^{N-1}\left[t c_i^\dagger c_{i+1} + \Delta c_i^\dagger c_{i+1}^\dagger + \text{h.c.}\right] - \mu \sum_{i=1}^{N} n_i$$

$$\Downarrow$$

$$H = -i\sum_{i=1}^{N-1}[\omega_+ \gamma_i^2 \gamma_{i+1}^1 - \omega_- \gamma_i^1 \gamma_{i+1}^2] - i\frac{\mu}{2}\sum_{i=1}^{N} \gamma_i^2 \gamma_i^1$$

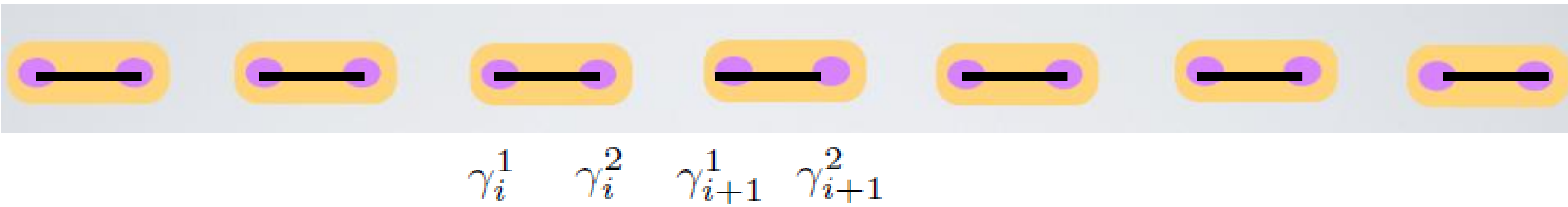
$$\omega_+ = \frac{1}{2}(t+\Delta)$$

$$\omega_- = \frac{1}{2}(t-\Delta)$$

$$\mu$$

Kitaev's model

t= Δ=0



$$H = -i \sum_{i=1}^{N-1} [\omega_+ \gamma_i^2 \gamma_{i+1}^1 - \omega_- \gamma_i^1 \gamma_{i+1}^2] - i\frac{\mu}{2} \sum_{i=1}^{N} \gamma_i^2 \gamma_i^1$$

$\omega_+ = t = \Delta = 0$

$\omega_- = 0$

Kitaev's model

$t = \Delta = 0$

$\gamma_i^1 \quad \gamma_i^2 \quad \gamma_{i+1}^1 \quad \gamma_{i+1}^2$

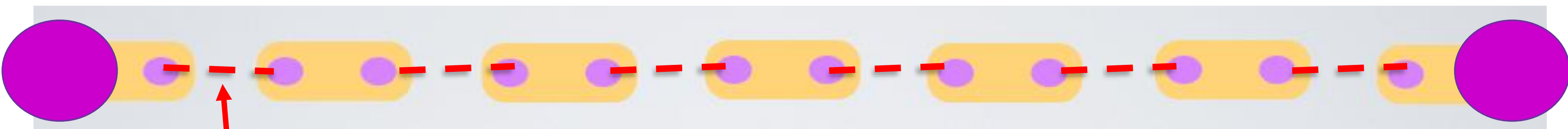
$$H = -i\frac{\mu}{2}\sum_{i=1}^{N}\gamma_i^2\gamma_i^1 = -\mu\sum_{i=1}^{N}(c_i^\dagger c_i - \frac{1}{2})$$

TRIVIAL NONINTERACTING FERMIONS ON THE LATTICE

$|\mu| > 2t$

Kitaev's model

t= Δ ; μ=0 |μ| < 2t



$\gamma_i^1 \quad \gamma_i^2 \quad \gamma_{i+1}^1 \quad \gamma_{i+1}^2$

fuse Majorana fermions across nearest neighbor bonds

$$a_i = \frac{1}{2}(\gamma_i^2 + i\gamma_{i+1}^1)$$

$$a_i^\dagger = \frac{1}{2}(\gamma_i^2 - i\gamma_{i+1}^1)$$

→

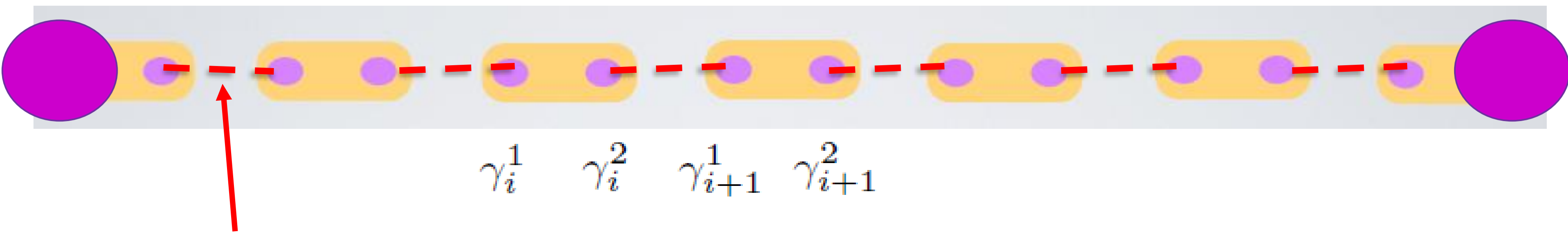
$$H = 2t\sum_{i=1}^{N-1}(a_i^\dagger a_i - \frac{1}{2})$$

$$H = -i\sum_{i=1}^{N-1}[\omega_+\gamma_i^2\gamma_{i+1}^1 - \omega_-\gamma_i^1\gamma_{i+1}^2] - i\frac{\mu}{2}\sum_{i=1}^{N}\gamma_i^2\gamma_i^1$$

$$\omega_+ = t = \Delta$$

Kitaev's model

t= Δ ; μ=0 |μ| < 2t



fuse Majorana fermions across nearest neighbor bonds

$$a_i = \frac{1}{2}(\gamma_i^2 + i\gamma_{i+1}^1)$$

$$a_i^\dagger = \frac{1}{2}(\gamma_i^2 - i\gamma_{i+1}^1)$$

$$H = 2t \sum_{i=1}^{N-1} (a_i^\dagger a_i - \frac{1}{2})$$

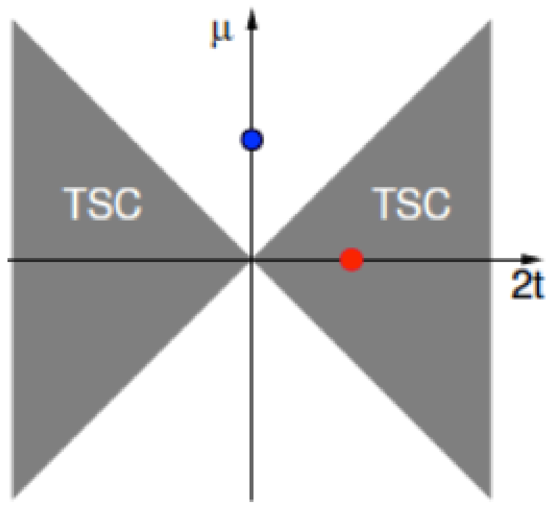
GAPPED SPECTRUM+ZERO-ENERGY MAJORANAS AT THE END OF THE WIRE
(DECOUPLED FROM THE BULK OF THE CHAIN)!!!

Phase diagram of the 1D Kitaev model

$$H = -\sum_{i=1}^{N-1}\left[tc_i^\dagger c_{i+1} + \Delta c_i^\dagger c_{i+1}^\dagger + \text{h.c.}\right] - \mu\sum_{i=1}^{N} n_i$$ + periodic boundary conditions

$$\Rightarrow \quad H_{\text{BdG}} = \frac{1}{2}\sum_p \Psi_p^\dagger \begin{pmatrix} -2t\cos p - \mu & 2i|\Delta|\sin p \\ -2i|\Delta|\sin p & 2t\cos p + \mu \end{pmatrix} \Psi_p$$

The gap closes for $|\mu| = 2t$



System is topologically non trivial (topological SC) for $|\mu| < 2t$

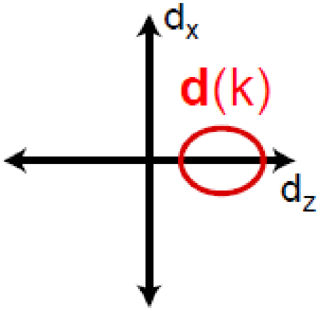
Z Bulk invariant

$$H - \mu N = \sum_i t(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) - \mu c_i^\dagger c_i + \Delta(c_i c_{i+1} + c_{i+1}^\dagger c_i^\dagger)$$

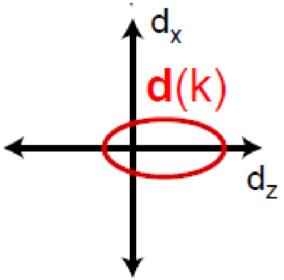
$$H_{BdG}(k) = \tau_z(2t\cos k - \mu) + \tau_x \Delta \sin k = \mathbf{d}(k) \cdot \vec{\tau}$$

Only two components of **d** appear !

$|\mu|>2t$: Strong pairing phase
trivial superconductor



$|\mu|<2t$: Weak pairing phase
topological superconductor



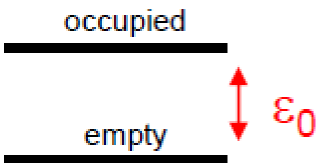
⇒ Winding number = Z invariant (emergent chiral symmetry)

BDI class

About Majorana fermions'properties

Two Majorana fermions define a single two level system

$$\begin{cases} \gamma_1 = \Psi + \Psi^\dagger \\ \gamma_2 = -i(\Psi - \Psi^\dagger) \end{cases} \Rightarrow H = 2i\varepsilon_0 \gamma_1 \gamma_2 = \varepsilon_0 \Psi^\dagger \Psi$$



- 2 degenerate states (full/empty) = 1 qubit

- 2N separated Majoranas = N qubits
- Quantum Information is stored non locally : Immune from local decoherence

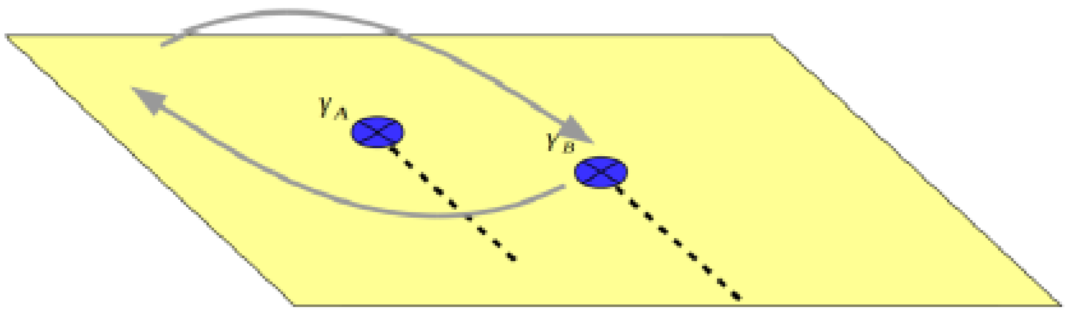
Braiding performs unitary operations:

Non-Abelian statist

Interchange rule

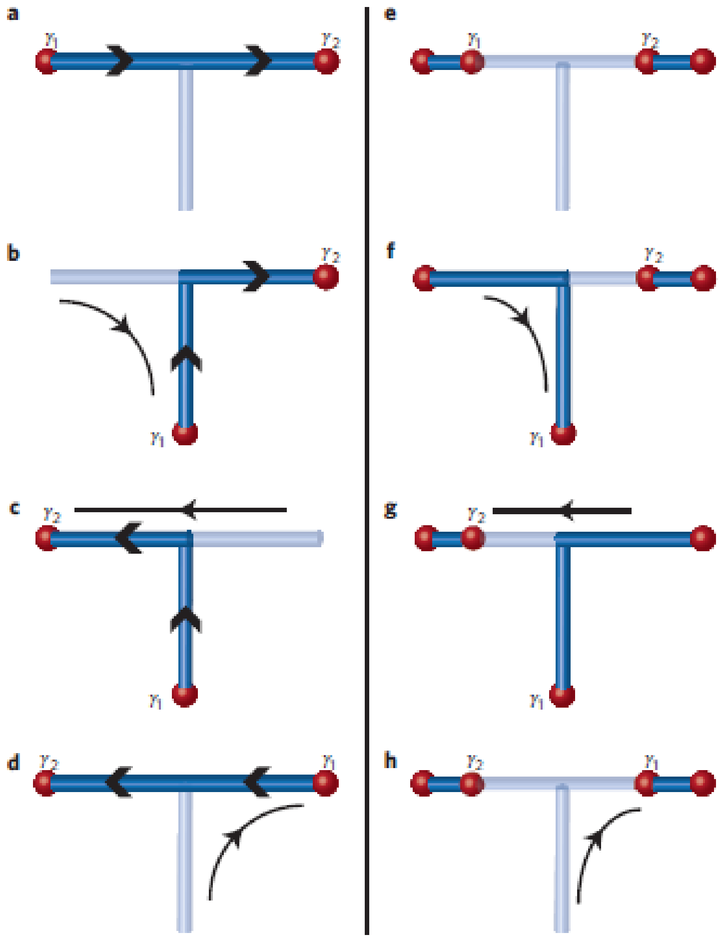
$$\gamma_i \rightarrow \gamma_j$$

$$\gamma_j \rightarrow -\gamma_i$$



- exchange (=π rotation): $\gamma_b \rightarrow \gamma_a \qquad \gamma_a \rightarrow -\gamma_b$
- braid around (=2π rotation): $\gamma_a \rightarrow -\gamma_a \qquad \gamma_b \rightarrow -\gamma_b$

Braiding of Majorana fermions



T-junctions shows non-Abelian statistics

Alicea et al., Nature Physics, 2010

II) Some different material strategies to obtain a p-wave topological superconductor

Challenges

- Electrons are spin-degenerate so we must freeze out half of the degrees of freedom to have an effective spinless system.
- p-wave superconductors seem rather rare in nature

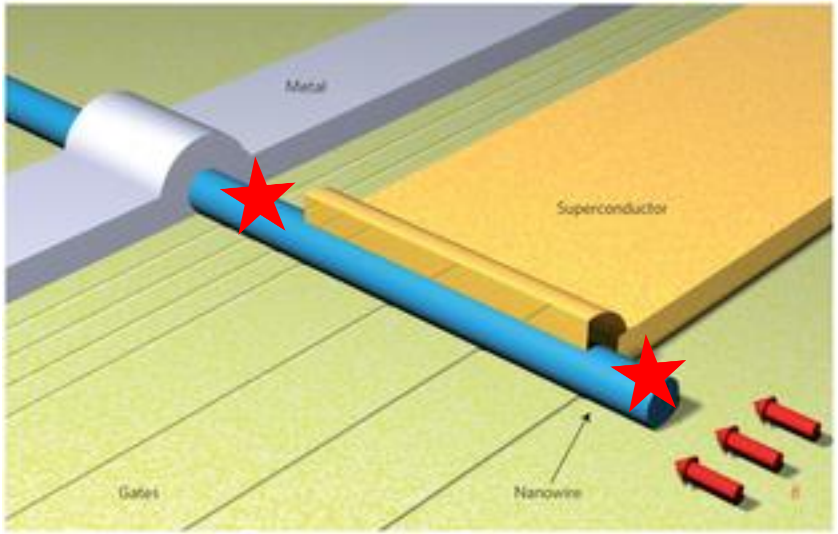
→ Clever proposals that overcome these challenges have the same three main ingredients:

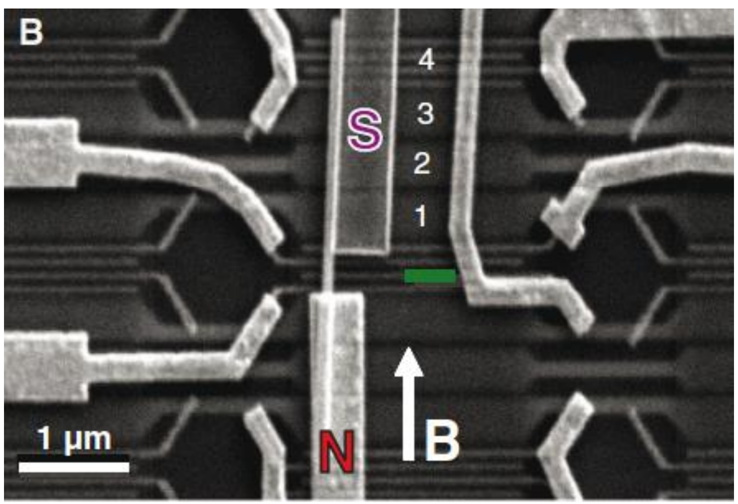
1. Instead of using intrinsic superconductivity use the superconducting proximity effect.

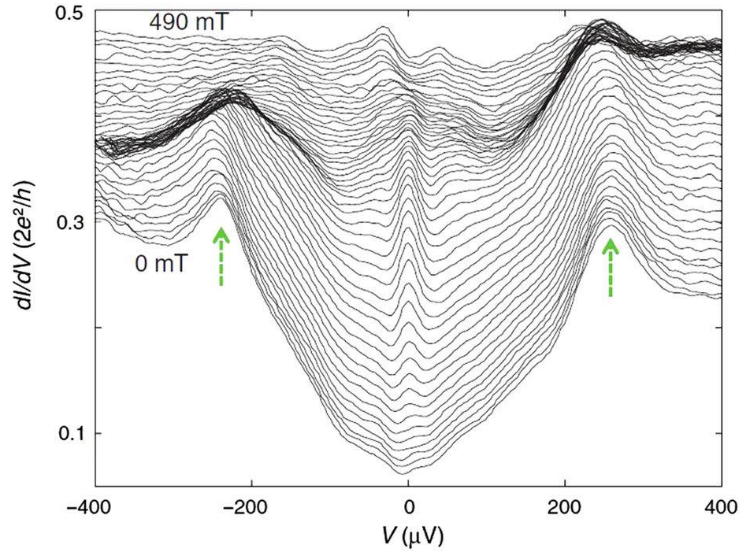
2. Time-reversal symmetry breaking

3. Spin-orbit coupling or magnetic texture

Majorana end states in semiconducting nanowire devices





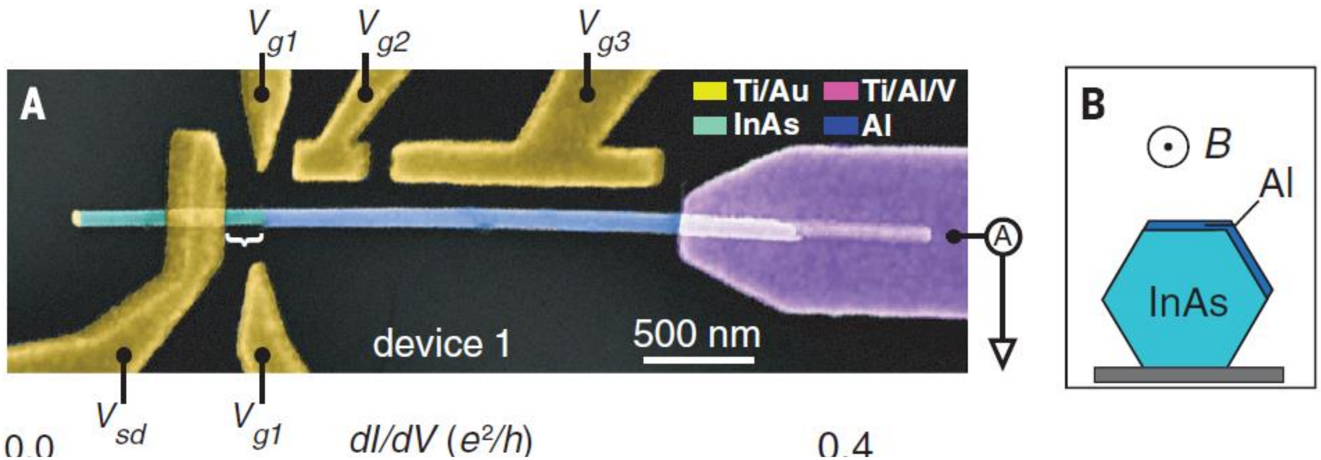


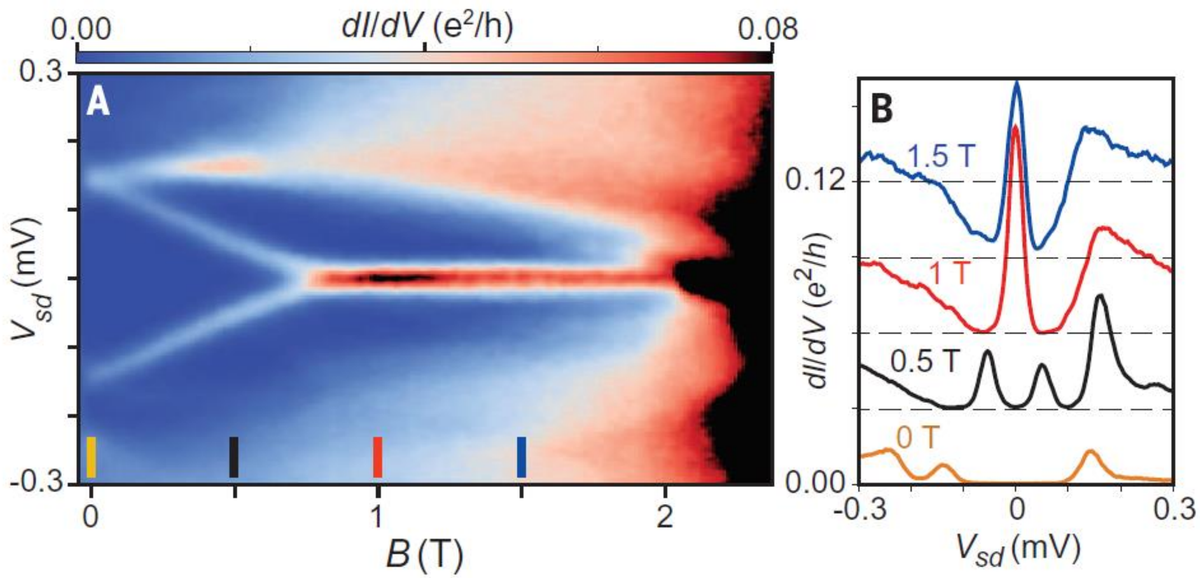
experiment (Kouwenhoven)

Resonant Andreev reflection in the NS interface owing to the presence of the Majorana peak at $V = 0$.

Mourik V, Zuo K, Frolov SM, Plissard SR, Bakkers EPAM, Kouwenhoven LP., *Science* 336, 1003, (2012).

Majorana end states in semiconducting nanowire devices



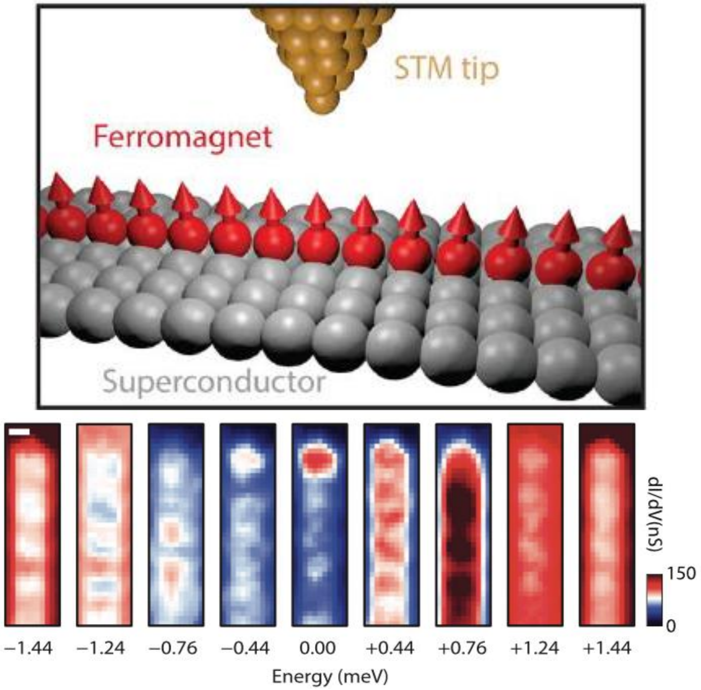


Marcus' group, Copenhagen

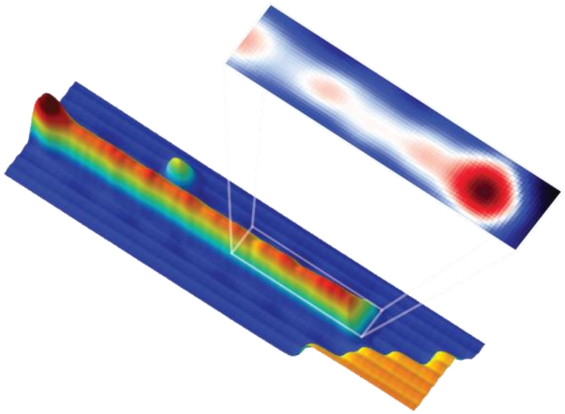
Majorana end states in one-dimension structures

Chain/wire of magnetic adatoms

Possible experimental realizations



Zero-bias anomaly localized on the last atoms of the Fe chain, almost no extension into the Pb substrate



S. Nadj-Perge et al., Science **346,** 6209 (2014)
B. E. Feldman et al., Nature Physics (2016)
S. Jeon et al., Science (2017) **(Princeton)**

see also

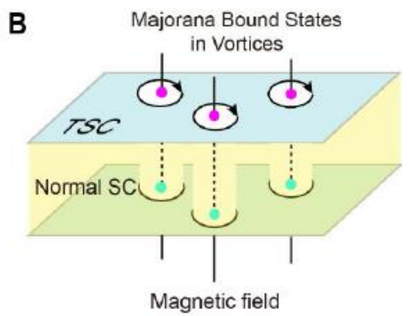
M. Ruby et al., PRL 2015 **(Berlin)**
R. Pawlak et al., NPJ QI (2016) **(Basel)**
H. Kim et al. Science Advances (2018) **(Hamburg)**

Majorana fermions in 2D-like structures

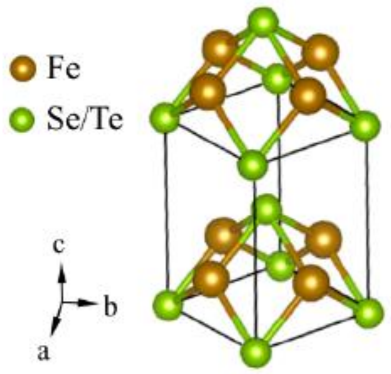
Iron-based bulk superconductor $FeTe_{1-x}Se_x$:

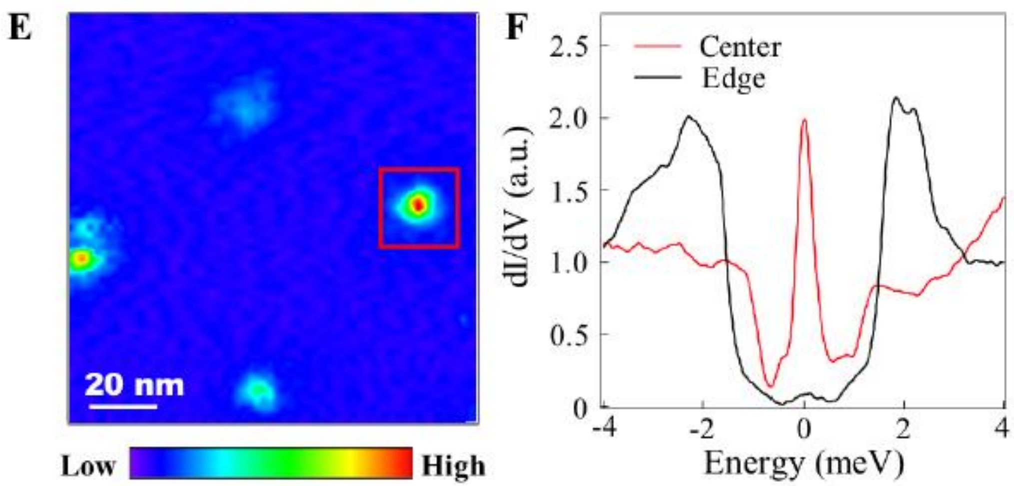
- spin-helical Dirac surface state
- small Fermi energy

S. Jeon et al., Science **358**, 772 (2017)



Bound states in vortex cores
in iron-based superconductor $FeTe_{1-x}Se_x$





D. Wang et al., arxiv 1706.06074

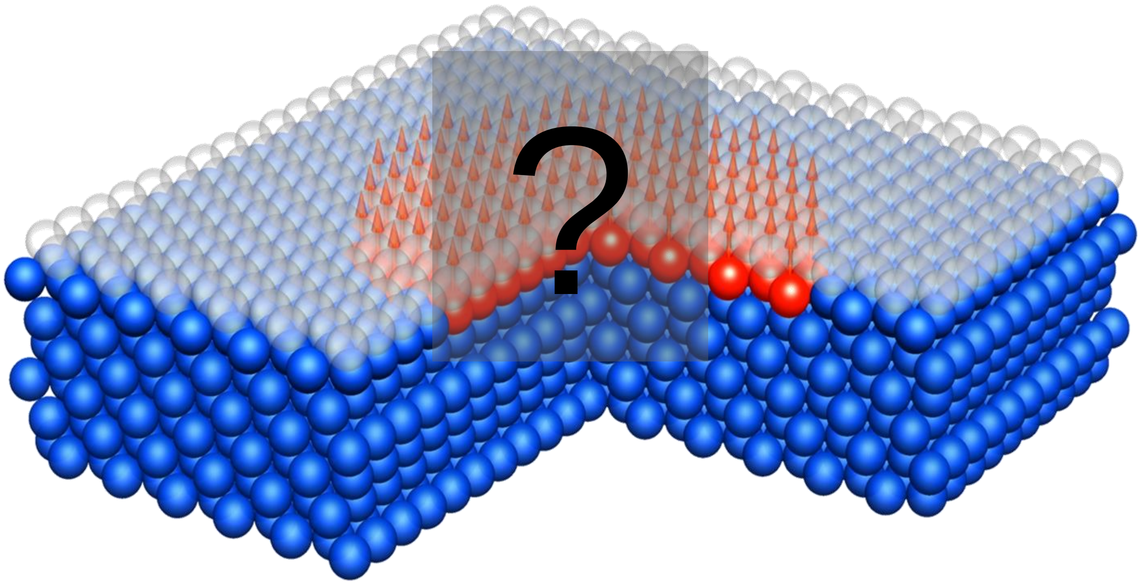
See also related study in (Li0.84Fe0.16)OHFeSe

Q. Liu et al., arxiv 1807.01278

our strategy:

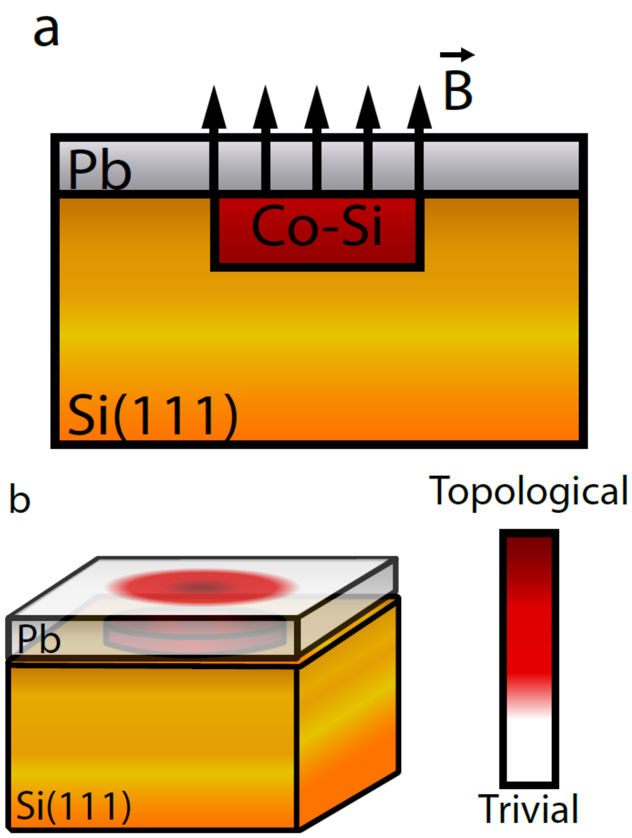
Engineering **2D topological superconductivity**
with
self-assembled magnetic clusters
covered by a monolayer of Pb

Interplay between a magnetic cluster and 2D superconductivity

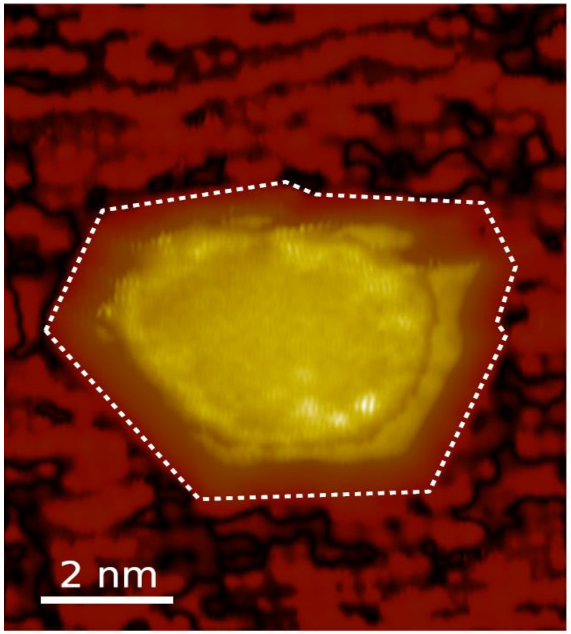


What happens above a magnetic cluster sitting below a Pb monolayer ?

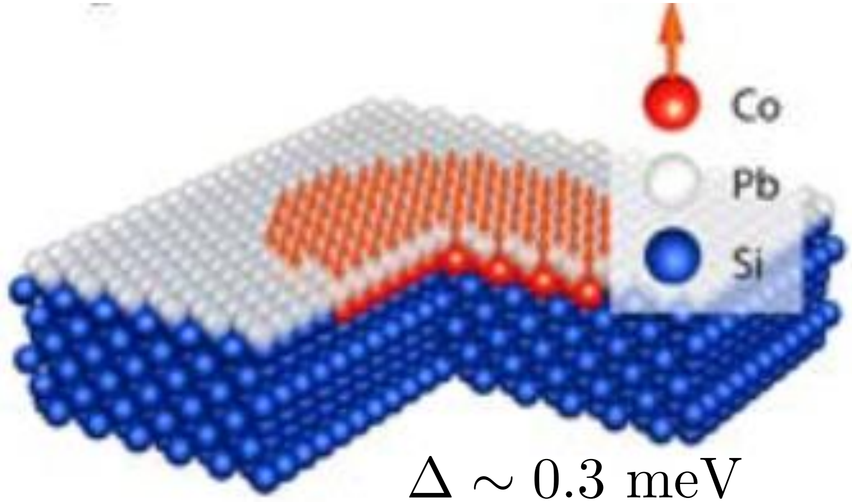
System studied: Magnetic nano-cluster embedded in Pb/Si(111)



Magnetic clusters under the Pb layer to create topological superconductivity over the cluster

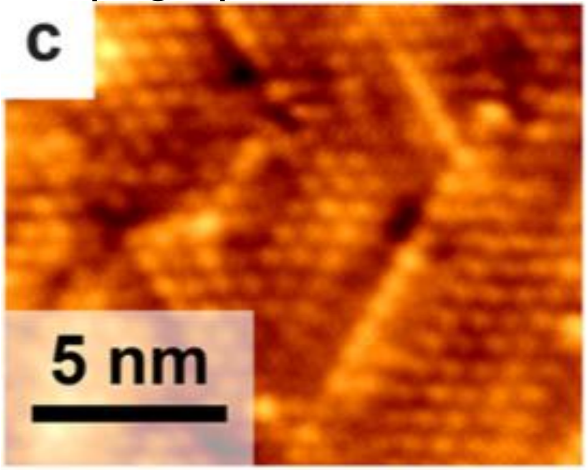


Pb/Co/Si(111) system



$$\Delta \sim 0.3\ \text{meV}$$
$$\alpha \sim 300\ \text{meV}$$

Topograph 16nm × 13nm

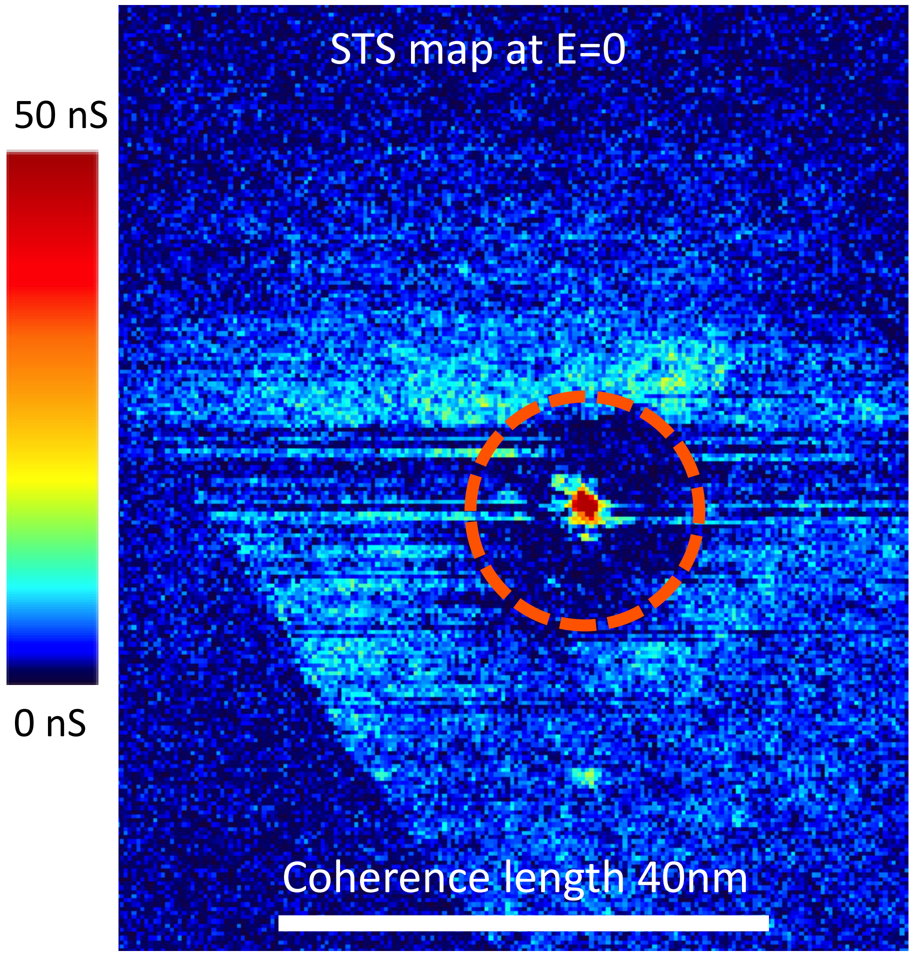


- Island radius: $R \sim 10nm$
- Coherence length: $\xi \sim 40nm$

➡ **Lengthscale separation** $L \gg \xi > R \gg l_F \sim l_{so}$

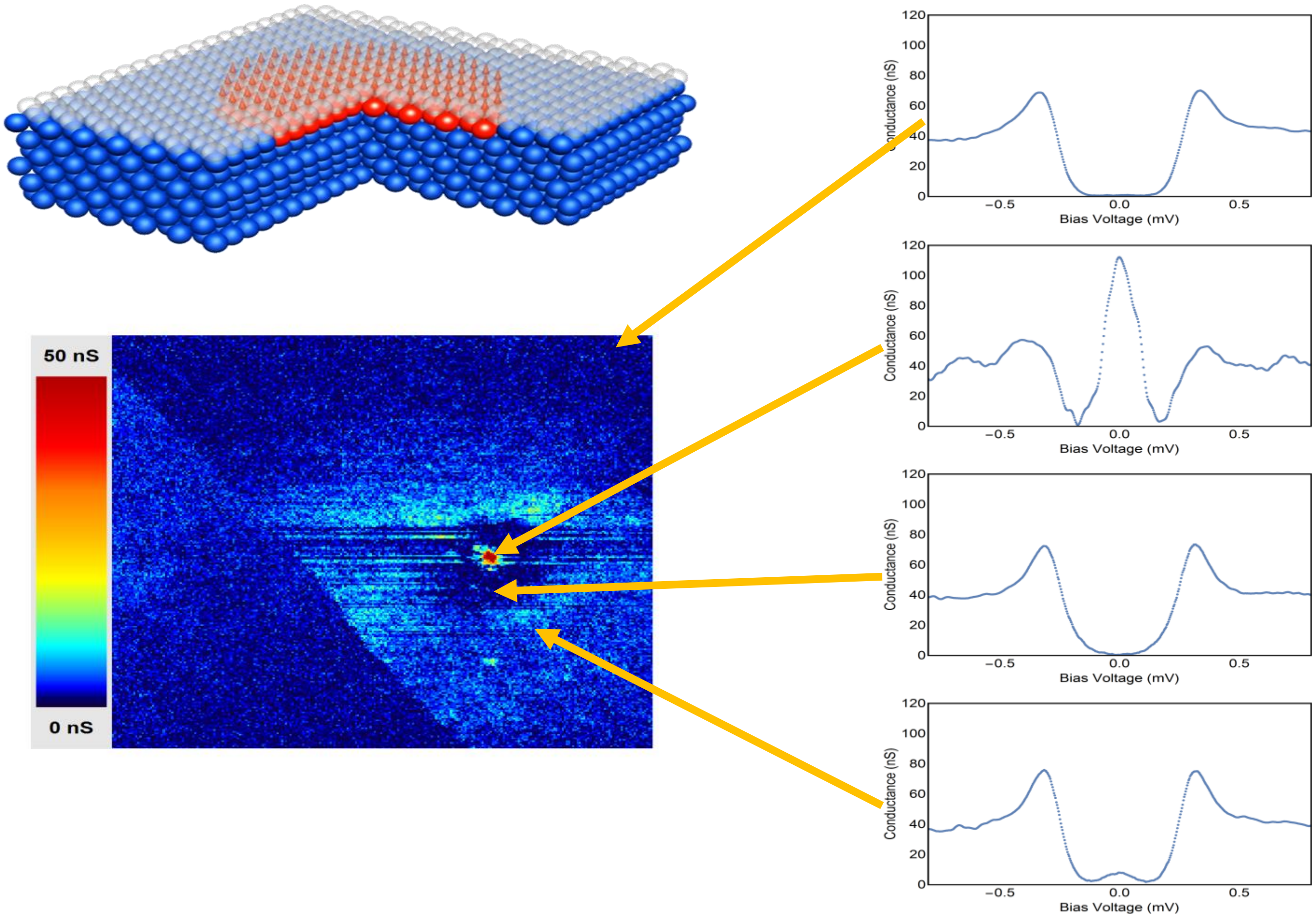
Extraordinary state at E=0 in some larger cluster

(1) Strongly localized+edge state



Lengthscale separation $L \gg \xi > R \gg l_F \sim l_{so}$

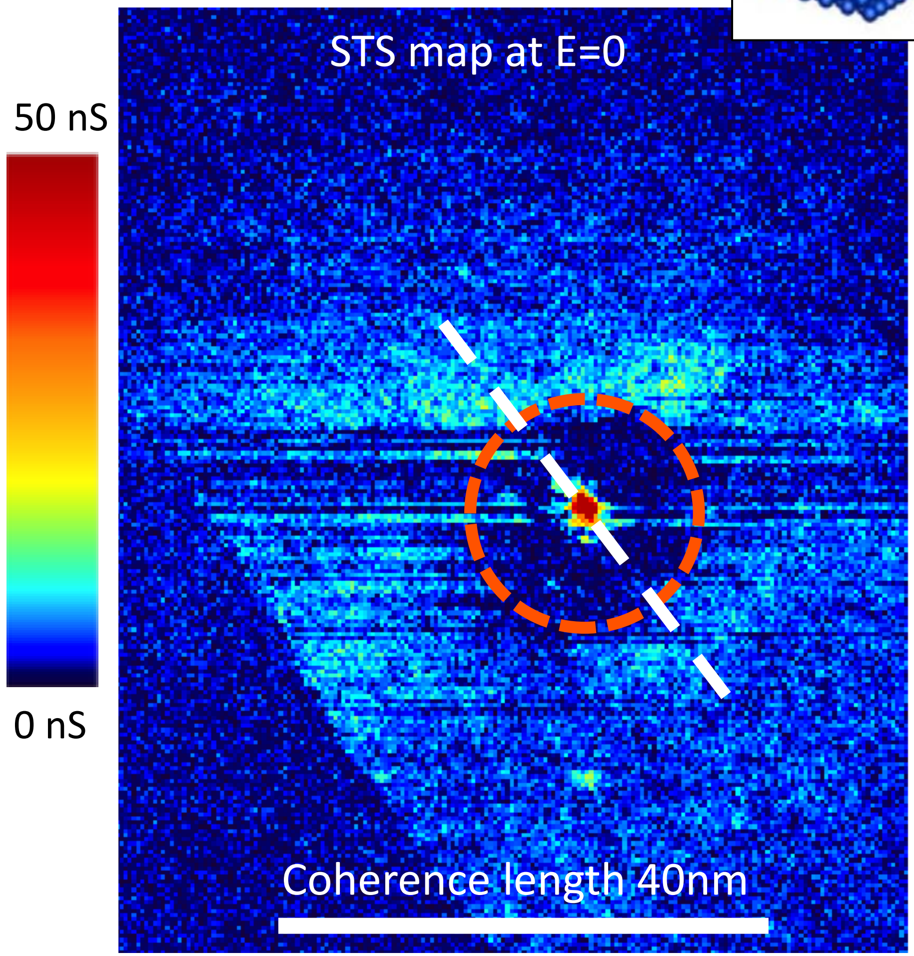
Majorana bound state in a vortex core ?

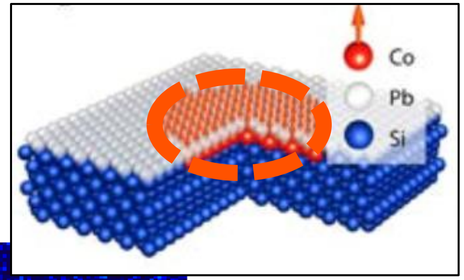


G. Ménard et al., Nature Comm 2019

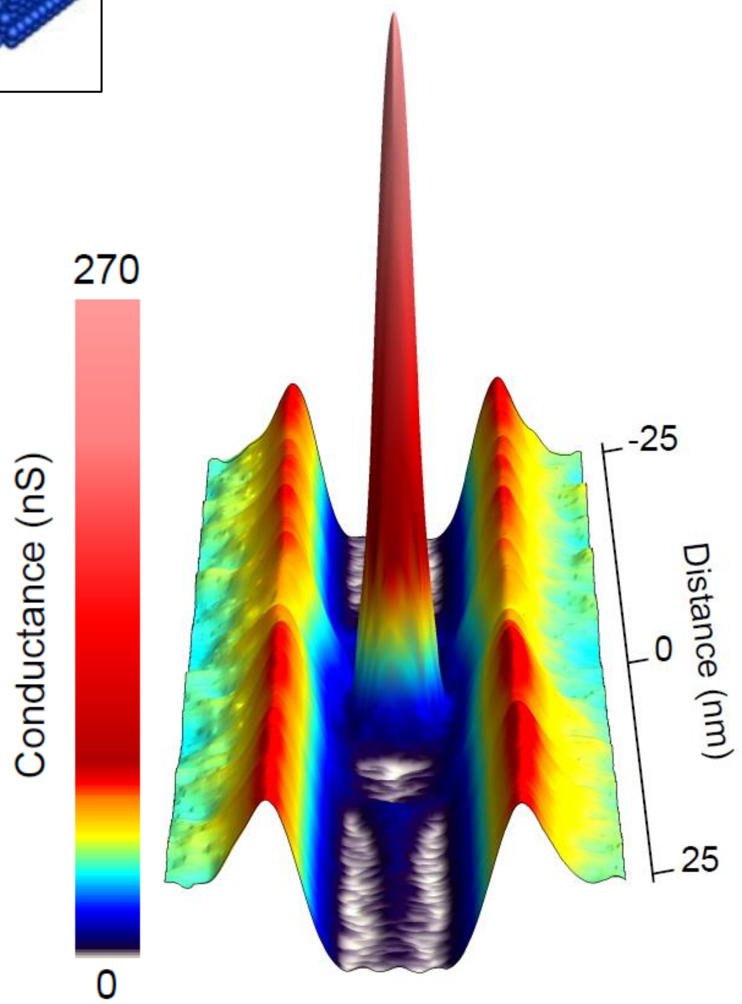
Extraordinary state at E=0

(1) Strongly localized
+ edge state





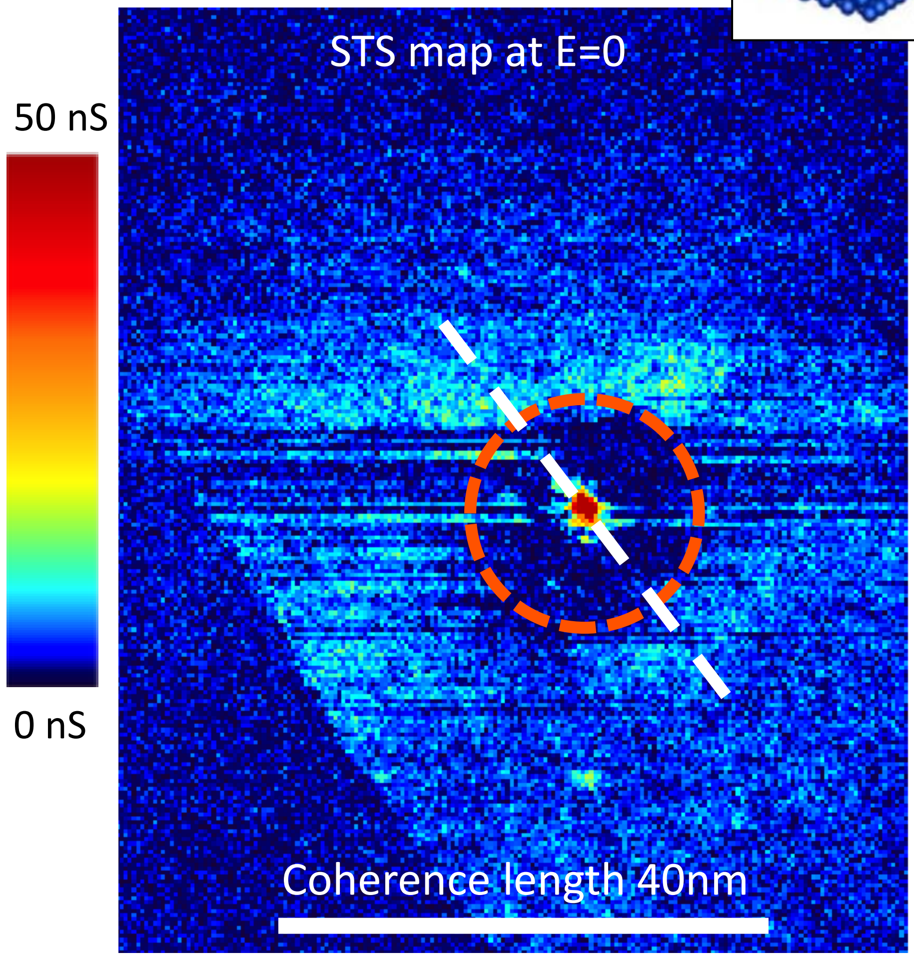
(2) Isolated in energy

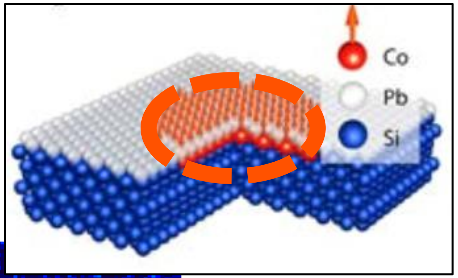


G. Ménard et al., Nature Comm 2019

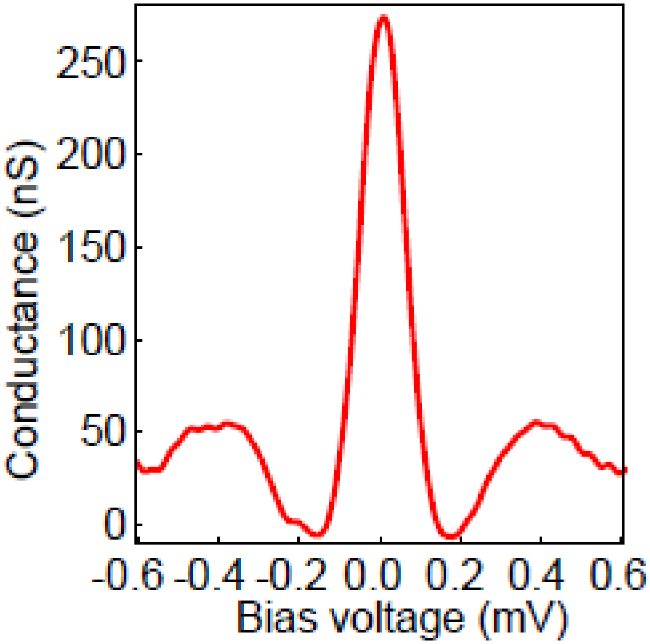
Extraordinary state at E=0

(1) Strongly localized + edge state





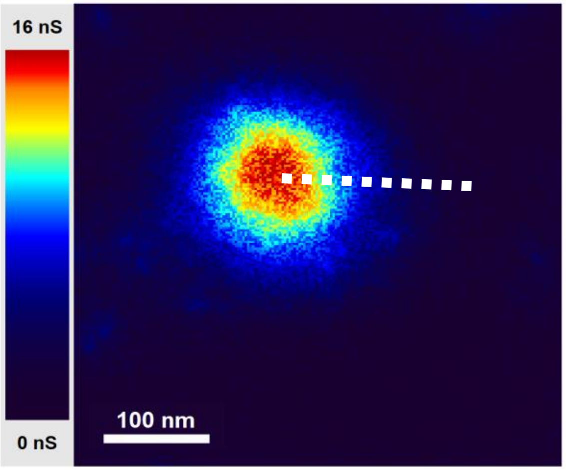
(2) Isolated in energy

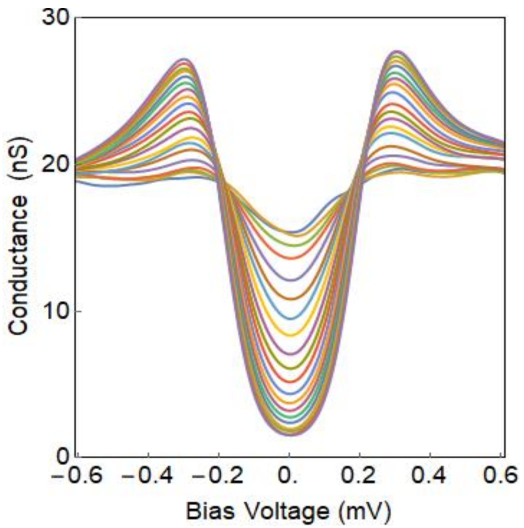


G. Ménard et al., Nature Comm 2019

Contrast to superconducting vortex

(1) Large lengthscale

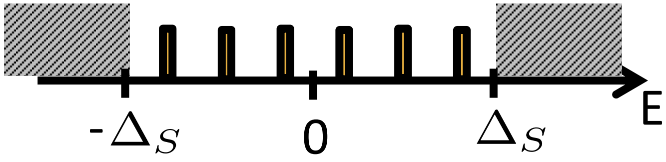




(2) Filled energy gap

Trivial SC *(spin singlet)*

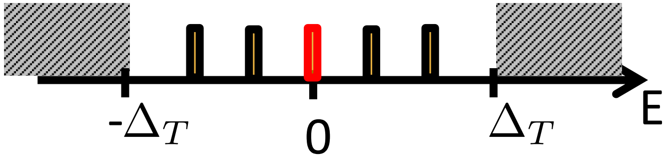
$$H_0 = \sum_{\sigma=\uparrow\downarrow} (\varepsilon_k - \mu) c^\dagger_{k\sigma} c_{k\sigma} + \Delta_S c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + H.c.$$



Caroli *et al.* PhysLett'64

Topological SC *(triplet/spinless)*

$$H_T = (\varepsilon_k - \mu) c^\dagger_k c_k + \frac{\Delta_T}{k_F}(k_x - i k_y) c^\dagger_k c^\dagger_{-k} + H.c.$$



Kopnin&Salomaa'91
Read&Green *PRB'00*

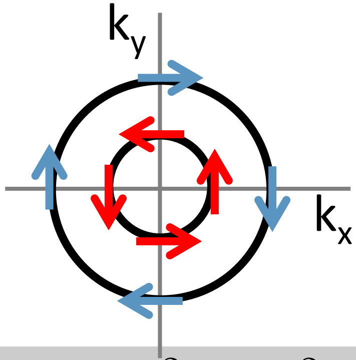
"Mini-gap":

$$\delta \sim \frac{\Delta}{k_F \xi} \sim \Delta \left(\frac{\Delta}{E_F} \right) \longrightarrow \text{many in-gap states}$$

Spin-orbit defect

Metal + Vz exchange + s-wave SC + Spin-Orbit

$$H = \sum_{\sigma=\pm} (\varepsilon_k - \mu + V_z \sigma) c^\dagger_{k\sigma} c_{k\sigma} + \Delta_S c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + H_{so} + H.c.$$



Rashba Spin-Orbit:

$$H_{so} = \alpha\, c^\dagger_{k\uparrow}(k_y + i k_x) c_{k\downarrow} \longrightarrow \mathcal{H}_{so} = \alpha\, \vec{\boldsymbol{\sigma}} \times \vec{\boldsymbol{k}} \cdot \hat{z}$$

Topo SC: $V_z^2 > \Delta_S^2 + \mu^2$

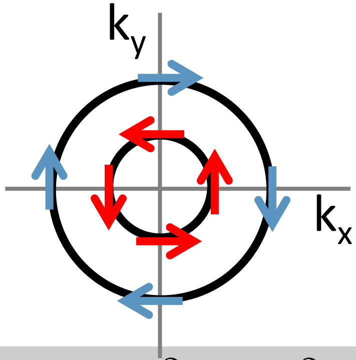
Spin-orbit defect

Metal + Vz exchange + s-wave SC + Spin-Orbit

$$H = \sum_{\sigma=\pm} (\varepsilon_k - \mu + V_z\sigma) c^\dagger_{k\sigma} c_{k\sigma} + \Delta_S c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + H_{so} + H.c.$$

Rashba Spin-Orbit:

$$H_{so} = \alpha\ c^\dagger_{k\uparrow}(k_y + ik_x)c_{k\downarrow} \longrightarrow \mathcal{H}_{so} = \alpha\ \vec{\boldsymbol{\sigma}} \times \vec{\boldsymbol{k}} \cdot \hat{z}$$

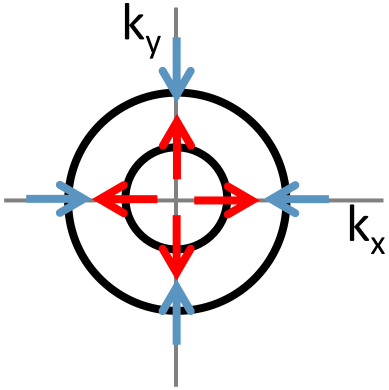


Topo SC: $V_z^2 > \Delta_S^2 + \mu^2$

Spin-Orbit defect:

$$H_{SOV} = |\alpha| e^{i\theta(\vec{r})}\ c^\dagger_{k\uparrow}(k_y + ik_x)c_{k\downarrow} \longrightarrow \mathcal{H}_{so} = \alpha \left[\cos(\theta)\vec{\boldsymbol{\sigma}} \times \vec{\boldsymbol{k}} - \sin(\theta)\vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{k}}\right]$$

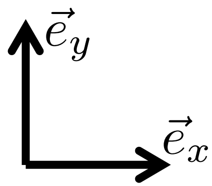
- **Topological defect entirely in band structure!**
- **Allowed by symmetry**



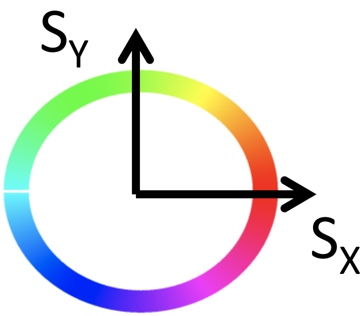
Spin-orbit defect: Tight binding

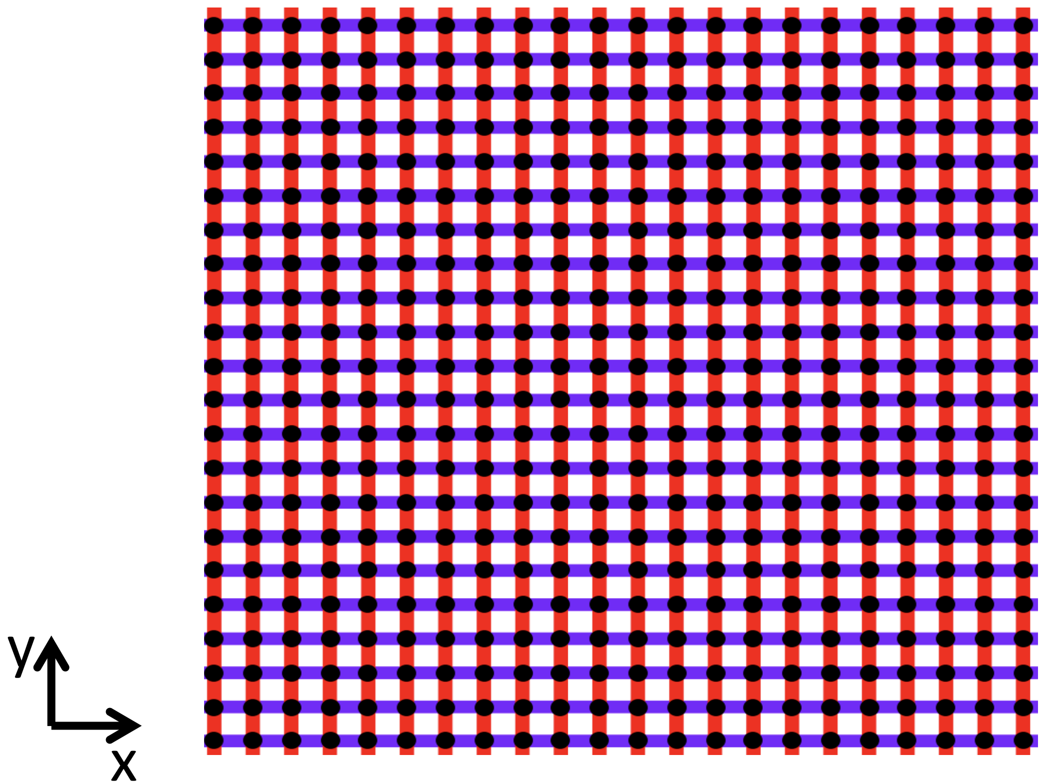
$$H = \sum_{R,\delta} \Psi^\dagger_{R+\delta} \left[-t + \frac{i\alpha}{2a} \hat{\lambda}_\delta \right] \tau_z \Psi_R + \sum_R \Psi^\dagger_R \left[-\mu \tau_z + \Delta_S \tau_x + V_z \sigma_z \right] \Psi_R$$

- Neighbors: δ =



Rashba:





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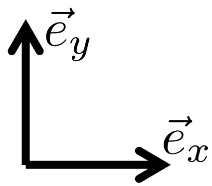
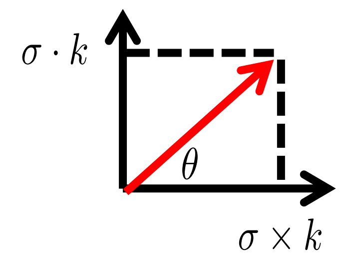
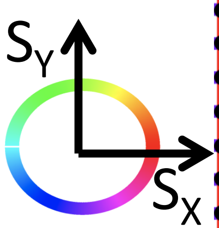
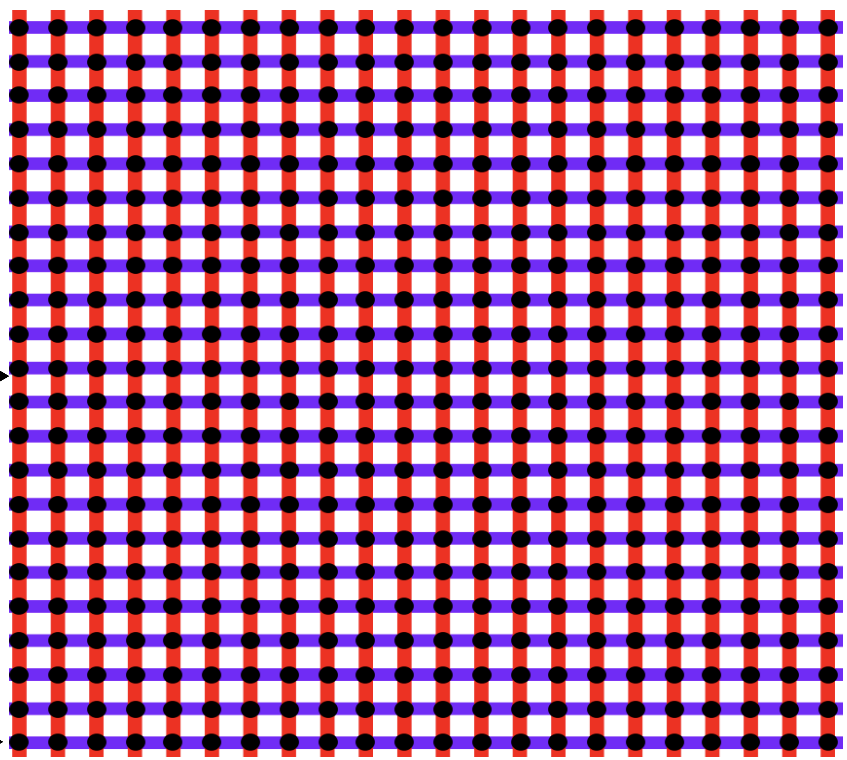


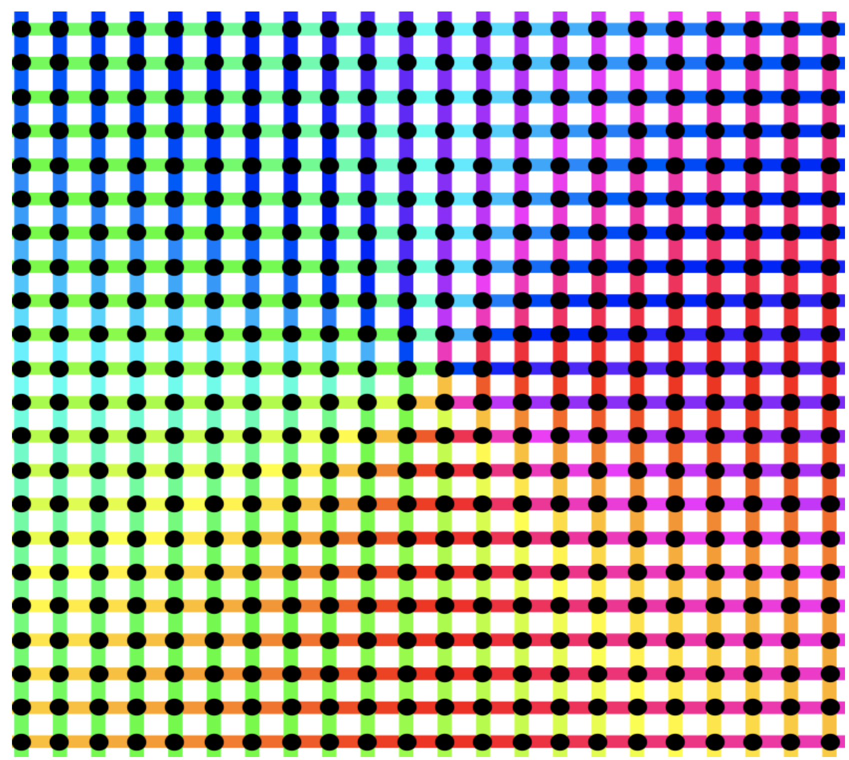


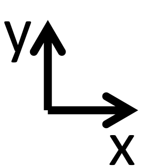
Rashba:

Spin-Orbit vortex:









Spin-orbit defect: Majorana Zero Mode

- At angular momentum **m**=0

$$\left[\begin{array}{c|c} -\partial_x^2 - \frac{1}{4x^2} + \tilde{V}_z & \tilde{\alpha}\partial_x + 1 \\ \hline -\tilde{\alpha}\partial_x - 1 & -\partial_x^2 - \frac{1}{4x^2} - \tilde{V}_z \end{array}\right] \psi_0(x) = 0$$

$$\tilde{V}_z = V_z/\Delta_S$$

$$\tilde{\alpha} = \alpha k_F/\sqrt{E_F\Delta}$$

$$x = rk_F\sqrt{\Delta/E_F}$$

- MZM Ansätze

$$\psi_0(r) \sim e^{-r(V_z-\Delta_S)/\alpha}$$

$$E_{exc} \sim V_z - \Delta_S \ ?$$

M.Sato, Y.Takahashi, S.Fujimoto
PRL 103, 020401(2009)

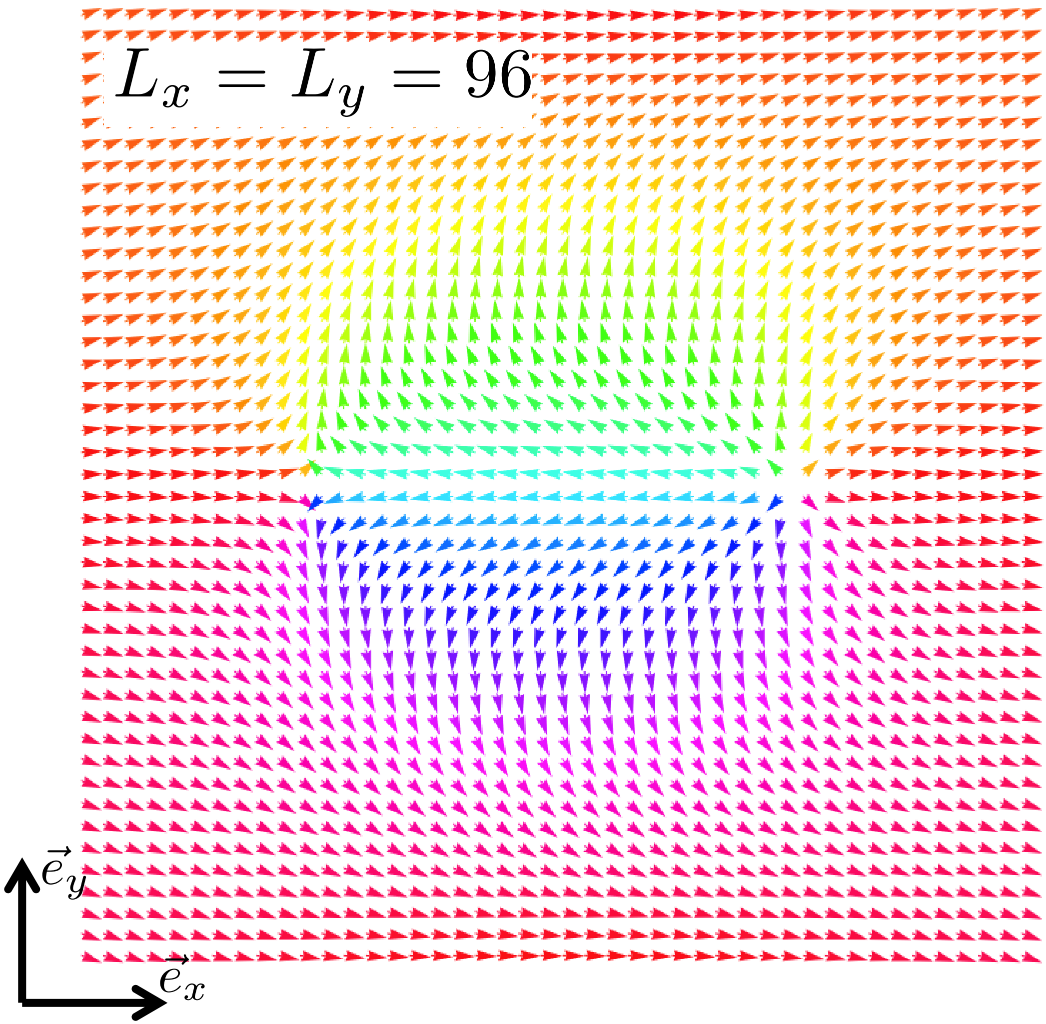
$$|\psi_0(r)| \sim e^{-r\Delta_S/\alpha}\frac{1}{\sqrt{r}}$$

$$E_{exc} \sim \Delta_S \ ?$$

J.D.Sau, S.Tewari, R.M.Lutchyn,
T.D.Stanescu, S.Das Sarma
PRB 82, 214509(2010)

Spin-orbit defect excitations: Numerics (I)

Tight-binding on 2d square lattice torus LxL: ***Vortex – anti-Vortex pair***



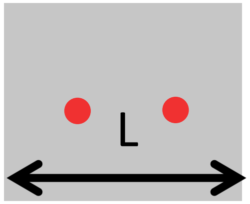
Full spectrum?

Majorana localization?

BUT!

- No island
- Finite size effects:

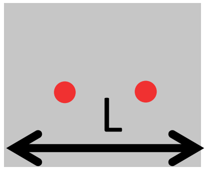
$$L \sim \xi \sim 80 \gg l_F \sim 1$$

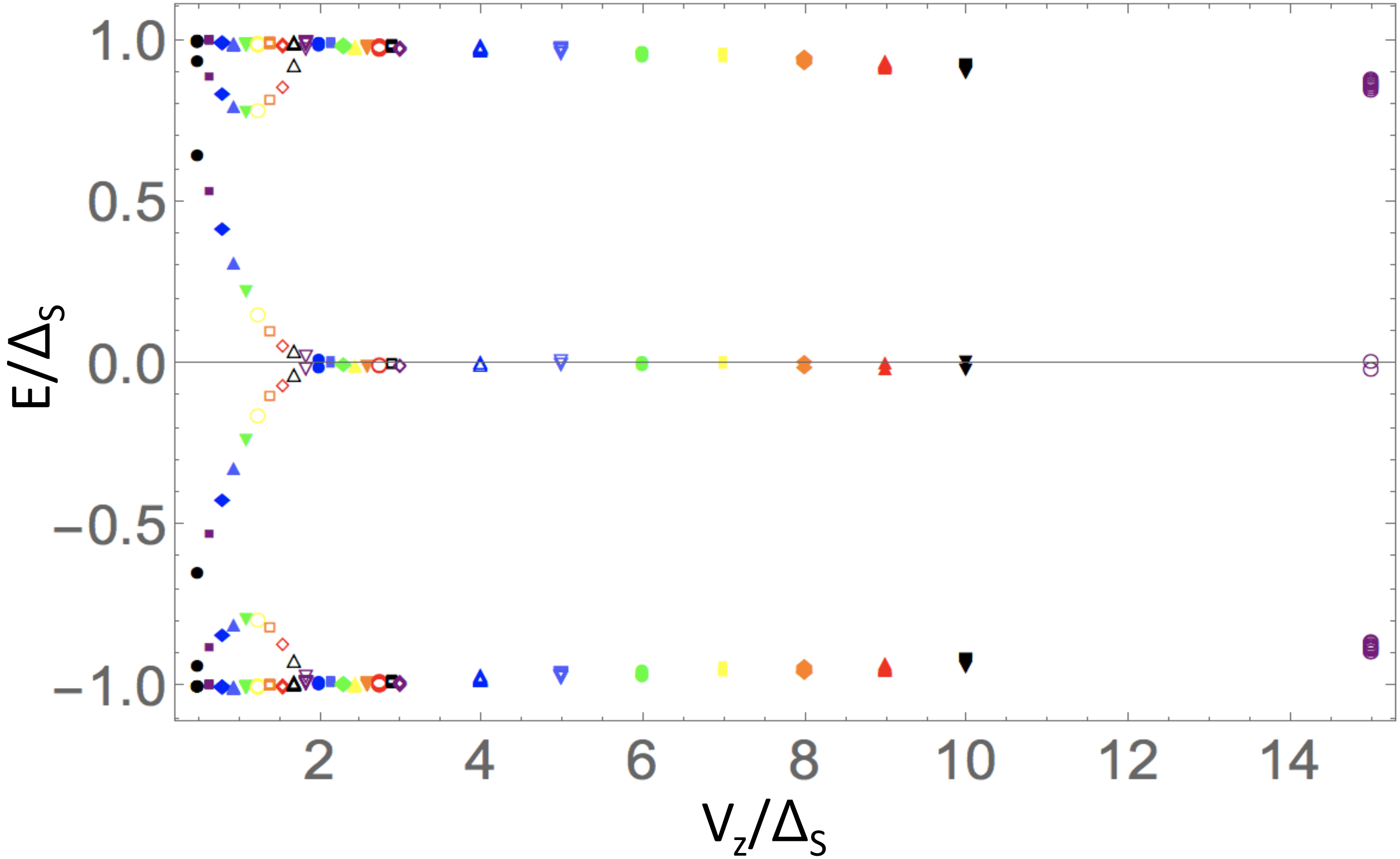


Spin-orbit defect excitations: Numerics (I)

Spectrum: 2d square lattice torus L=400

$E_F/\Delta_S \simeq 40,\ \alpha/E_F \simeq 3$





G. Ménard, A. Mesaros et al., submitted

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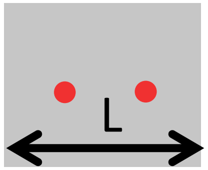
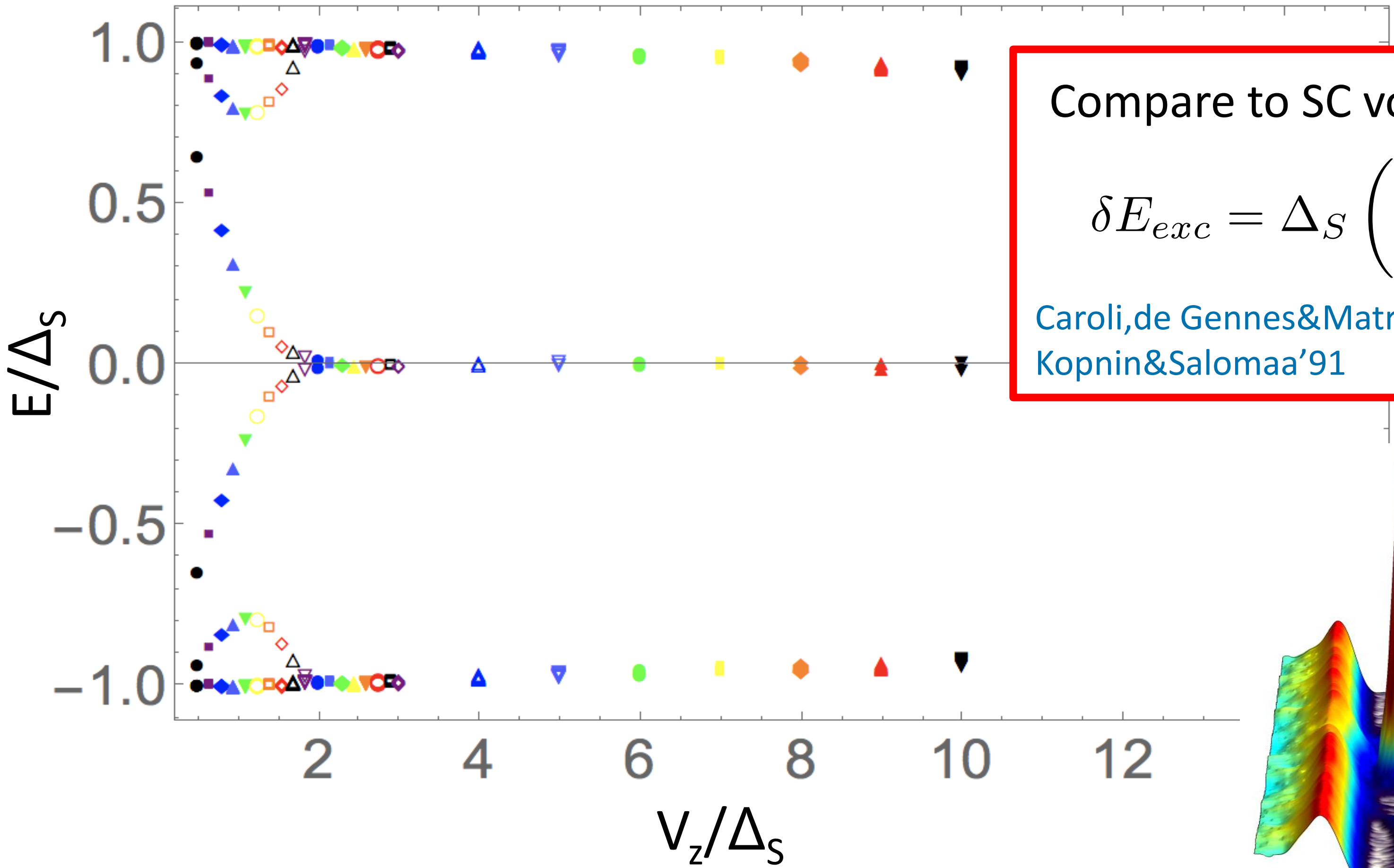
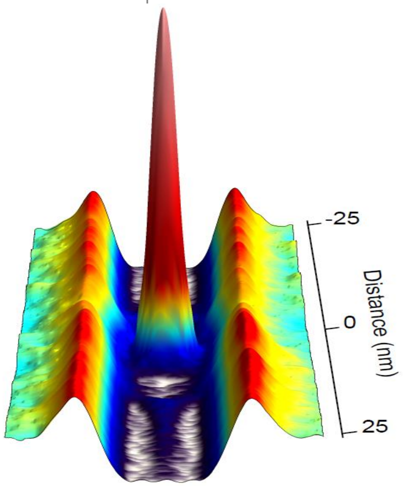




Compare to SC vortex

$$\delta E_{exc} = \Delta_S \left(\frac{\Delta_S}{E_F} \right)$$

Caroli,de Gennes&Matricon'64
Kopnin&Salomaa'91



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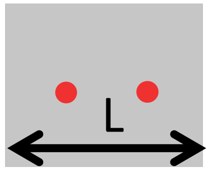

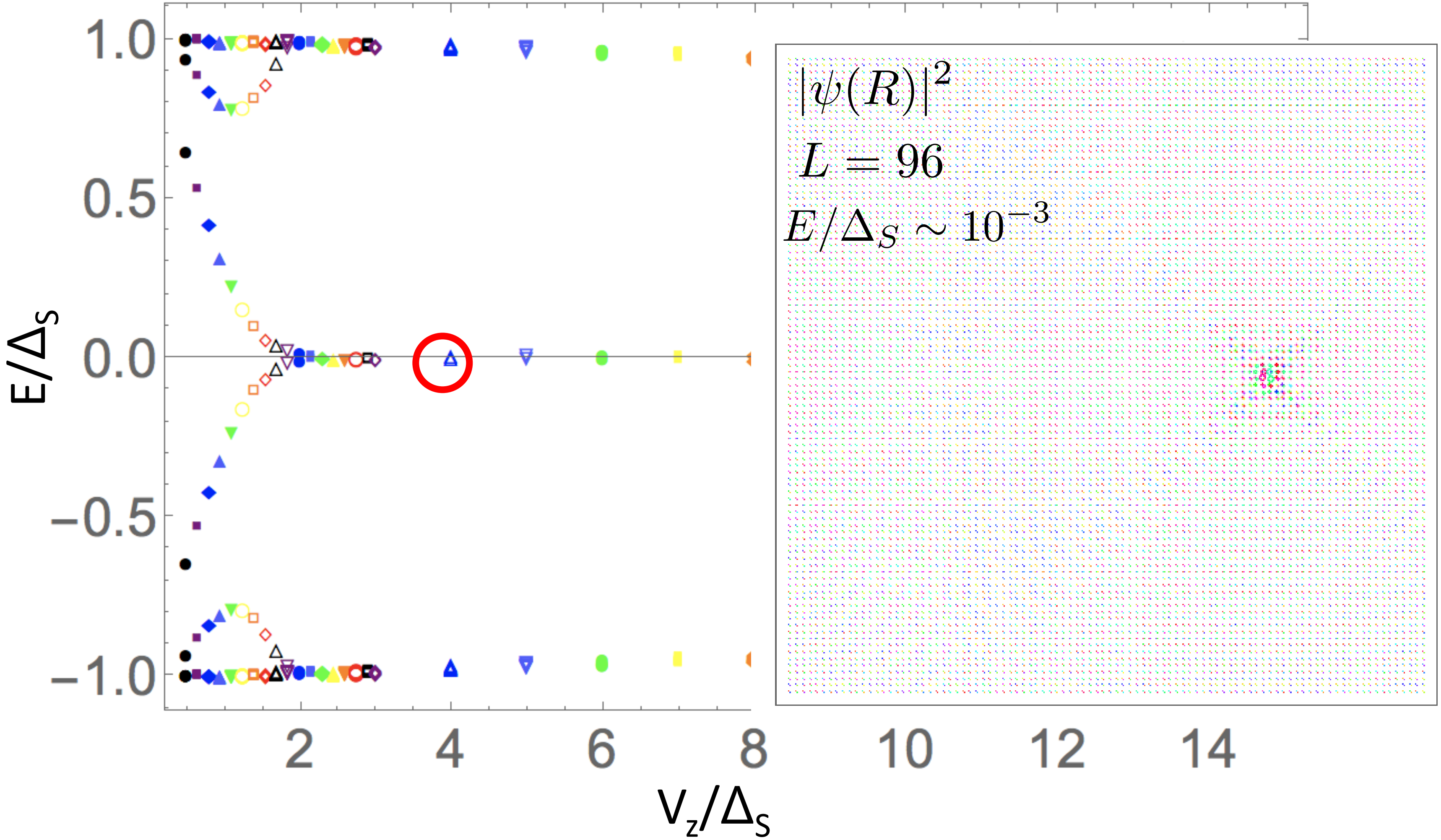

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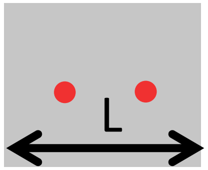
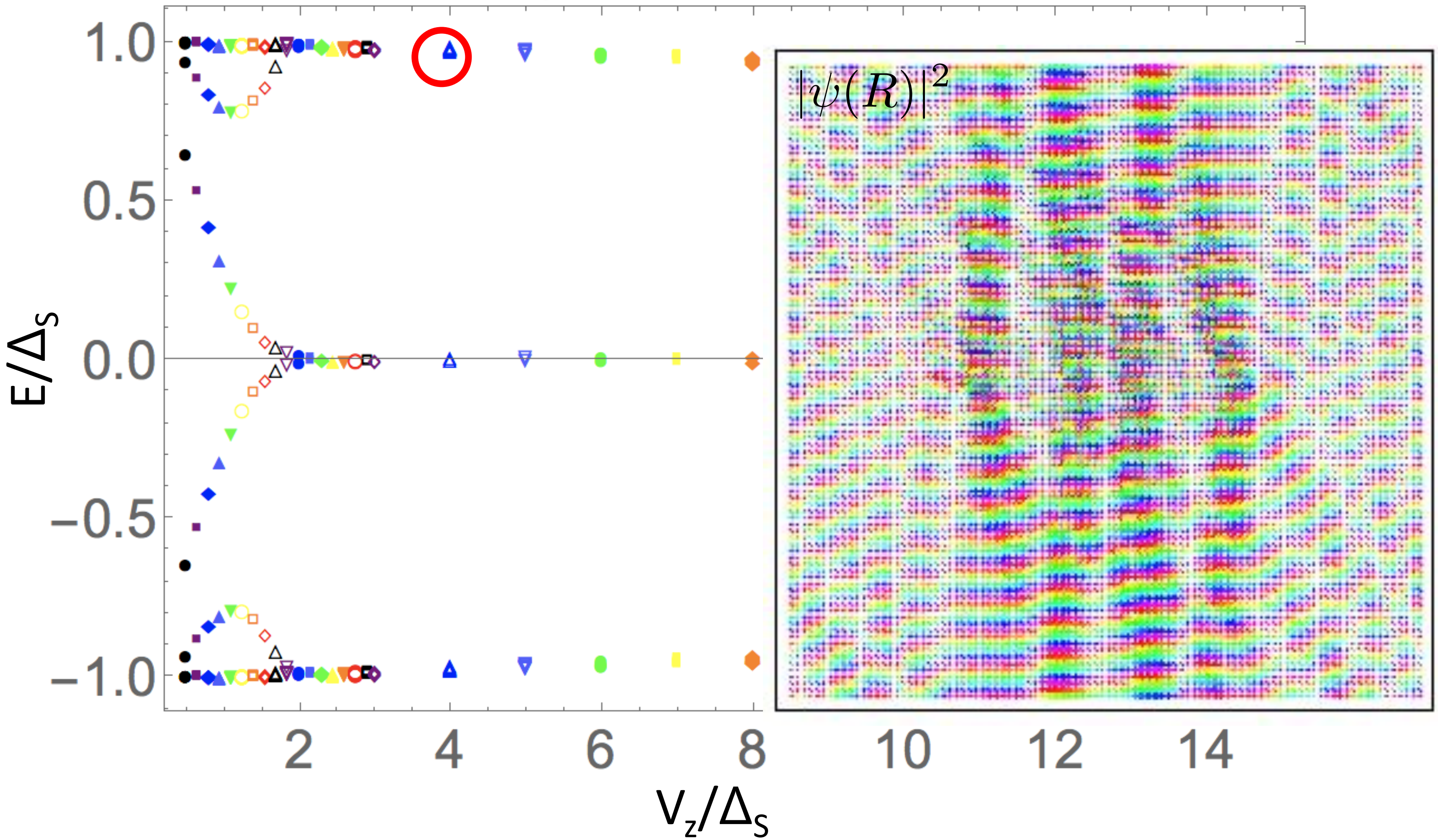
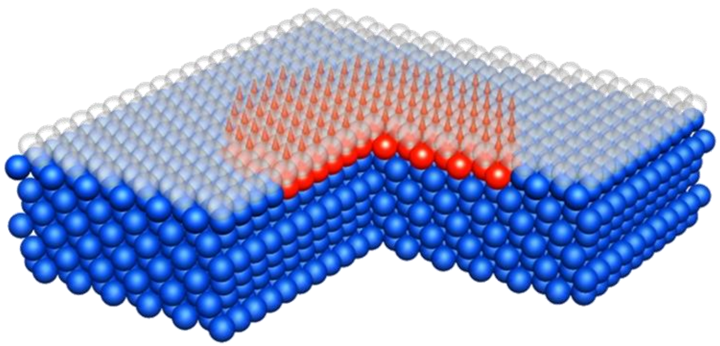
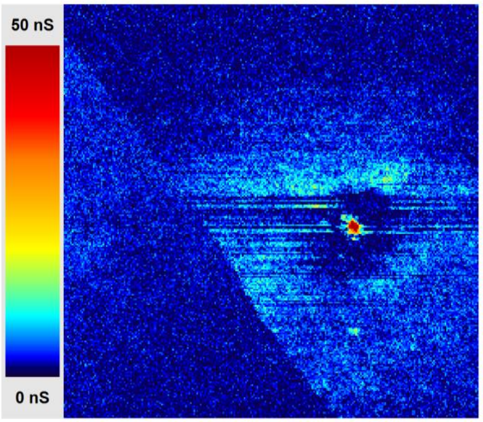




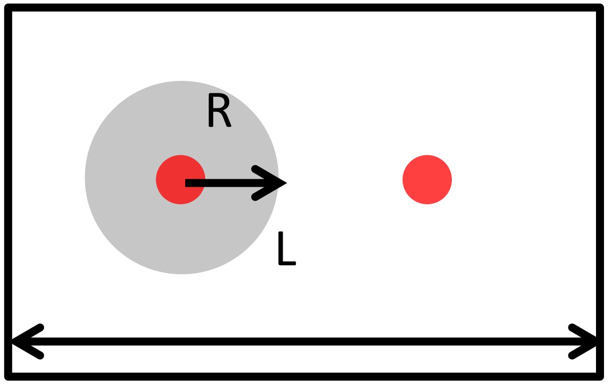
Magnetic island: Numerics (II)





Tight-binding 2d square lattice torus

Topological SC on island (V_z strong)



Defect on island vs. edges?

BUT!

- Finite size effects:

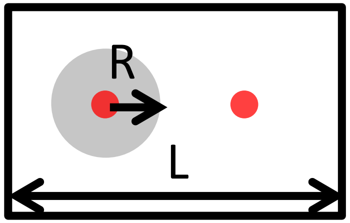
$$L \sim \xi \sim R \sim 80 \gg l_F \sim l_{so} \sim 1$$

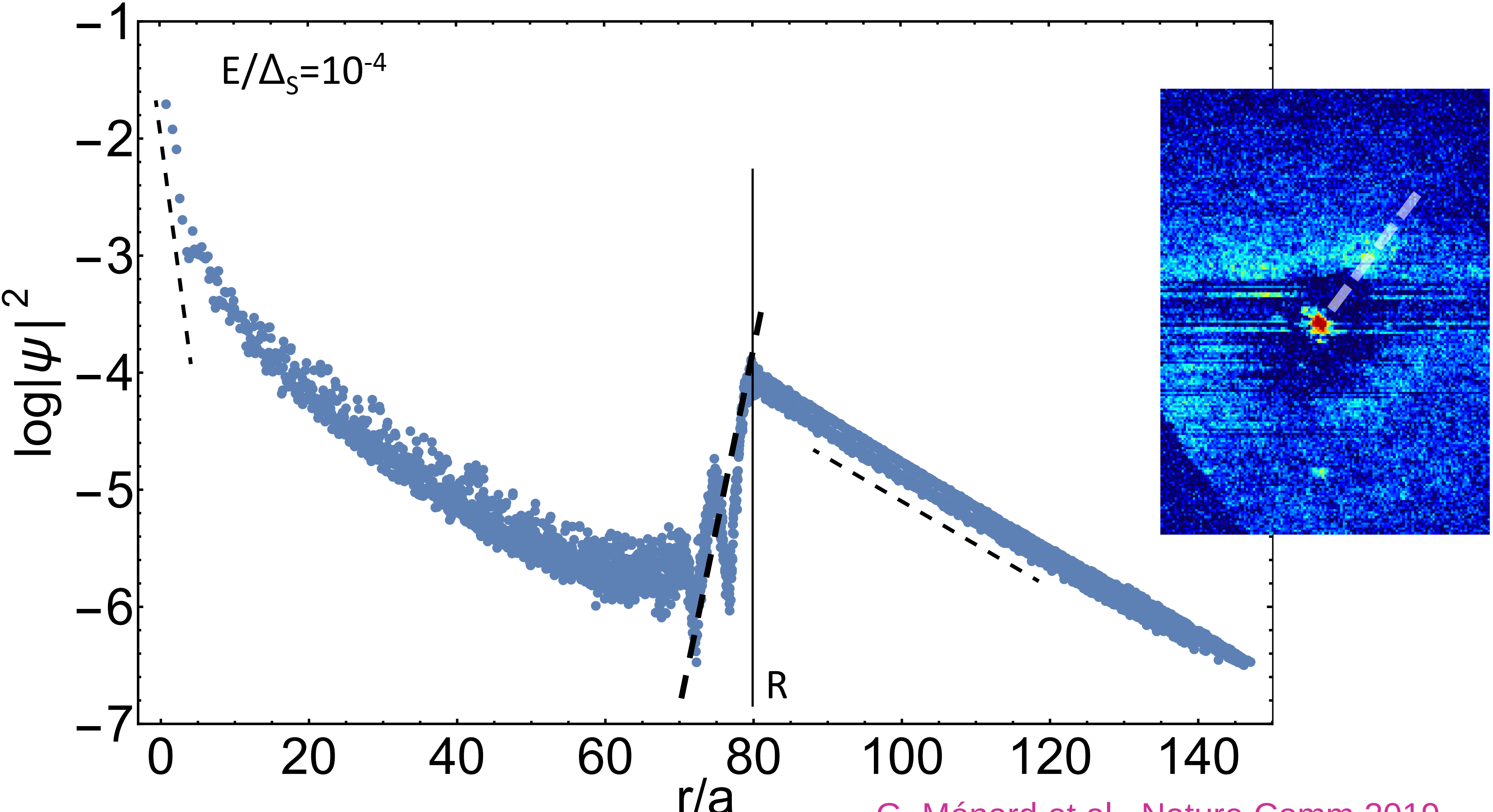
Magnetic island: Numerics (II)

Tight-binding 2d square lattice torus L=300

Topological island: $V_z^{island} = 5\Delta_S$

Majorana state





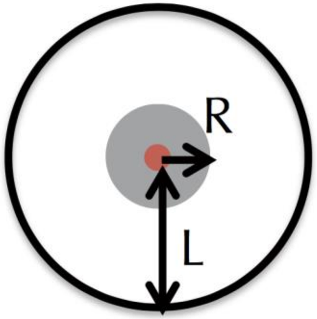
G. Ménard et al., Nature Comm 2019

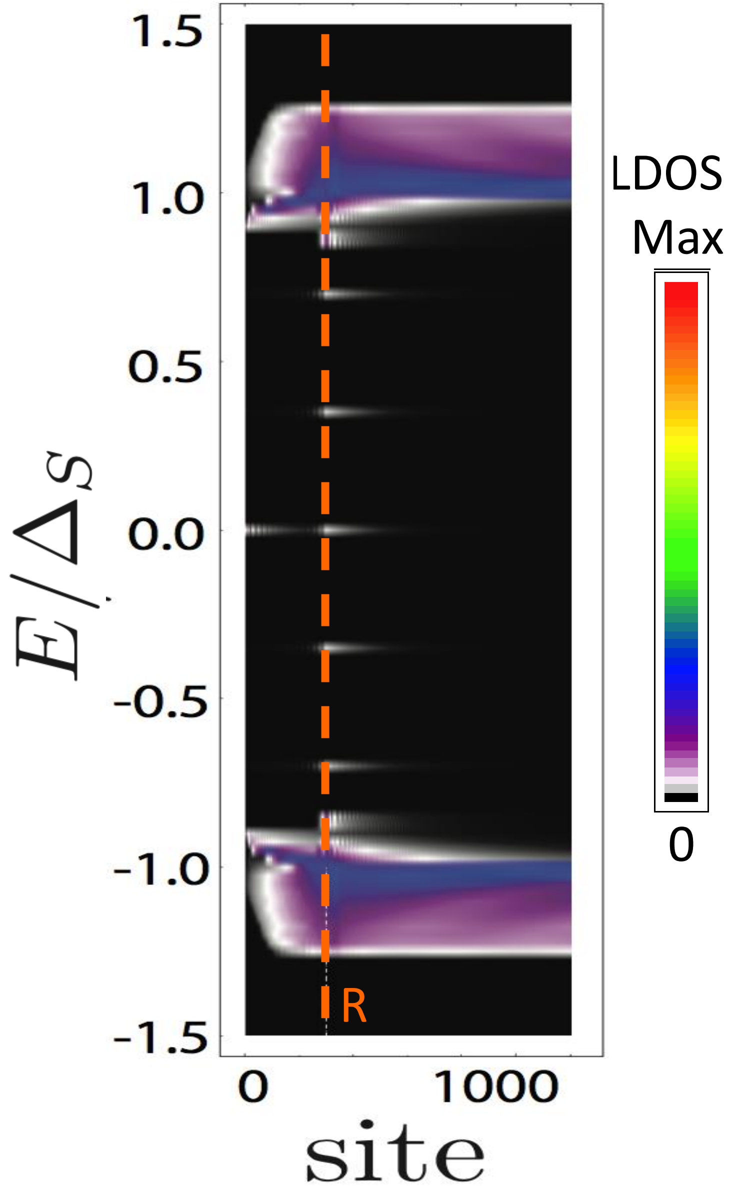
Lengthscale separation regime: Numerics (III)

Huge system size: 1d radial problem

Regime: $L \gg \xi > R \gg l_F \sim l_{so}$

BUT! Singularity…





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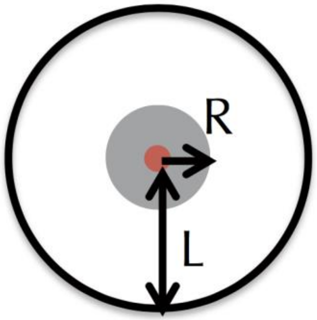
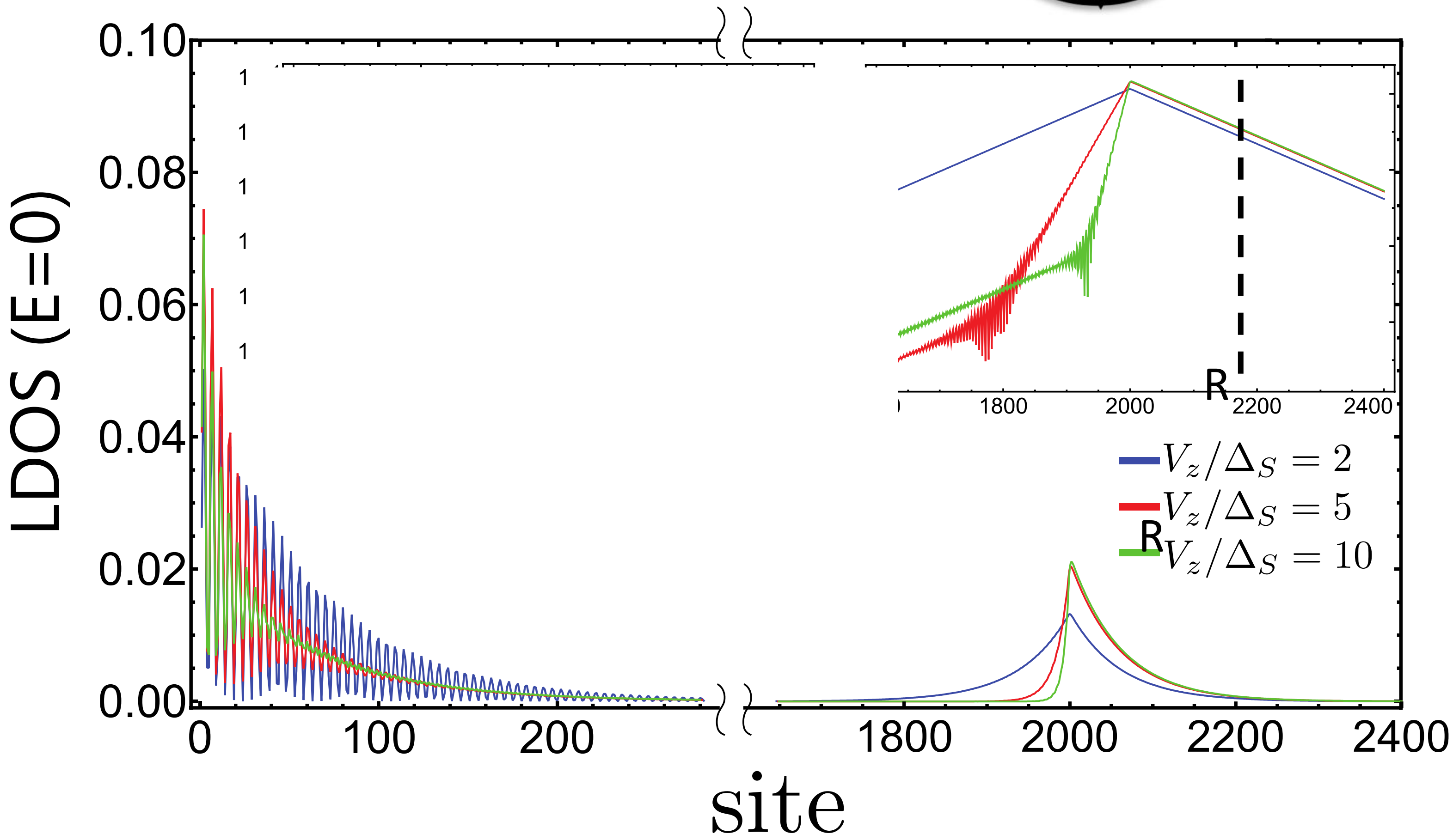
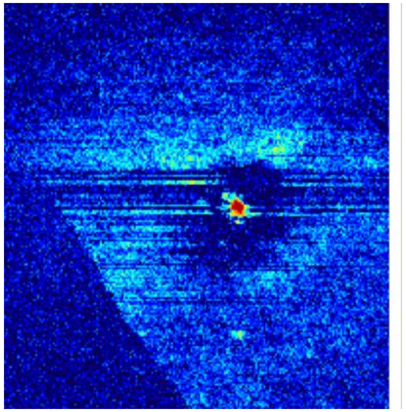




G. Ménard et al., Nature Comm 2019

Summary of the results

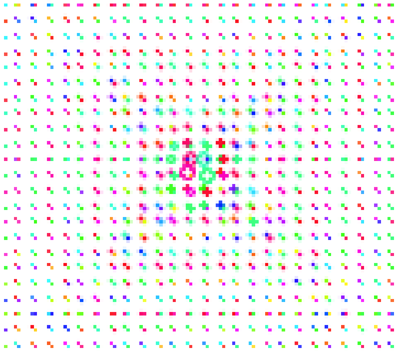


E=0 state in Pb/Co/Si(111):

(1) Localized + edges,

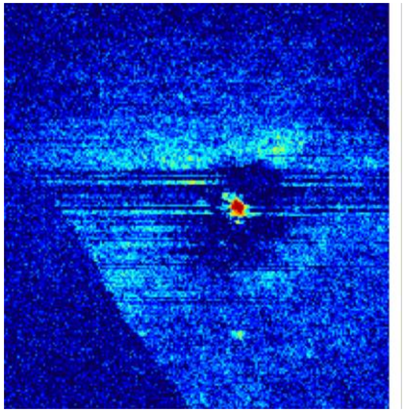
(2) Isolated in energy

↕



Spin-Orbit defect in topological superconductor

Summary of the results



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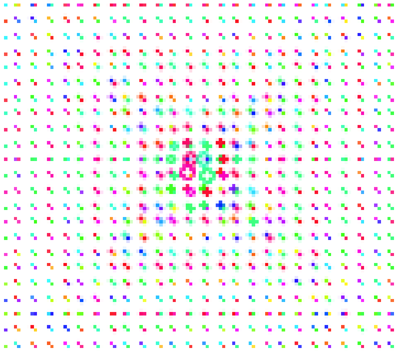


Spin-Orbit vortex in

topological superconductor

Gauging out the SO defect ?

Can we gauge the SO coupling ?

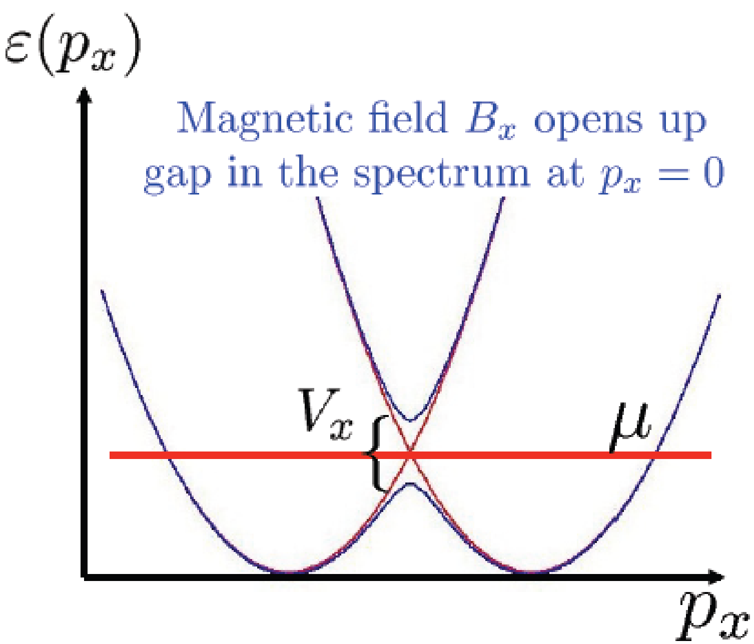
Example: 1D semi-conducting wire+SO+Zeeman

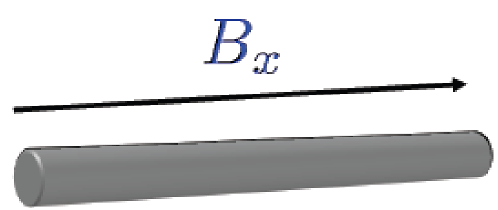
$$H_0 = \int_{-L}^{L} dx \psi_\sigma^\dagger(x) \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'}(x)$$

single channel nanowire

spin-orbit coupling

Zeeman splitting





InAs, InSb nanowires

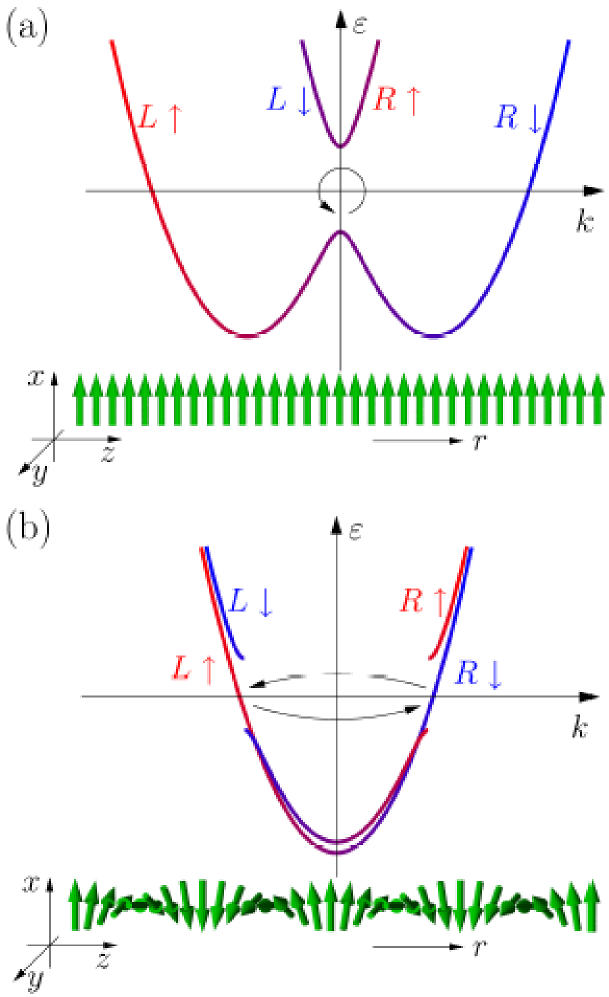
large spin-orbit ($\alpha \sim 0.1 eV \AA$)

large g-factor ($g \sim 10 - 50$)

good contacts with metals

Can we gauge the SO-vortex ?

Example: 1D semi-conducting wire+SO+Zeeman



In 1D systems

Change of frame: Gauge transform

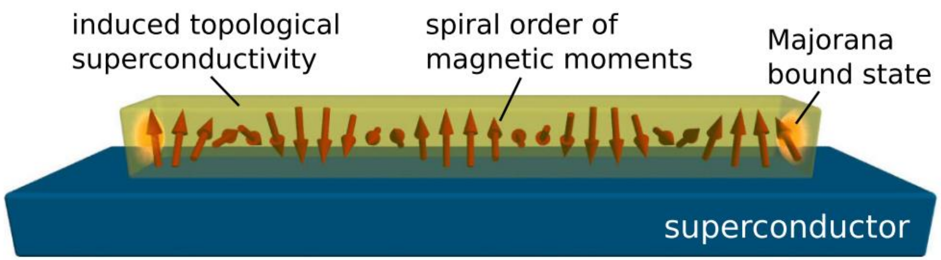
$$\psi_\sigma(r) \rightarrow e^{i\sigma k_{so} r}\psi_\sigma(r)$$

Equivalent to a helical magnetic field

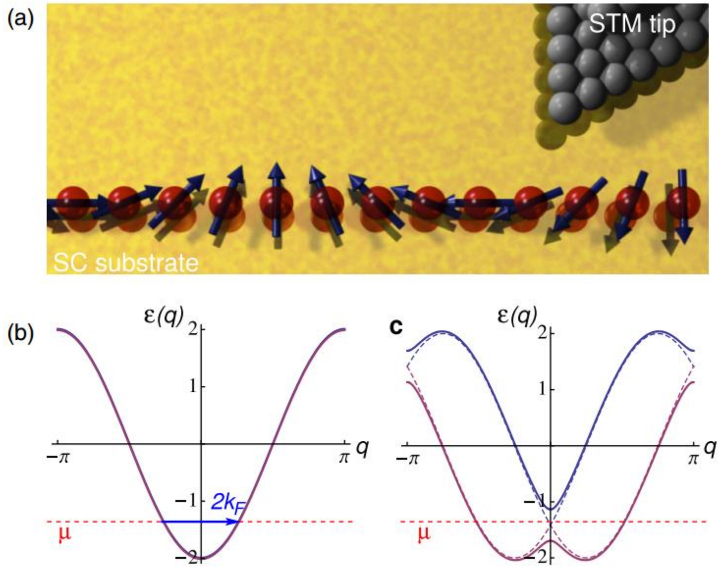
B. Braunecker et al., PRB 2010

Can we gauge the SO-defect?

Example: Prediction of Majorana end-states on the edges of helical spin chains



Braunecker and PS, PRL (2013)
Klinovaja, Stano, Yazdani and Loss, PRL (2013)



Vazifeh and Franz, PRL (2013)

SO-defect vs. magnetic texture

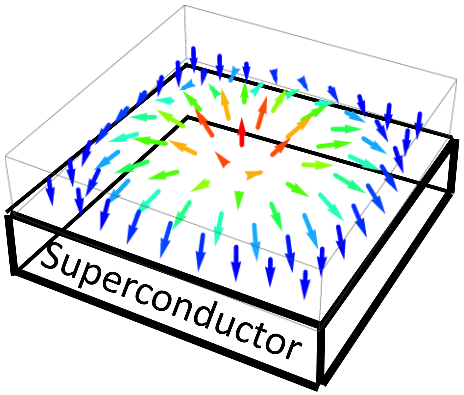
Example:
Skyrmion(n,p) magnetic field texture

$$H_{skyr} = \sum_{\sigma=\pm}\left(\frac{k^2}{2m} - \mu\right) c^\dagger_{k\sigma} c_{k\sigma} + \sum_{a,b=\pm} c^\dagger_{r,a} \vec{V}(\vec{r}) \cdot \vec{\sigma}_{ab} c_{r,b}$$

$$\vec{V} = [\cos(pr)\cos(n\varphi), \cos(pr)\sin(n\varphi), \sin(pr)]$$

Gauge transformation:

$$\tilde{c}(r,\varphi) \equiv U(r,\varphi)c(r,\varphi)$$



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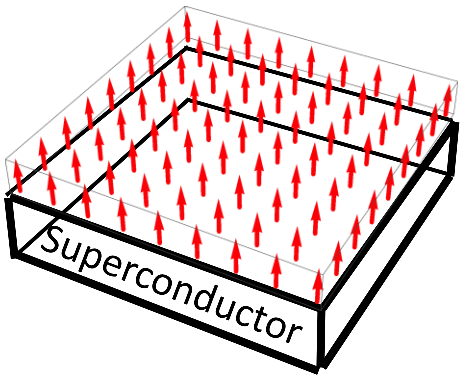


\+ Spin-Orbit texture

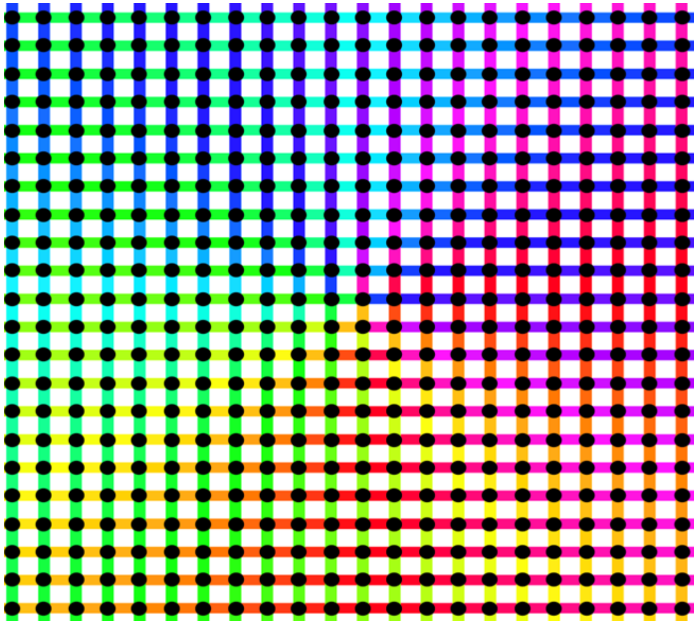
Magnetic field texture

Ferromagnet

$$H_{tex} = \sum_{\mathbf{R}} \vec{V}(\mathbf{R}) \cdot \vec{S}_{el}(\mathbf{R})$$

Gauging out the SO-defect ?

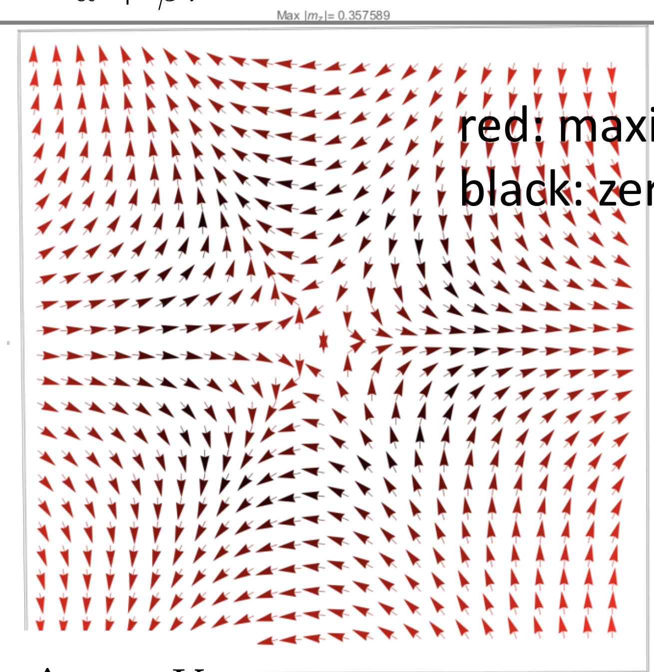
SO-defect



Skyrmion(n=2) + term

$$\vec{V} = [\cos(f_r)\cos(n\theta), \cos(f_r)\cos(n\theta), \sin(f_r)]$$
$$f_r = a + \beta r$$

$\vec{V}$:



$$H_0 = -\eta\left(\partial_r^2 + \frac{1-4m^2}{4r^2}\right)\tau_z + \Delta_S\tau_x + V_0\sigma_z$$

$$\delta H = \alpha\left[(i\partial_r)\sigma_y + \frac{m}{r}\sigma_x\right]\tau_z$$

$$\delta H_1 = \eta\beta\left[(i\partial_r)\sigma_y + \frac{\beta}{4}\right]\tau_z$$

$$\delta H_2 = \eta\beta\frac{m}{r}\left[\boxed{\sigma_x} + \frac{n}{r}\left(\sin(2f_r)\sigma_x - \cos(2f_r)\sigma_z\right)\right]\tau_z$$

Non-topological, entire system

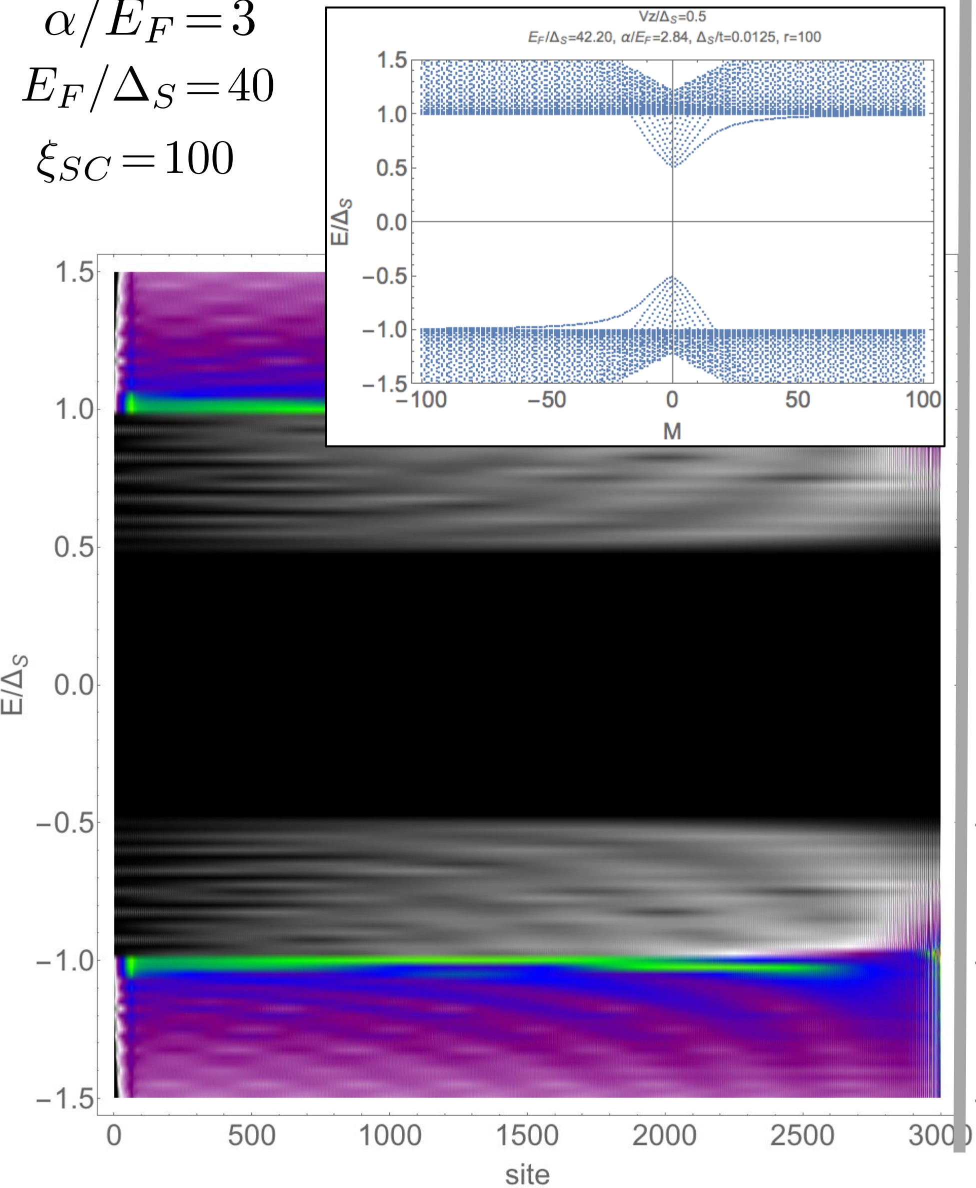
$V_0/\Delta_S = 0.5$

SO-defect

$\alpha/E_F = 3$

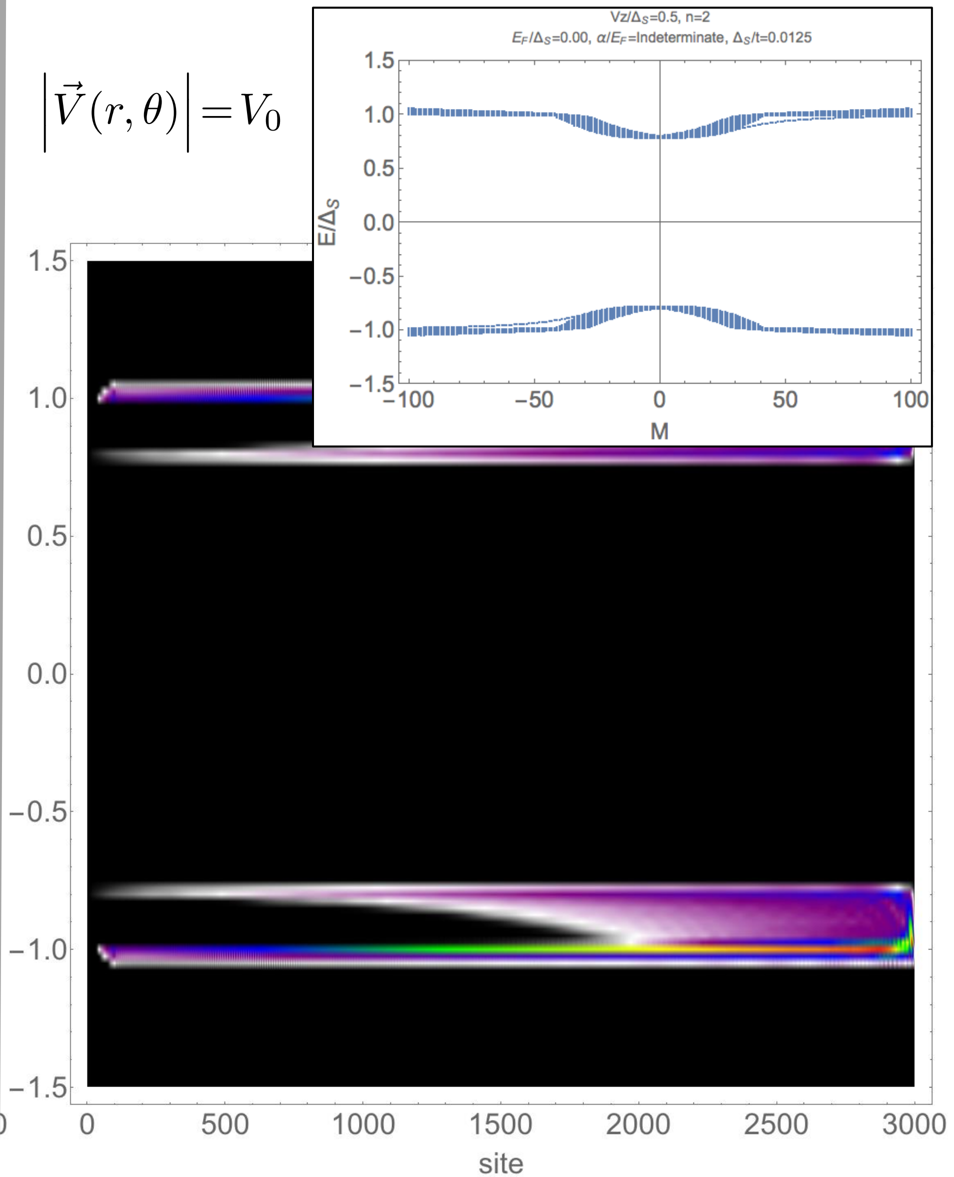
$E_F/\Delta_S = 40$

$\xi_{SC} = 100$



Skyrmion(n=2) + term

$\left|\vec{V}(r,\theta)\right| = V_0$



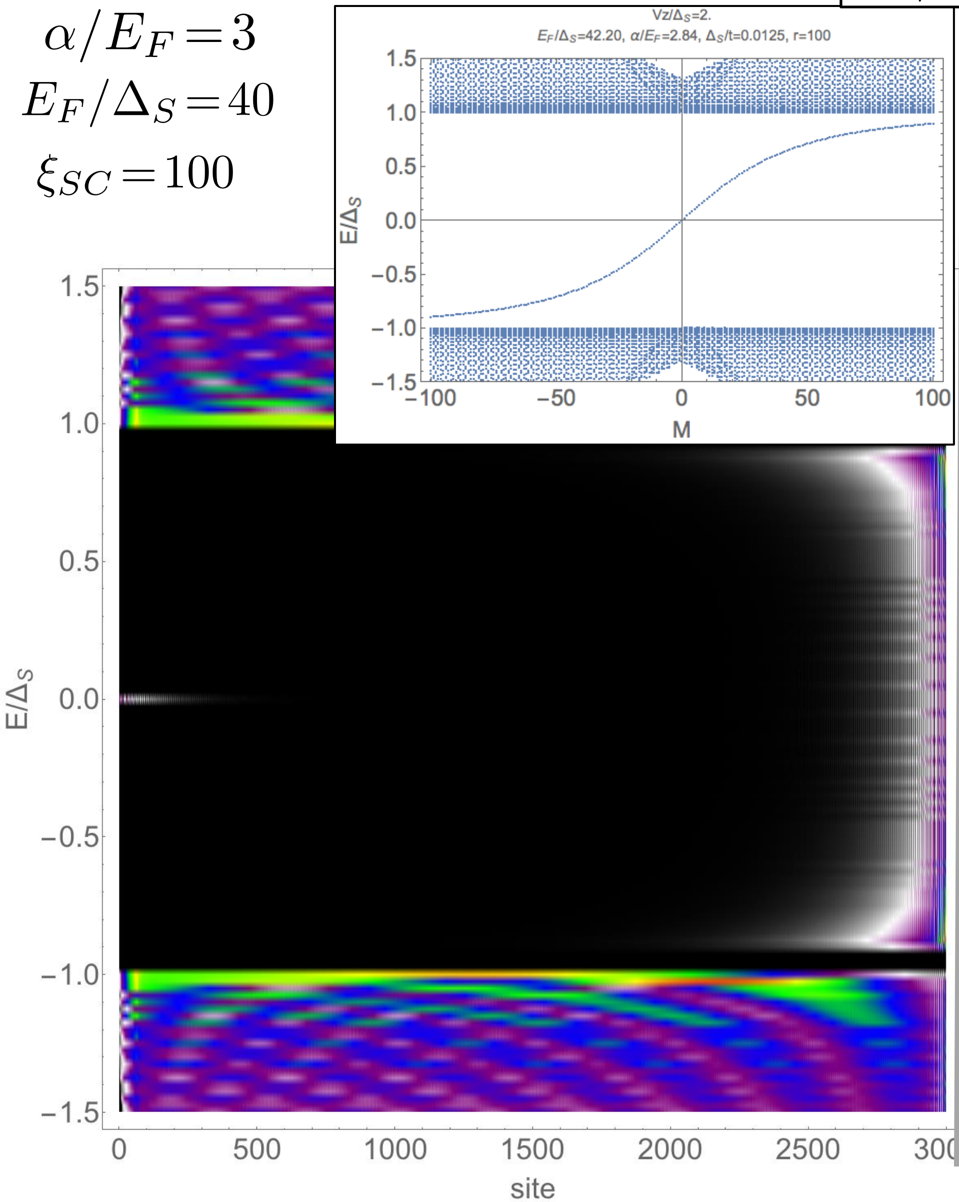
Topological, entire system

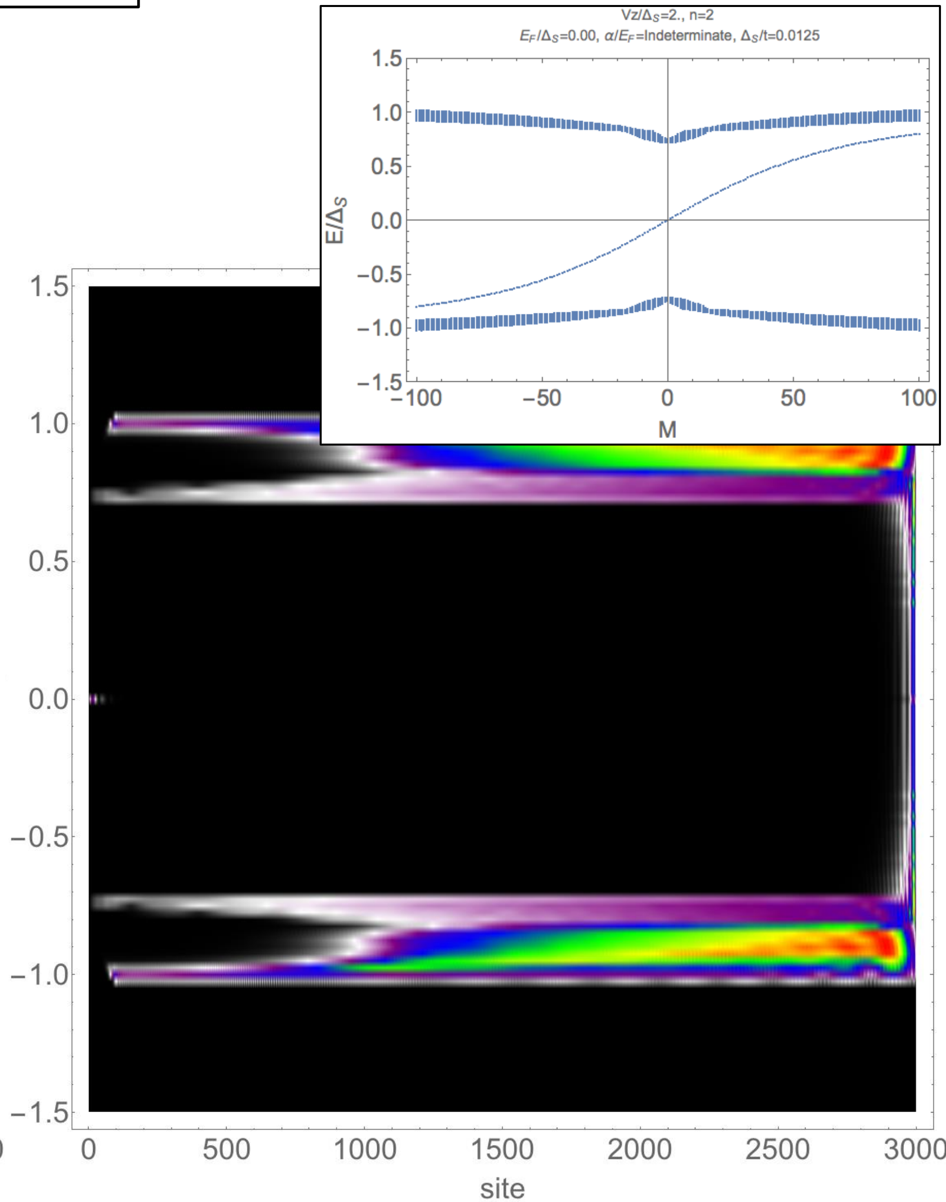
SO-defect

$V_0/\Delta_S = 2$

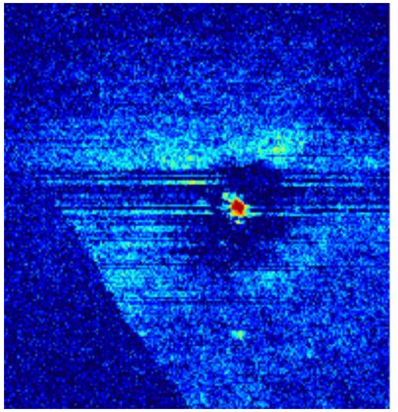
Skyrmion(n=2) + term

$\alpha/E_F = 3$
$E_F/\Delta_S = 40$
$\xi_{SC} = 100$





Summary of the results



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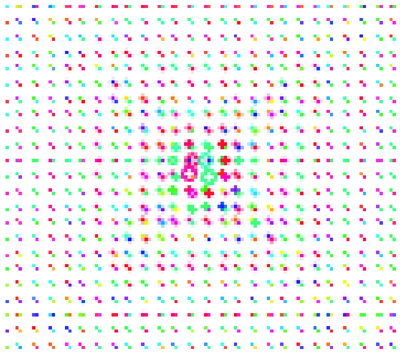
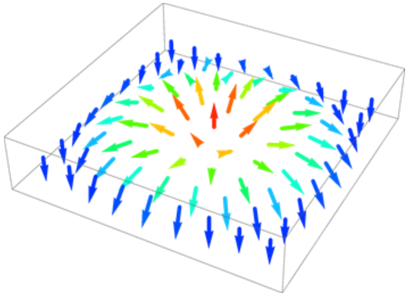
(2) Isolated in energy

↕



Spin-Orbit defect in topological superconductor

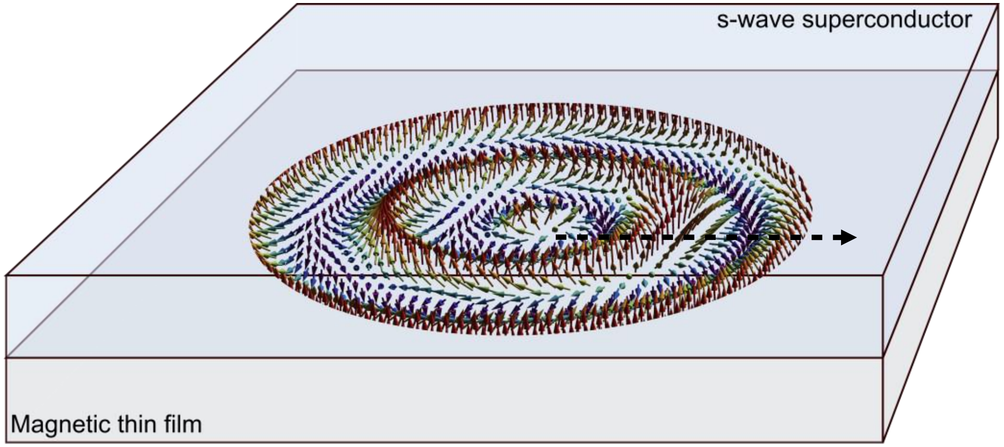
↕



Magnetic 2D texture In topological superconductor

IV) Majorana zero modes in skyrmions

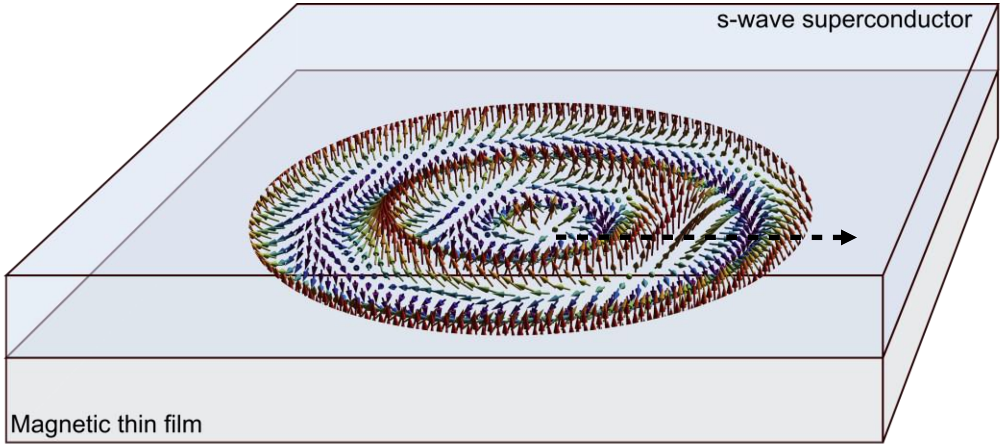
Model for a skyrmion



- Kinetic energy m
- chemical potential μ
- s-wave pairing Δ_0
- exchange interaction J

Polar coordinates $\mathbf{r} = (r, \theta)$

Model for a skyrmion



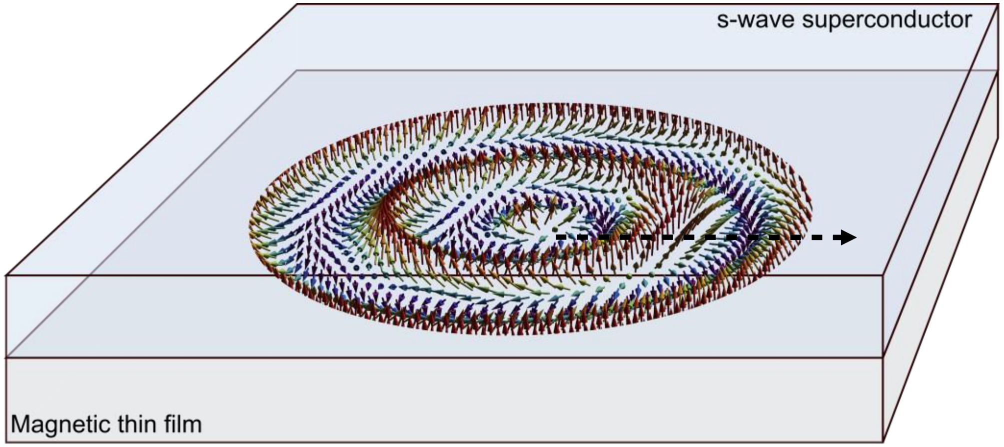
- Kinetic energy m
- chemical potential μ
- s-wave pairing Δ_0
- exchange interaction J

Polar coordinates $\mathbf{r} = (r, \theta)$

Bogoliubov- de Gennes Hamiltonian $H = \frac{1}{2} \int d\mathbf{r}\, \Psi^\dagger(\mathbf{r}) \mathcal{H}(\mathbf{r}) \Psi(\mathbf{r})$ $\Psi^\dagger(\mathbf{r}) = \left(\psi_\uparrow^\dagger(\mathbf{r}), \psi_\downarrow^\dagger(\mathbf{r}), \psi_\downarrow(\mathbf{r}), -\psi_\uparrow(\mathbf{r})\right)$

$$\mathcal{H}(\mathbf{r}) = \left(-\frac{\nabla^2}{2m} - \mu\right)\tau_z + J\,\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}) + \Delta_0\,\tau_x$$

Model for a skyrmion



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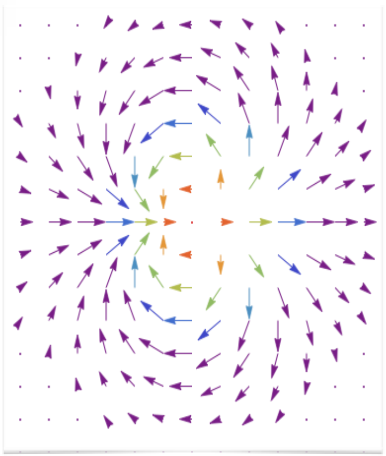
$$\mathcal{H}(\mathbf{r}) = \left(-\frac{\nabla^2}{2m} - \mu\right) \tau_z + J\, \boldsymbol{\sigma} \cdot \mathbf{n}(\mathbf{r}) + \Delta_0\, \tau_x$$

with the magnetic texture

$$\mathbf{n}(\mathbf{r}) = (\sin f(r) \cos(q\theta), \sin f(r) \sin(q\theta), \cos f(r))$$



Two winding numbers: p (radial) q (azimuthal)

Topological charge: $N_{\mathrm{sk}} = \frac{q}{2}(1 - (-1)^p)$

Length of a radial spin flip: λ

Radial Hamiltonian

« Rotational symmetry » ⟶ polar coordinates

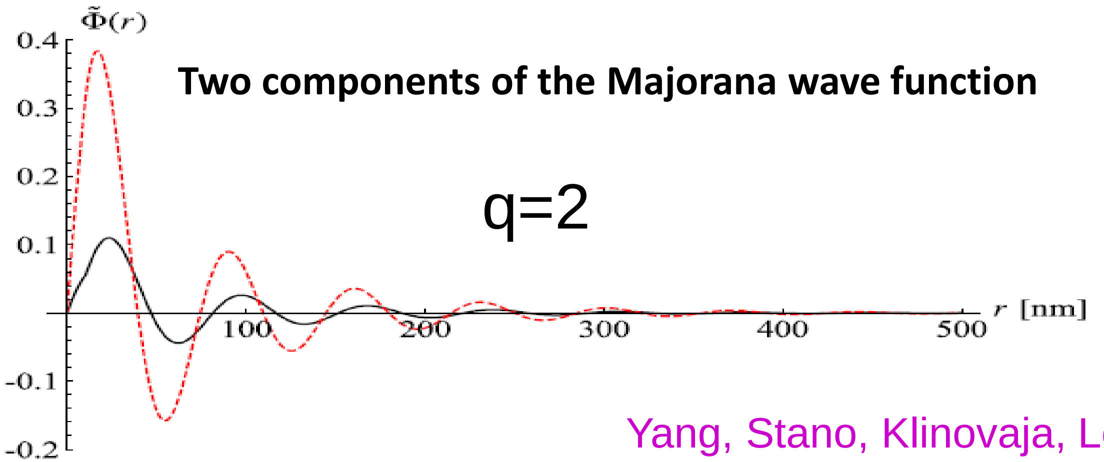
$$J_z = L_z + \frac{n}{2}\sigma_z \longrightarrow m_J$$

$$\mathcal{H}_{m_J}(r) = \left(-\frac{[\nabla^2]_r}{2m} - \mu\right)\tau_z + g\,\sigma_z \cos f + g\,\sigma_x \sin f + \Delta_0\,\tau_x$$

$$[\nabla^2]_r = \partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\left(m_J - \frac{n}{2}\sigma_z\right)^2$$

Majorana condition:

$$\begin{pmatrix} -\frac{[\nabla^2]_r}{2m} - \mu + \alpha\cos f & \alpha\sin f + \eta\Delta_0 \\ \alpha\sin f - \eta\Delta_0 & -\frac{[\nabla^2]_r}{2m} - \mu - \alpha\cos f \end{pmatrix}\Phi = 0$$



Two components of the Majorana wave function

q=2

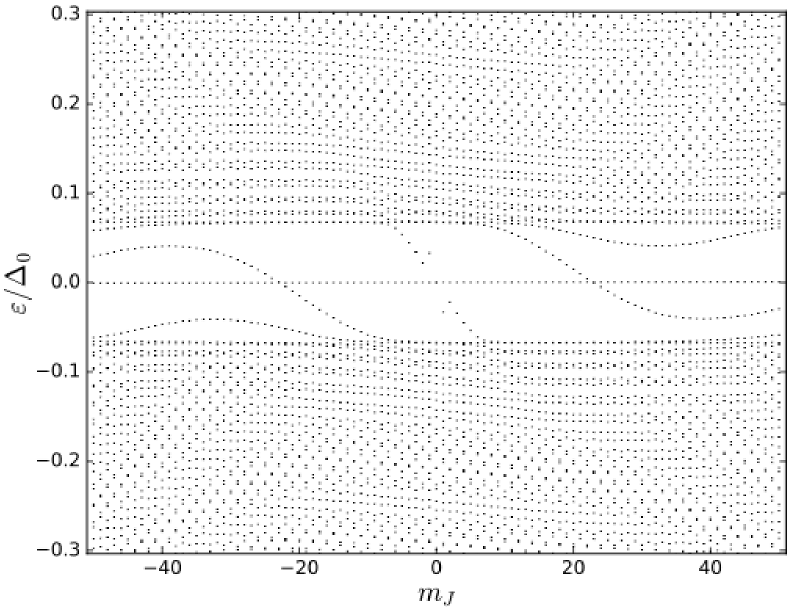
Yang, Stano, Klinovaja, Loss, PRB (2016)

Radial tight-binding model

Use the rotational symmetry $J_z = L_z + \frac{q}{2}\sigma_z$ with eigenvalues $m_J \in \begin{cases} \mathbb{Z} \text{ if } q \text{ even} \\ \mathbb{Z} + \frac{1}{2} \text{ if } q \text{ odd} \end{cases}$

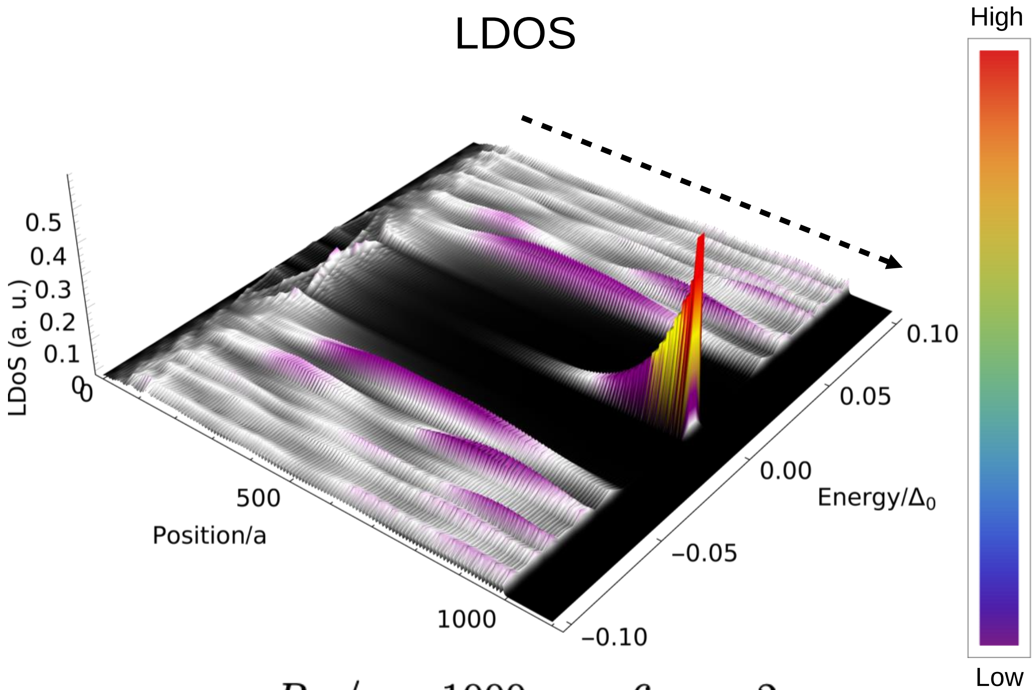
Radial Hamiltonian $\mathcal{H}_{m_J}(r) = e^{-i\left(m_J - \frac{q}{2}\sigma_z\right)\theta}\,\mathcal{H}(\mathbf{r})\,e^{i\left(m_J - \frac{q}{2}\sigma_z\right)\theta}$ ⟶ discretize

Spectrum



$p = 10$

LDOS



$R_{\rm sk}/a = 1000,\ p = 6,\ q = 2,$

$\Delta_0/t = 0.1,\ J/t = 0.2,\ \mu/t = 0$

Radial tight-binding model

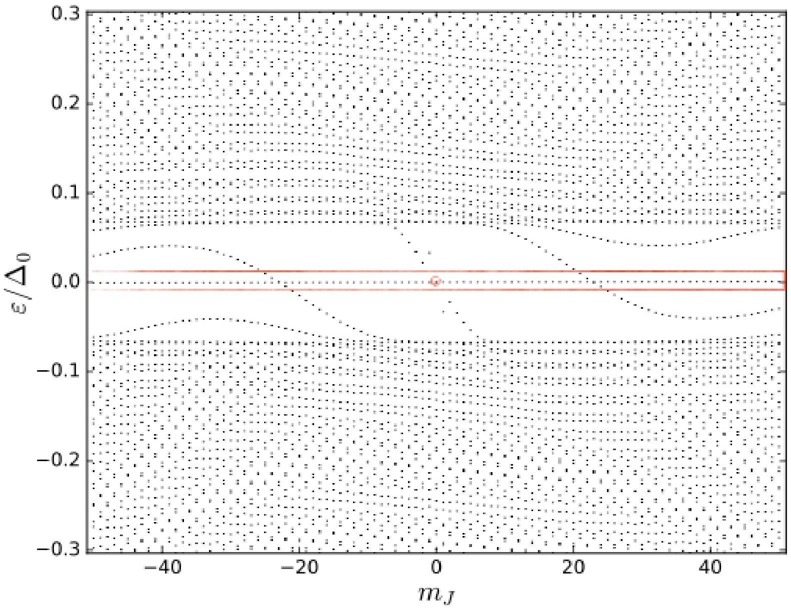
Use the rotational symmetry $J_z = L_z + \frac{q}{2}\sigma_z$ with eigenvalues $m_J \in \begin{cases} \mathbb{Z} \text{ if } q \text{ even} \\ \mathbb{Z} + \frac{1}{2} \text{ if } q \text{ odd} \end{cases}$

Radial Hamiltonian $\mathcal{H}_{m_J}(r) = e^{-i(m_J - \frac{q}{2}\sigma_z)\theta}\,\mathcal{H}(\mathbf{r})\,e^{i(m_J - \frac{q}{2}\sigma_z)\theta}$ ⟶ discretize

Spectrum

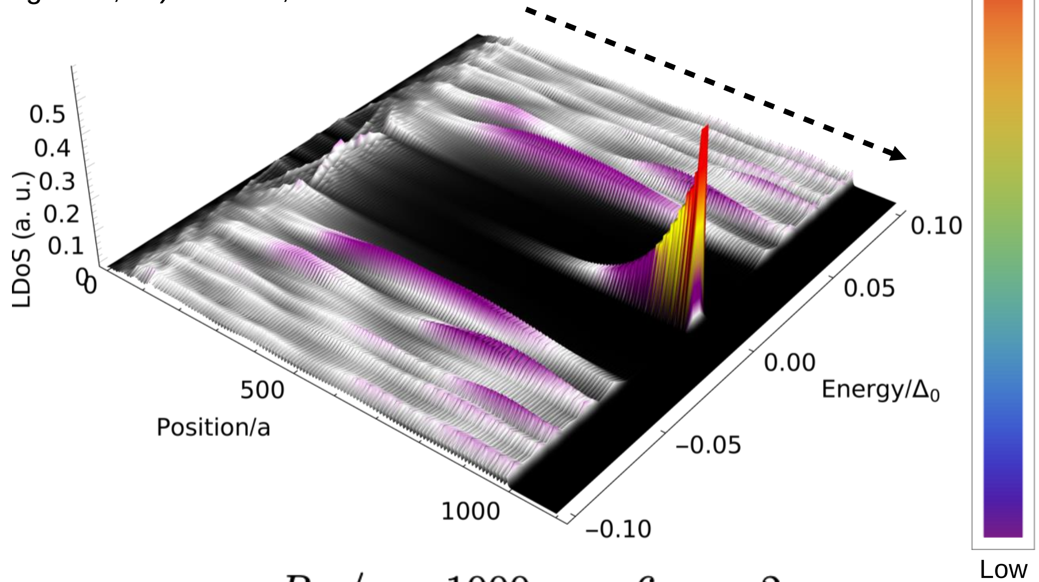
MZM (known) and flat band

LDOS



$p = 10$

Yang *et al.*, *Phys. Rev. B.*, 2016



$R_{\rm sk}/a = 1000,\ p = 6,\ q = 2,$

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Radial tight-binding model

Use the rotational symmetry $J_z = L_z + \frac{q}{2}\sigma_z$ with eigenvalues $m_J \in \begin{cases} \mathbb{Z} \text{ if } q \text{ even} \\ \mathbb{Z} + \frac{1}{2} \text{ if } q \text{ odd} \end{cases}$

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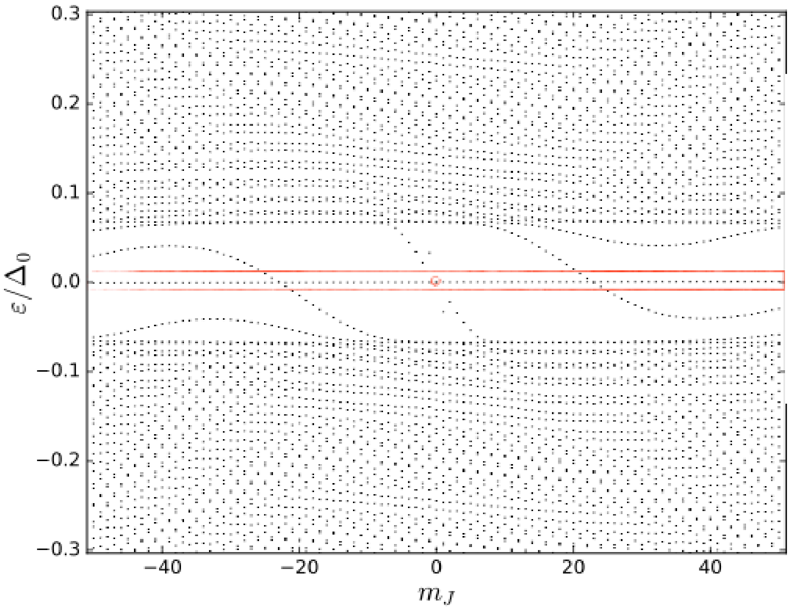
Spectrum

MZM (known) and flat band

LDOS



$p = 10$

identified. Inside the gap, we find two sets of localized fermionic states with finite angular momenta l [31], associated with the two MBSs. The localized states near the outer MBS have nearly zero energies and form an almost flat (yet distinctively quadratic) band, while those near the inner MBS

From Yang, Stano, Klinovaja, Loss, PRB (2016)

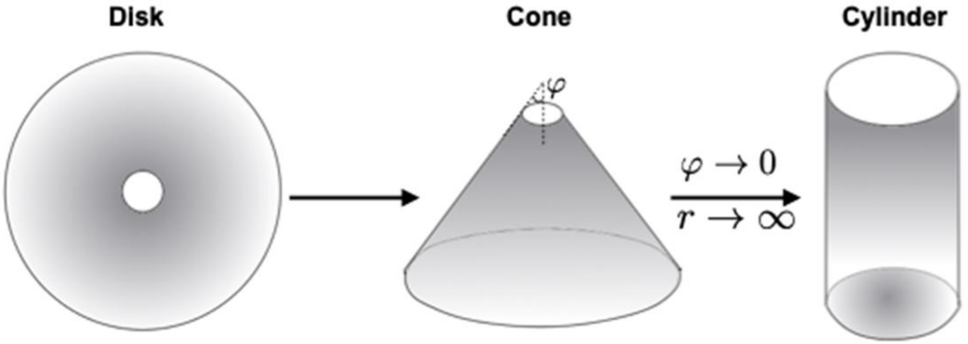
Origin of the Majorana flat band

Unitary transformation

$$U(r) = \exp\left(i\frac{\sigma_y}{2}f(r)\right)$$

$$\mathcal{H}_{m_J}(r) \longrightarrow \widetilde{\mathcal{H}}_{m_J}(r)$$

$$\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}) \longrightarrow \sigma_z$$



F. Wu, I. Martin, PRB (2017)

$$\widetilde{\mathcal{H}}^{\rm cyl}_{m_J}(r) = \mathcal{H}^{\rm wire}_{m_J}(r) + \mathcal{H}^{\rm slope}_{m_J}(r) + \mathcal{H}'_{m_J}(r)$$

$$\mathcal{H}^{\rm wire}_{m_J}(r) = \left[-\frac{1}{2m}\partial_r^2 - \mu\right]\tau_z + \frac{1}{2mR_{\rm sk}^2}\left(m_J^2 + \frac{q^2}{4}\right)\tau_z + \frac{f'}{2m}\partial_r i\sigma_y\tau_z + J\sigma_z + \Delta_0\tau_x$$

Renormalized kinetic term | SOC | Zeeman | s-wave SC

with corrections

$$\mathcal{H}^{\rm slope}_{m_J}(r) = -\frac{q\,m_J}{2mR_{\rm sk}^2}(-1)^p\sigma_z\tau_z$$

$$\mathcal{H}'_{m_J}(r) = \frac{f'^2}{8m}\tau_z + \frac{f''}{4m}i\sigma_y\tau_z$$

Unimportant ;
just renormalize parameters

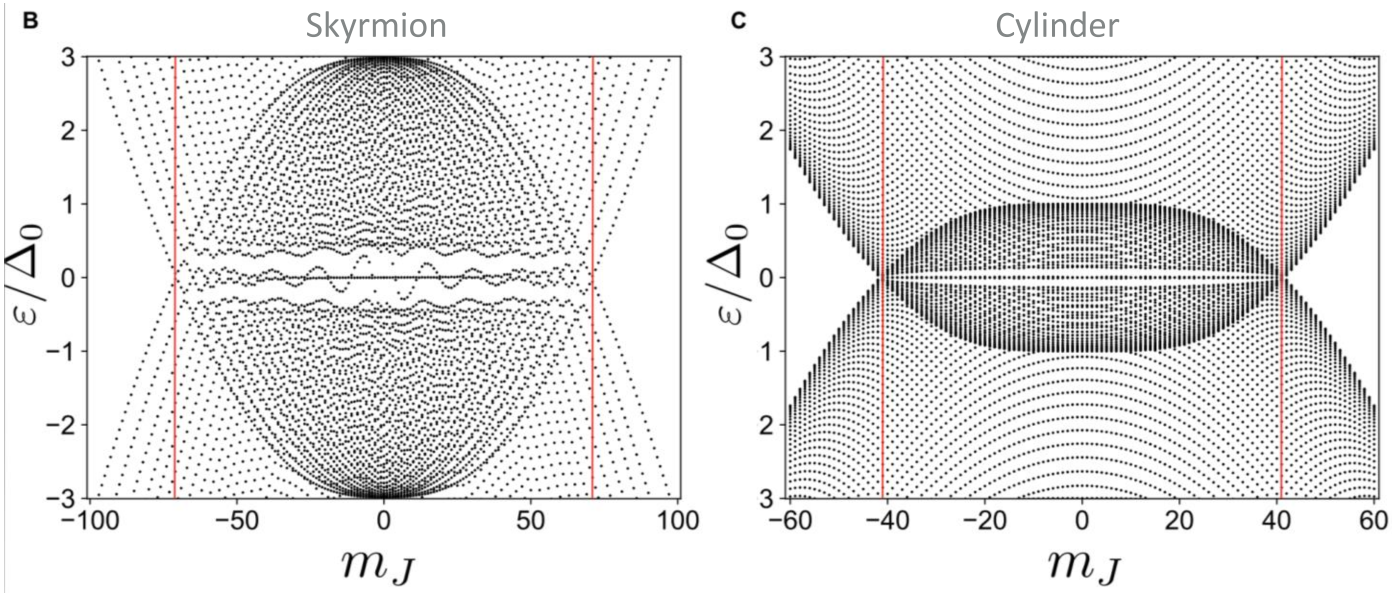
Interpretation of the flat band

Neglecting first the corrections, we obtain a set of Rashba wires for every m_J

A Rashba wire with $\mu,\ V_z,\ \Delta_0$ is in a topological phase when $V_z^2 > \Delta_0^2 + \mu^2$

$$\mu \to \mu(m_J)$$

$$|m_J^*| = R_{\rm sk}\sqrt{\left(\mu + \sqrt{J^2 - \Delta_0^2}\right)} + \mathcal{O}\left(R_{\rm sk}^{-1}\right)$$



Majorana flat band comes from the underlying Rashba wires

Interpretation of the flat band

Neglecting first the corrections, we obtain a set of Rashba wires for every m_J

A Rashba wire with μ, V_z, Δ_0 is in a topological phase when $V_z^2 > \Delta_0^2 + \mu^2$

$$\mu \to \mu(m_J)$$

$$|m_J^*| = R_{\rm sk}\sqrt{\left(\mu + \sqrt{J^2 - \Delta_0^2}\right)} + \mathcal{O}\left(R_{\rm sk}^{-1}\right)$$

Majorana flat band protected by a chiral operator

$$\Xi = \tau_y \sigma_y$$

Numerical expectations

$$t \sim \text{eV},\ \mu \sim \text{eV},\ \Delta_0 \sim 1\,\text{meV},\ g \sim 100\,\text{meV}\ \&\ 10\,\text{nm} \leq R_{\rm cyl} \leq 100\,\text{nm}$$

$$30 \leq |m_J^*| \leq 300$$

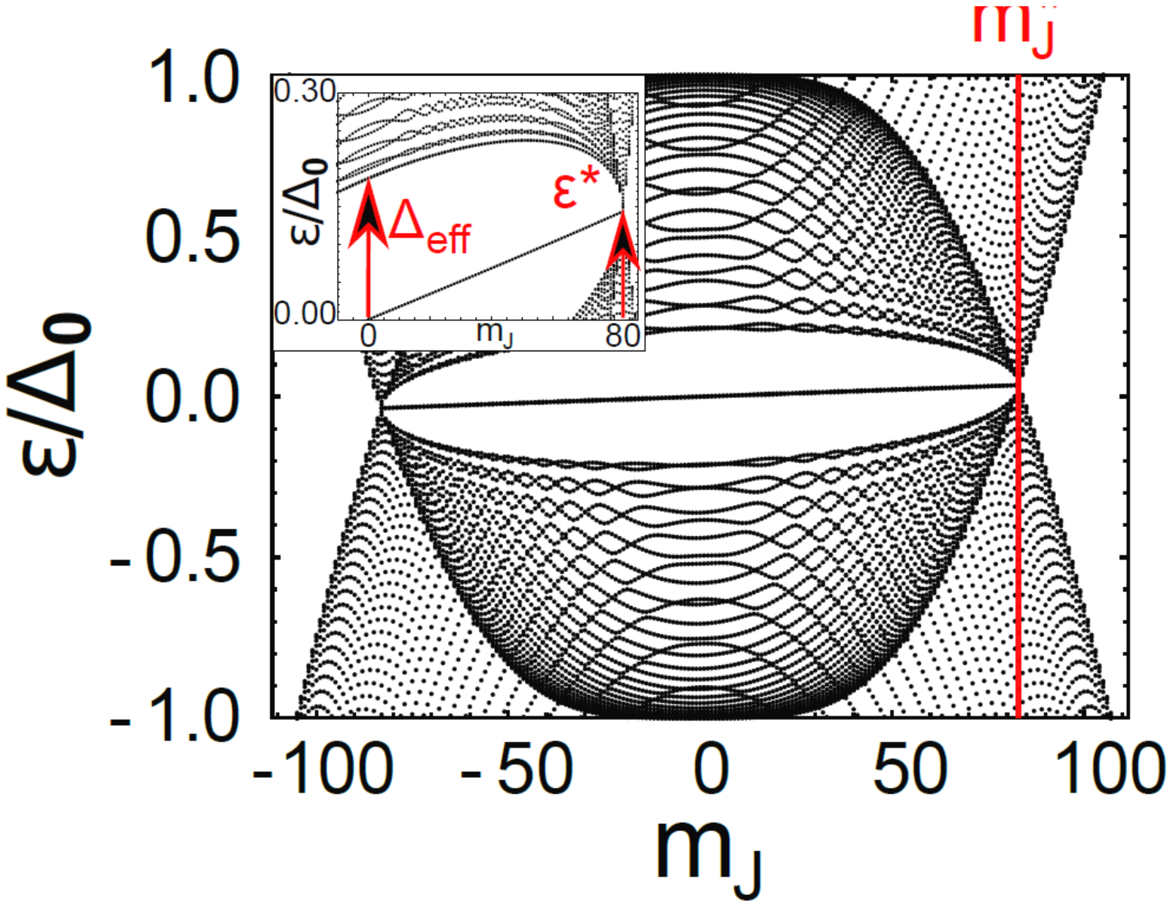
→ 1D Majorana flat band built from 0D-like Majorana edge modes

M. Garnier, A. Mesaros, PS, arXiv:1904.03005

Chiral symmetry breaking

$$\widetilde{\mathcal{H}}^{\text{cyl,eff}}_{m_J}(r) = \mathcal{H}^{\text{wire}}_{m_J}(r) + \mathcal{H}^{\text{slope}}_{m_J}(r)$$

$$\varepsilon^{\text{edgestate}}(m_J) \sim m_J$$



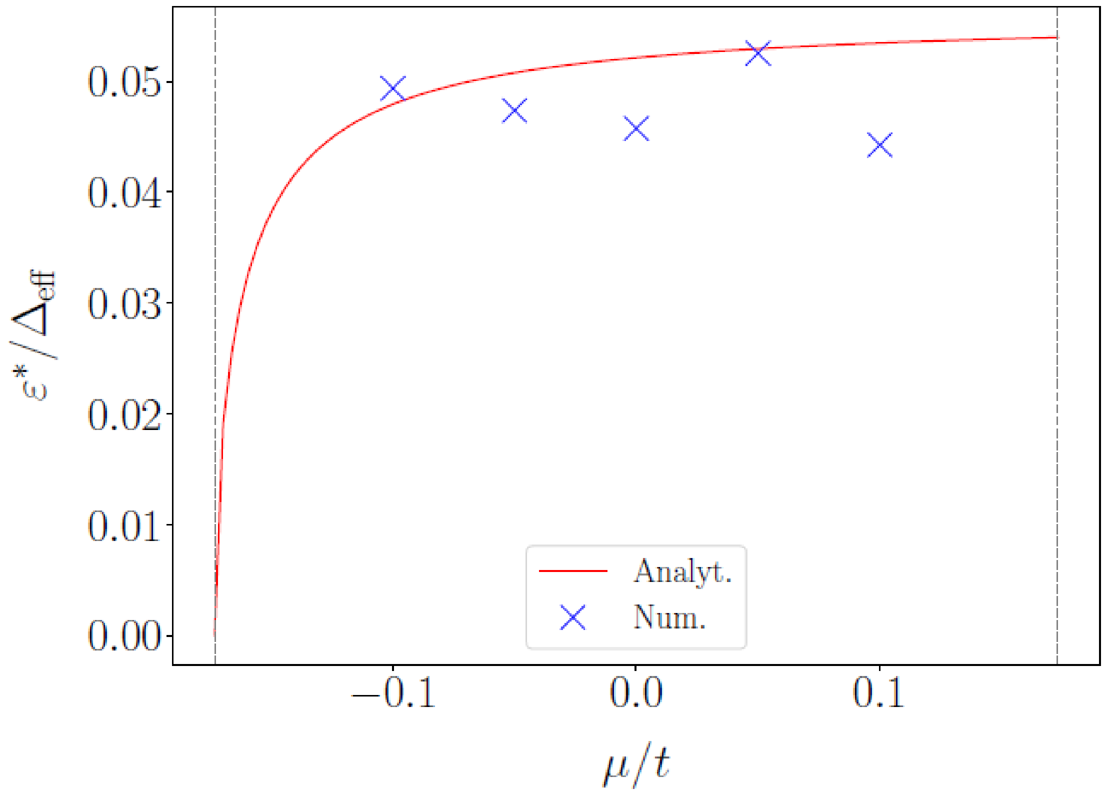
$$\varepsilon^* \equiv \left|\varepsilon^{\text{edgestate}}(|m_J^*|)\right|$$

$$\varepsilon^* = \frac{q\, m_J^*}{2mR_{\text{sk}}^2}$$

M. Garnier, A. Mesaros, PS, arXiv:1904.03005

Estimation of the slope w.r.t to the effective gap

$$\frac{\varepsilon^*}{\Delta_{\text{eff}}} = \frac{q}{p}\frac{J}{\pi\Delta_0}\sqrt{\frac{\mu + \sqrt{J^2 - \Delta_0^2}}{J+\mu}}$$



$R_{\text{sk}}/a = 1001$

$p = 10$

$q = 2$

$\Delta_0/t = 0.1$

$J/t = 0.2$

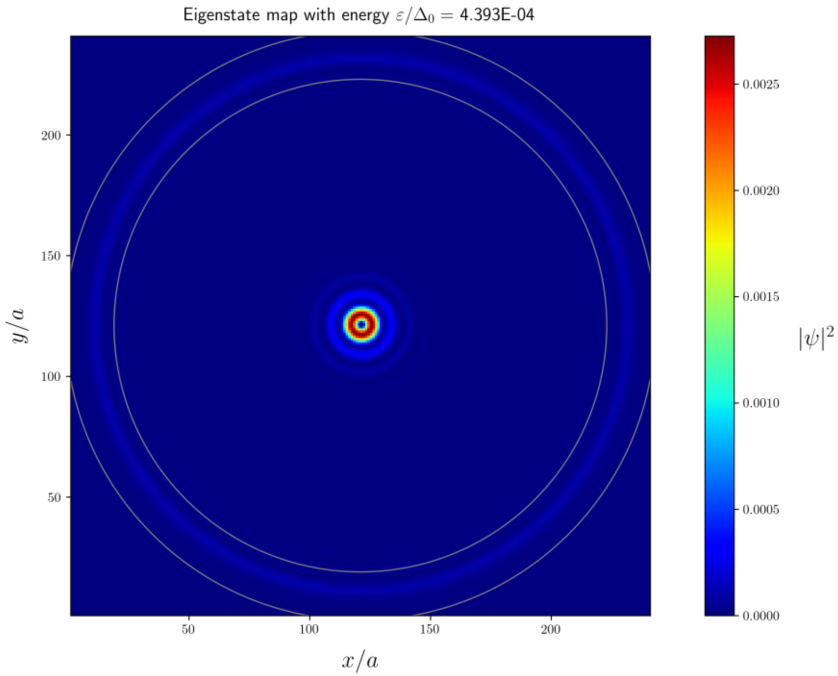
Slope of order of 0.5% of the SC gap (not experimentally visible) → Majorana Flat band

2D tight-binding calculations

2D tight-binding on the square lattice: breaks continuous rotation symmetry

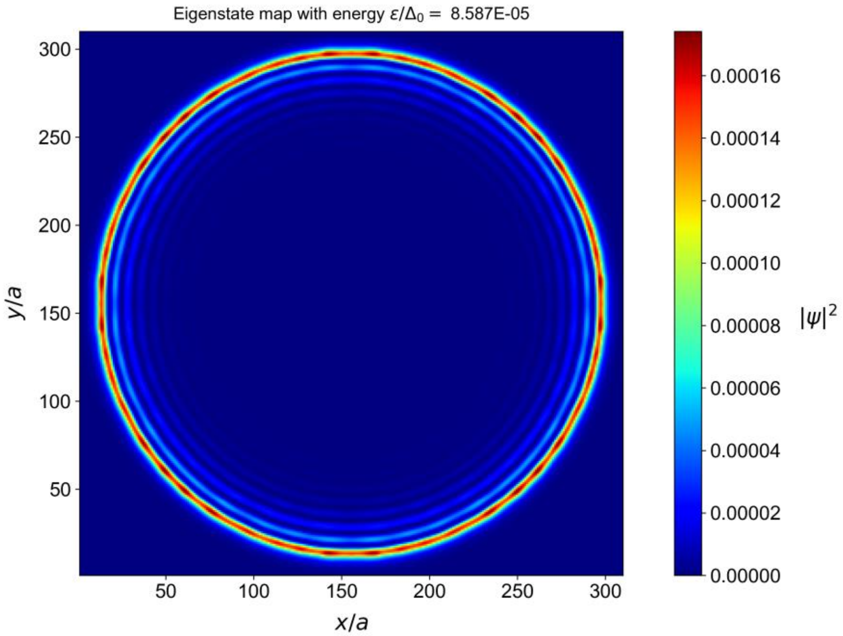
$$H^{\mathrm{2D\,TB}} = \sum_{\mathbf{r}=x,y} \left[\sum_{\sigma=\uparrow,\downarrow} -t\, c^{\dagger}_{\mathbf{r}+\hat{\mathbf{x}}\sigma} c_{\mathbf{r}\sigma} - t\, c^{\dagger}_{\mathbf{r}+\hat{\mathbf{y}}\sigma} c_{\mathbf{r}\sigma} + (4t-\mu)\, c^{\dagger}_{\mathbf{r}\sigma} c_{\mathbf{r}\sigma} + \Delta_0\, c^{\dagger}_{\mathbf{r}\uparrow} c^{\dagger}_{\mathbf{r}\downarrow} + \mathrm{h.c.} \right.$$
$$\left. + J \sum_{\sigma,\sigma'} c^{\dagger}_{\mathbf{r}\sigma} \left(\mathbf{n}(\mathbf{r}) \cdot \boldsymbol{\sigma}\right)_{\sigma\sigma'} c_{\mathbf{r}\sigma'} \right]$$

Majorana zero mode



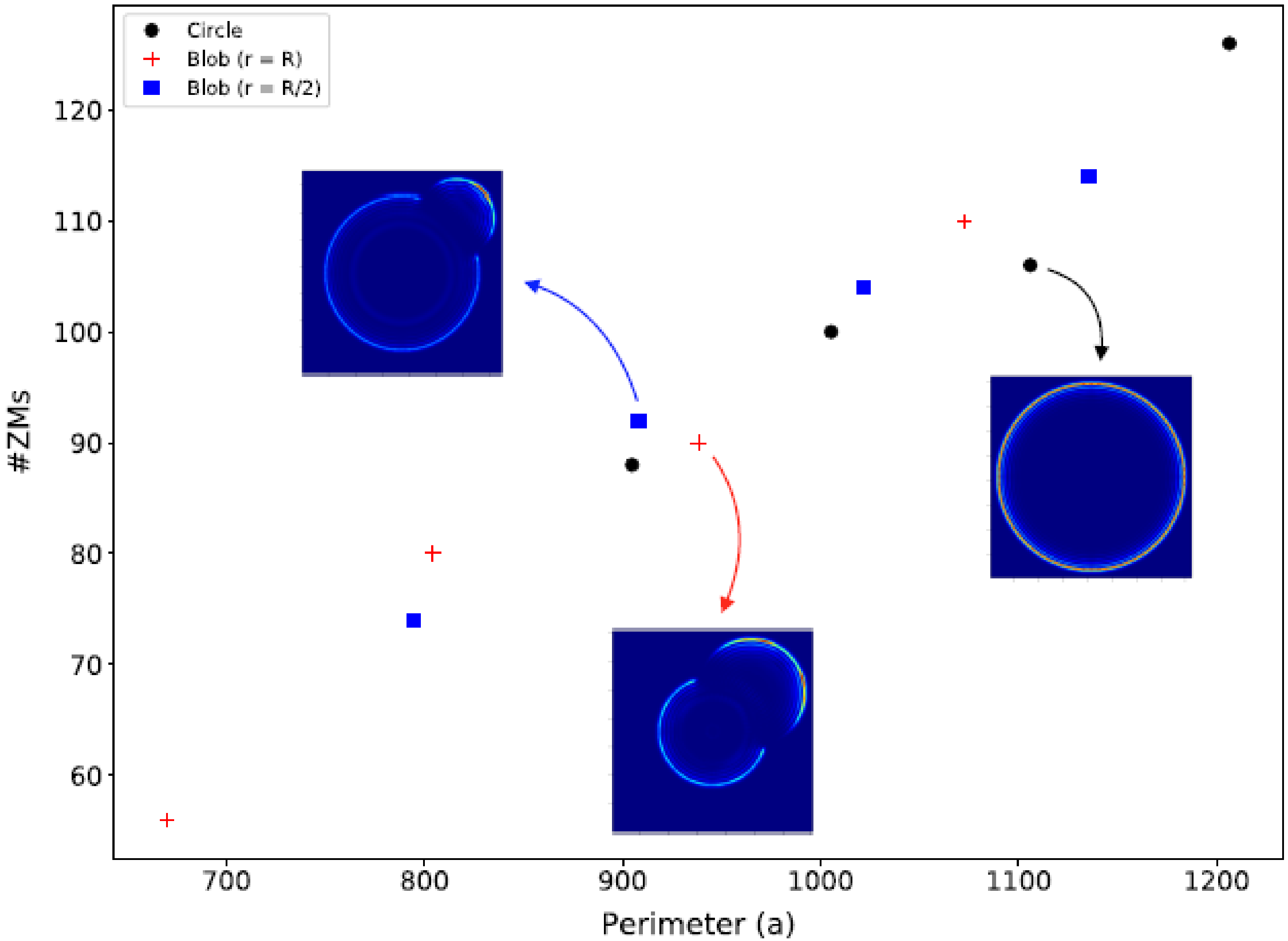
$N_x = N_y = 240,\ q=2,\ p=8,\ \lambda/a=14,$
$\mu/t=0,\ J/t=0.2,\ \Delta_0/t=0.1$

Flat band state



$N_x = N_y = 309,\ q=1,\ p=9,\ \lambda/a=16,$
$\mu/t=0,\ J/t=0.2,\ \Delta_0/t=0.1$

Controlling the nb of ``Majorana zero modes”



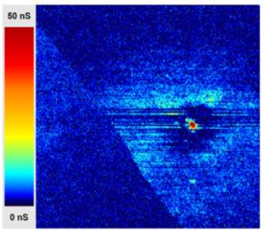
M. Garnier, A. Mesaros, PS, arXiv:1904.03005

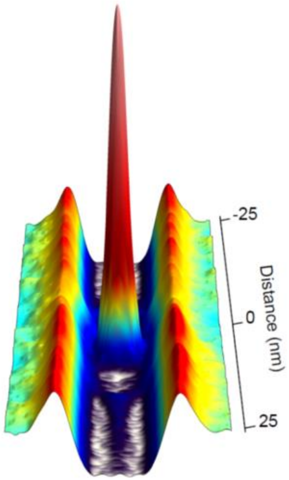
Conclusion

Majorana zero-energy bound states in defect core:

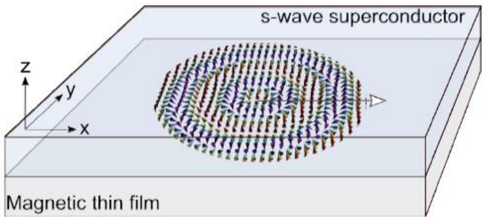
Interpreted with a defect in the SO phase
or non-trivial magnetic texture

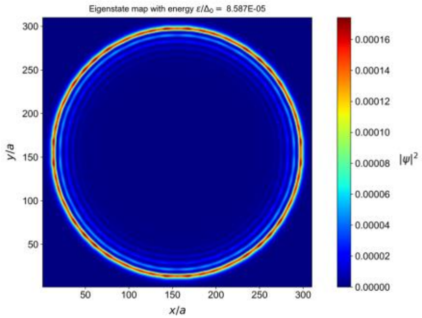
G. Ménard et al, Nature Comm (2019)





Almost Majorana flat band around skyrmions:



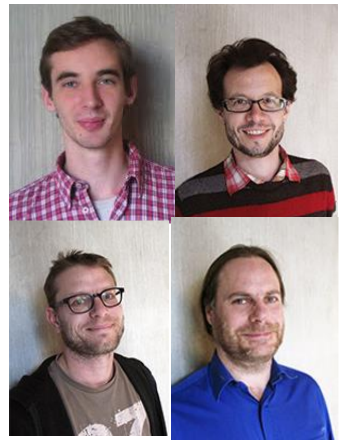


M. Garnier, A. Mesaros, PS, arXiv:1904.03005

Collaborators

Institut des NanoSciences de Paris
CNRS & Sorbonne University

- Gerbold Ménard
- Christophe Brun
- François Debontridder
- Tristan Cren



ESPCI

- Dimitri Roditchev

LPS, University Paris Sud

- Andrej Mesaros
- Maxime Garnier